1-D maps on a finite grid

Jason Barnett
Final project, PHYS 250
UC Davis, 06/04/09

Most of the following results are covered in:

"Simulating chaotic behavior with finite-state machines", Philippe M. Binder and Roderick V. Jensen, Phys. Rev. A 34, 4460 - 4463 (1986)

Why study discretized 1-D maps?

- Complex from simple:
  - Simple 1-D maps have complex behavior on the real interval
  - What if we simplify further to a finite grid?
    - Complex? Chaotic?
- Computer simulation:
  - Computers have finite precision
  - Simulations 'look' correct, but are we missing something?

Discretized maps

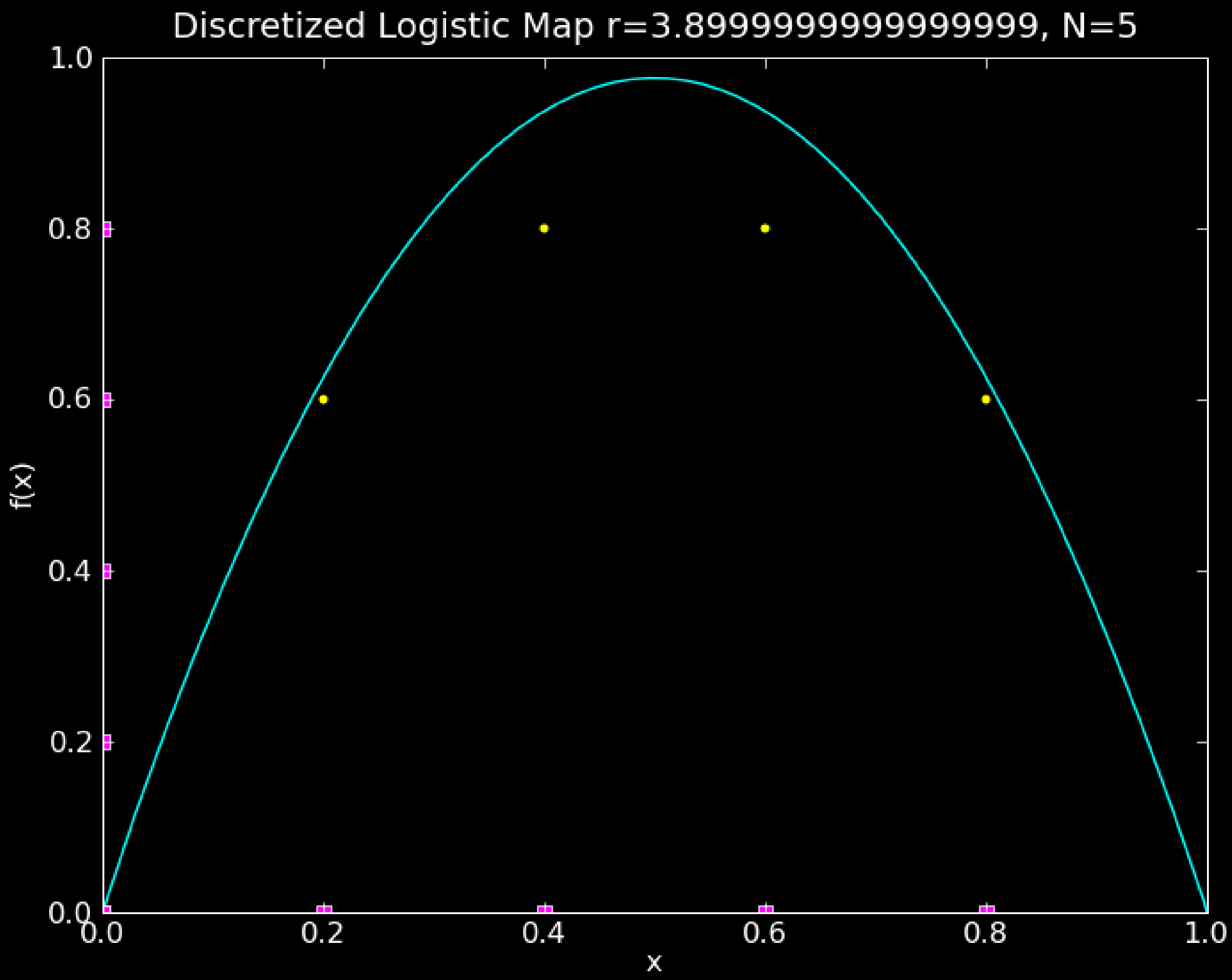
- Finite-state machine:
  - N states
  - Cycle length at most N
    - Turns out to scale more like $N^{1/2}$

Setup

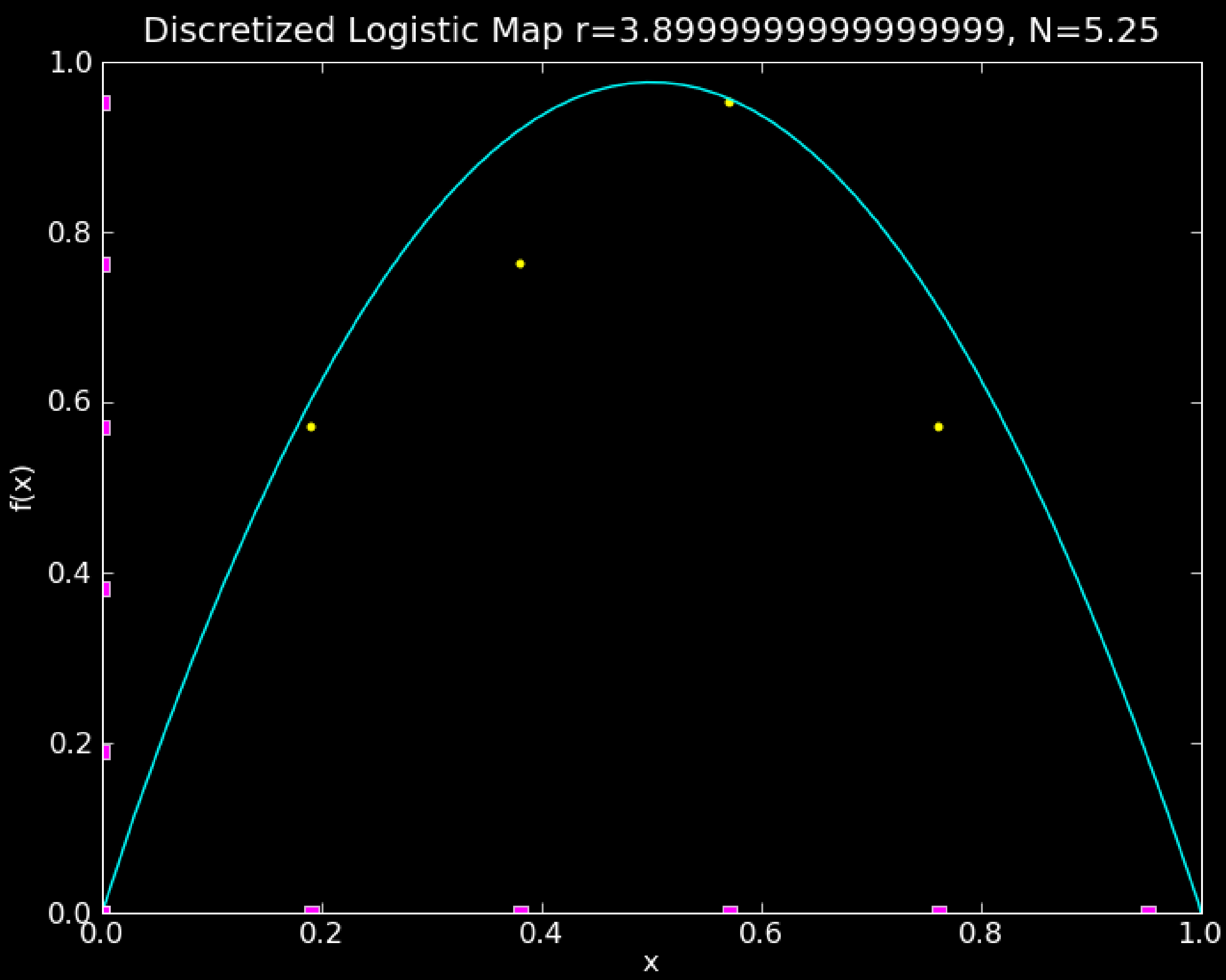
- Rounding down to a grid of 'N points':
  - N is actually a continuous grid scaling factor
  - Pseudo code for the discretized logistic map:

$$X = \text{floor}(Nrx(1-x))/N$$

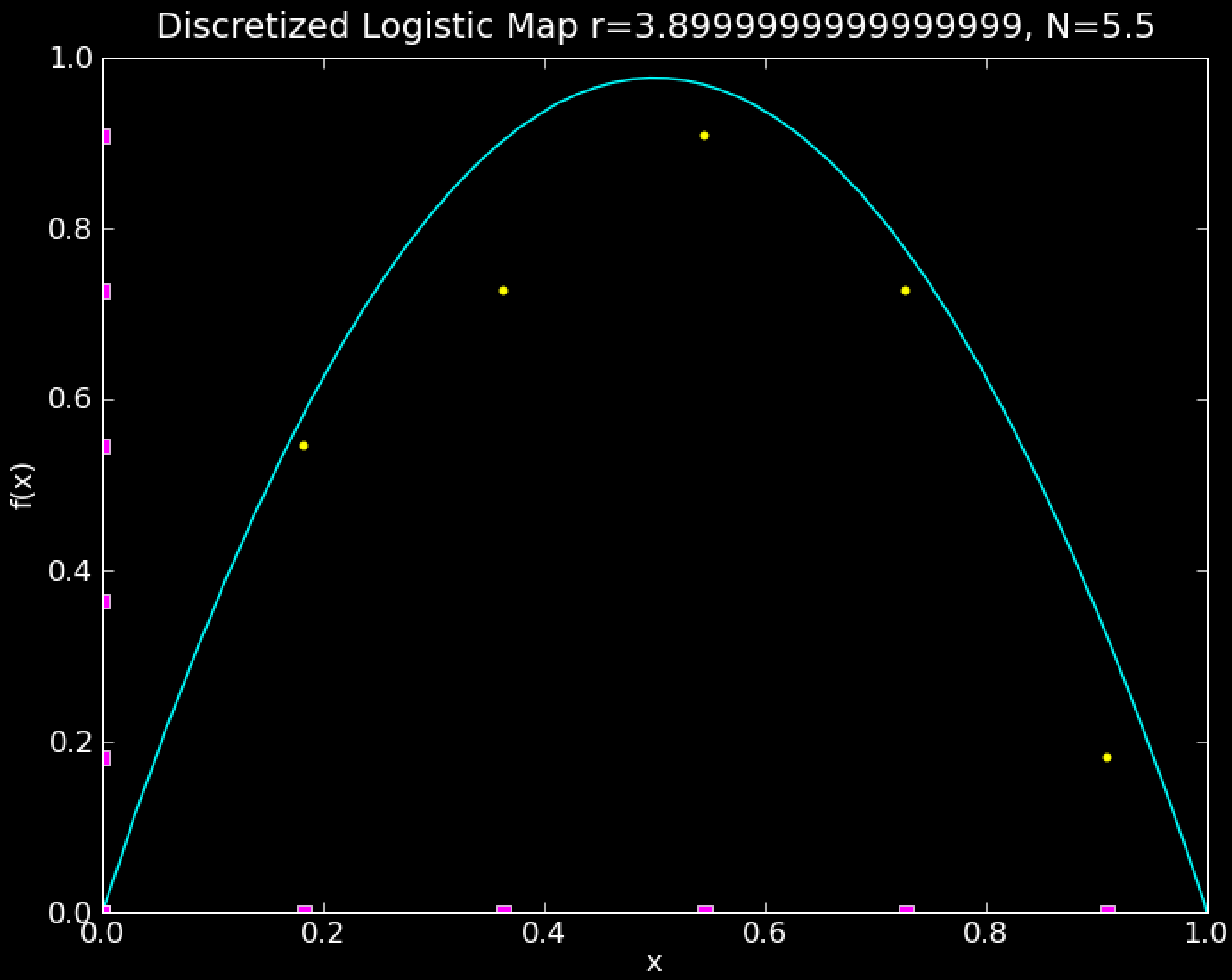
Scaling factor N



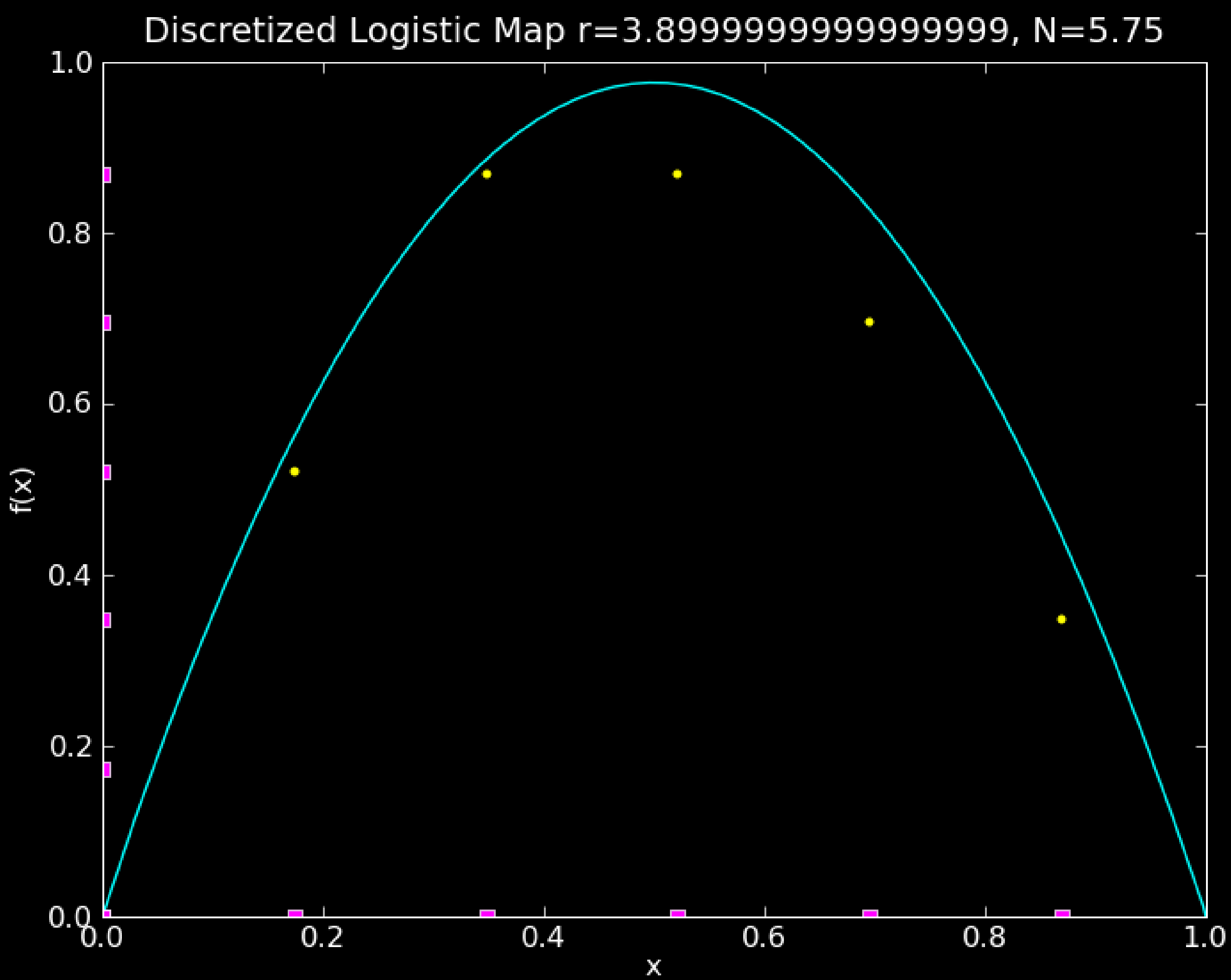
Scaling factor N



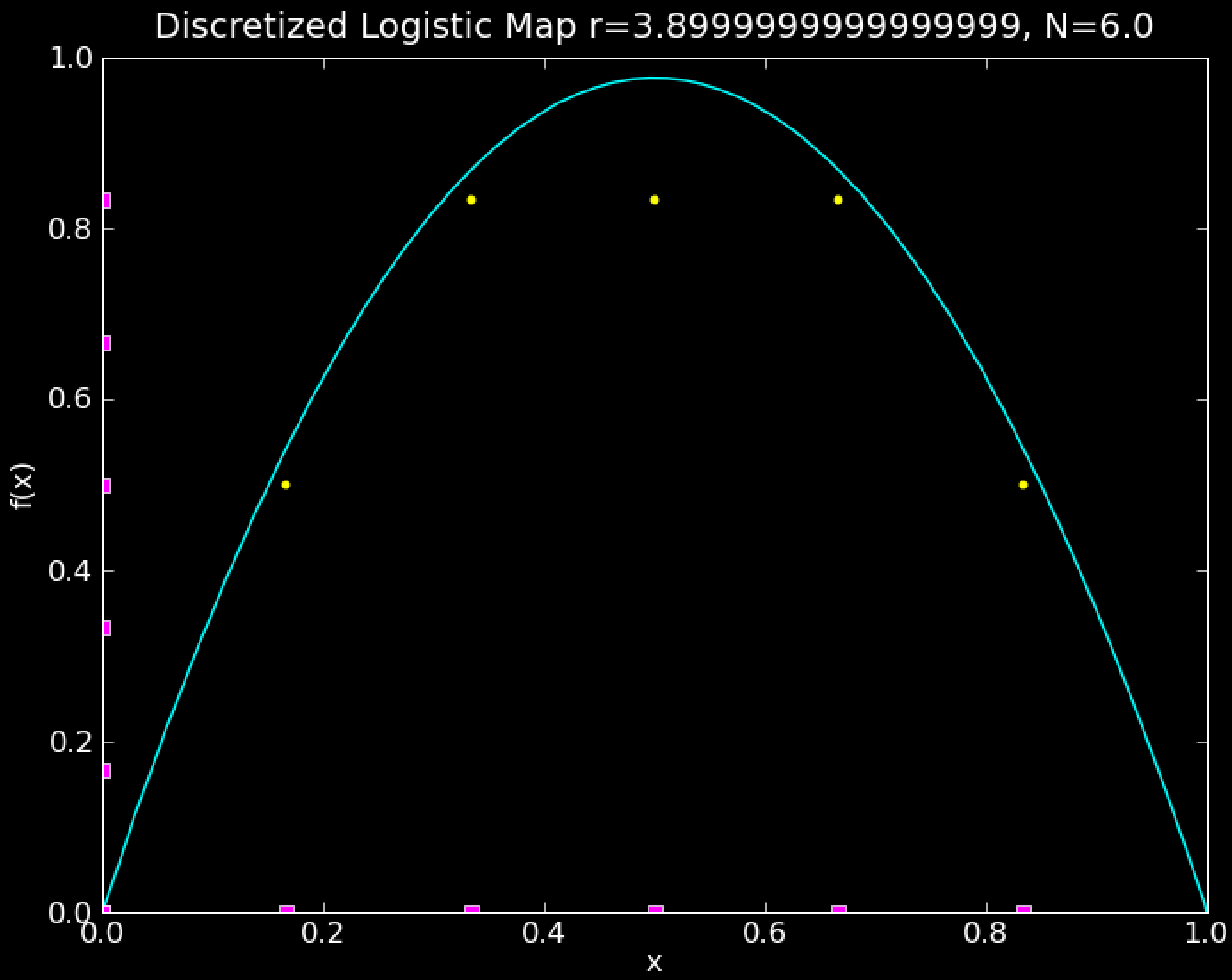
Scaling factor N



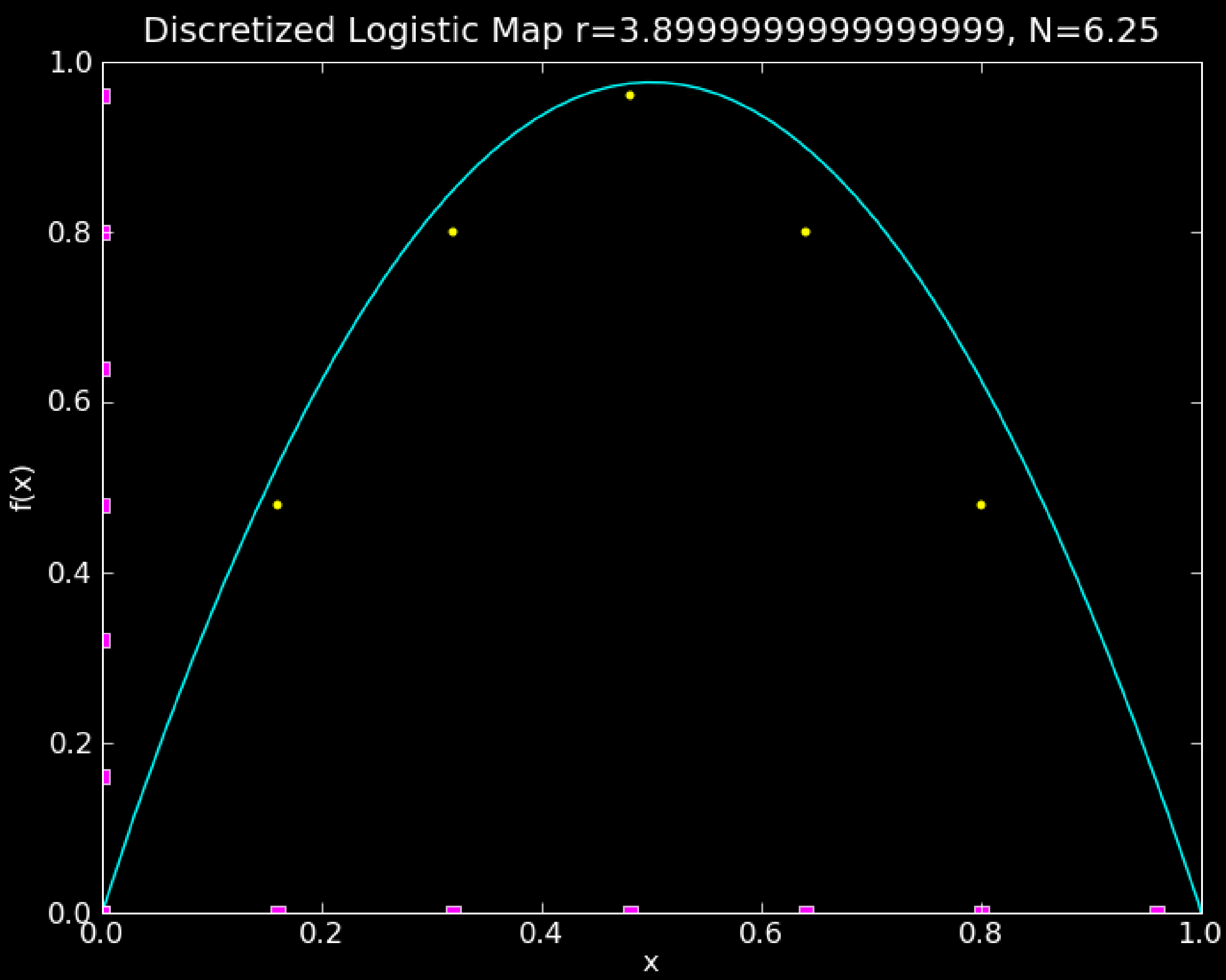
Scaling factor N



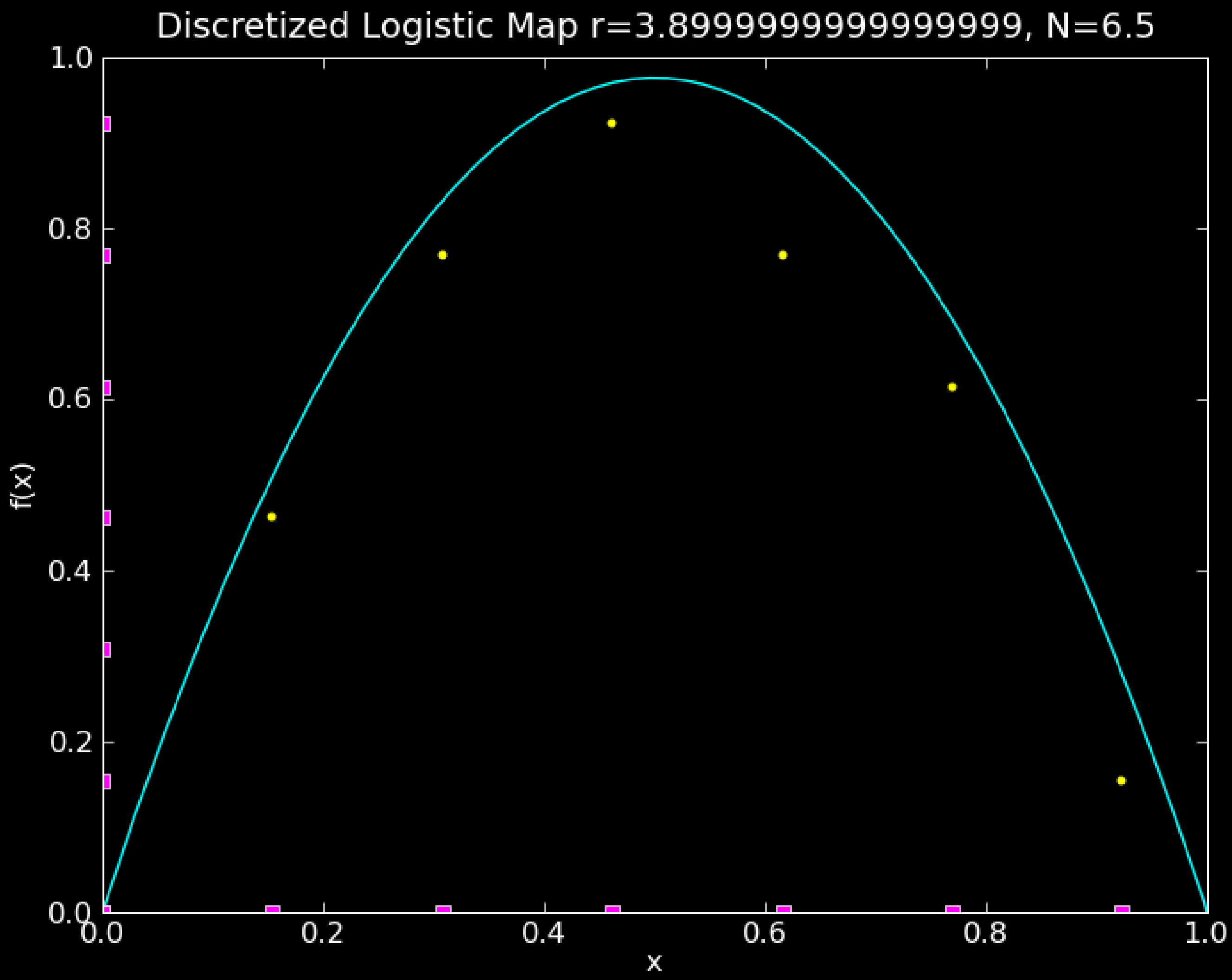
Scaling factor N



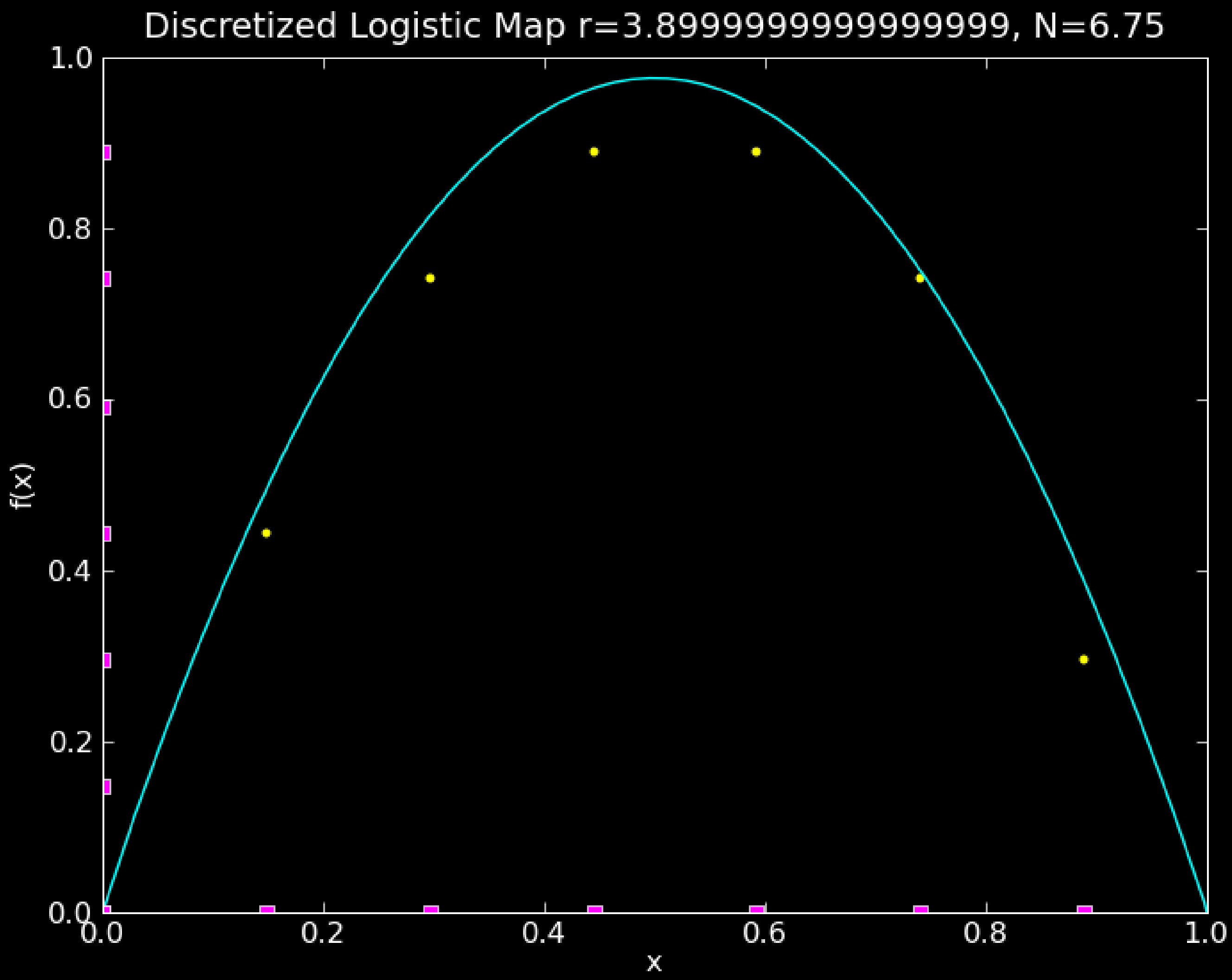
Scaling factor N



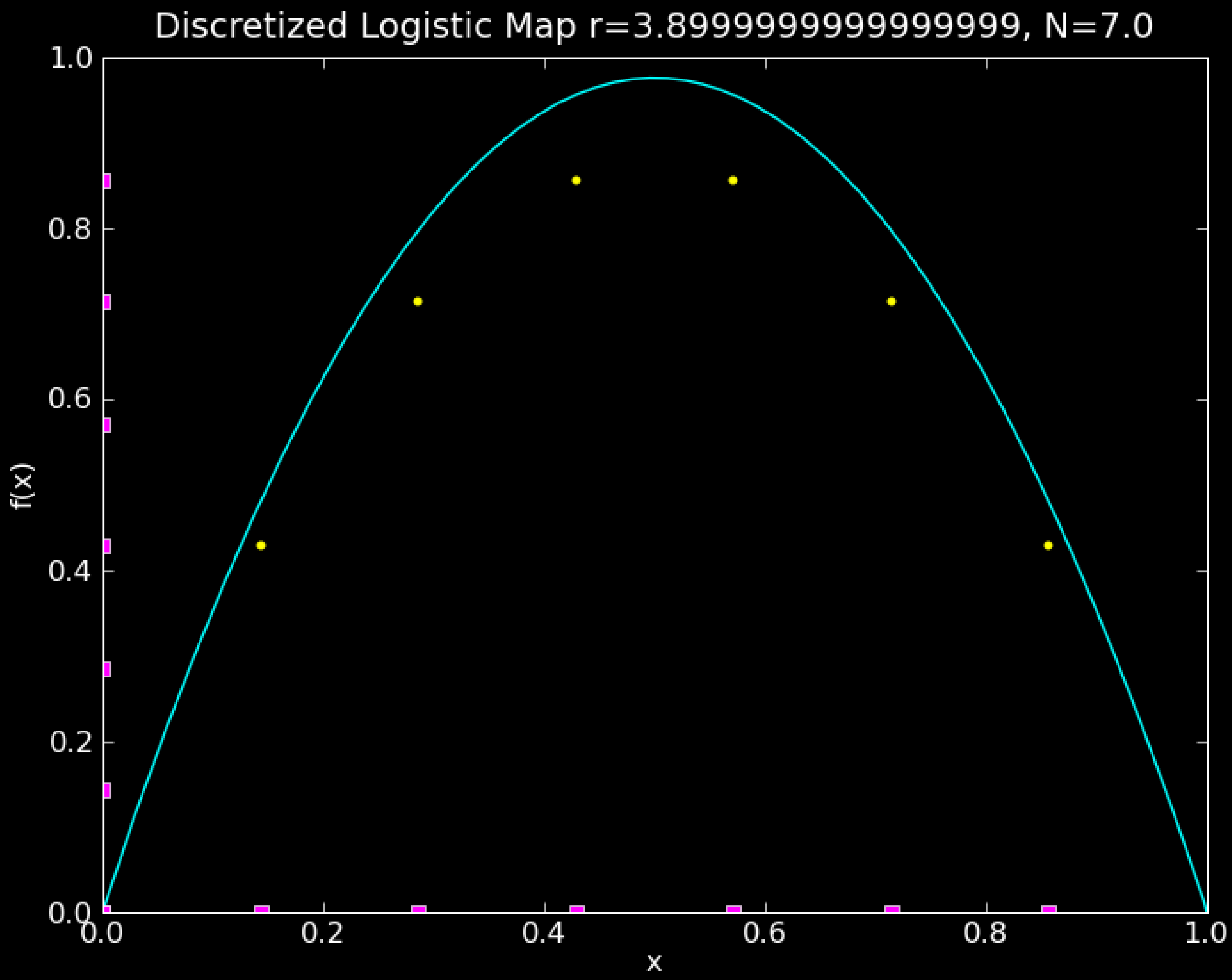
Scaling factor N



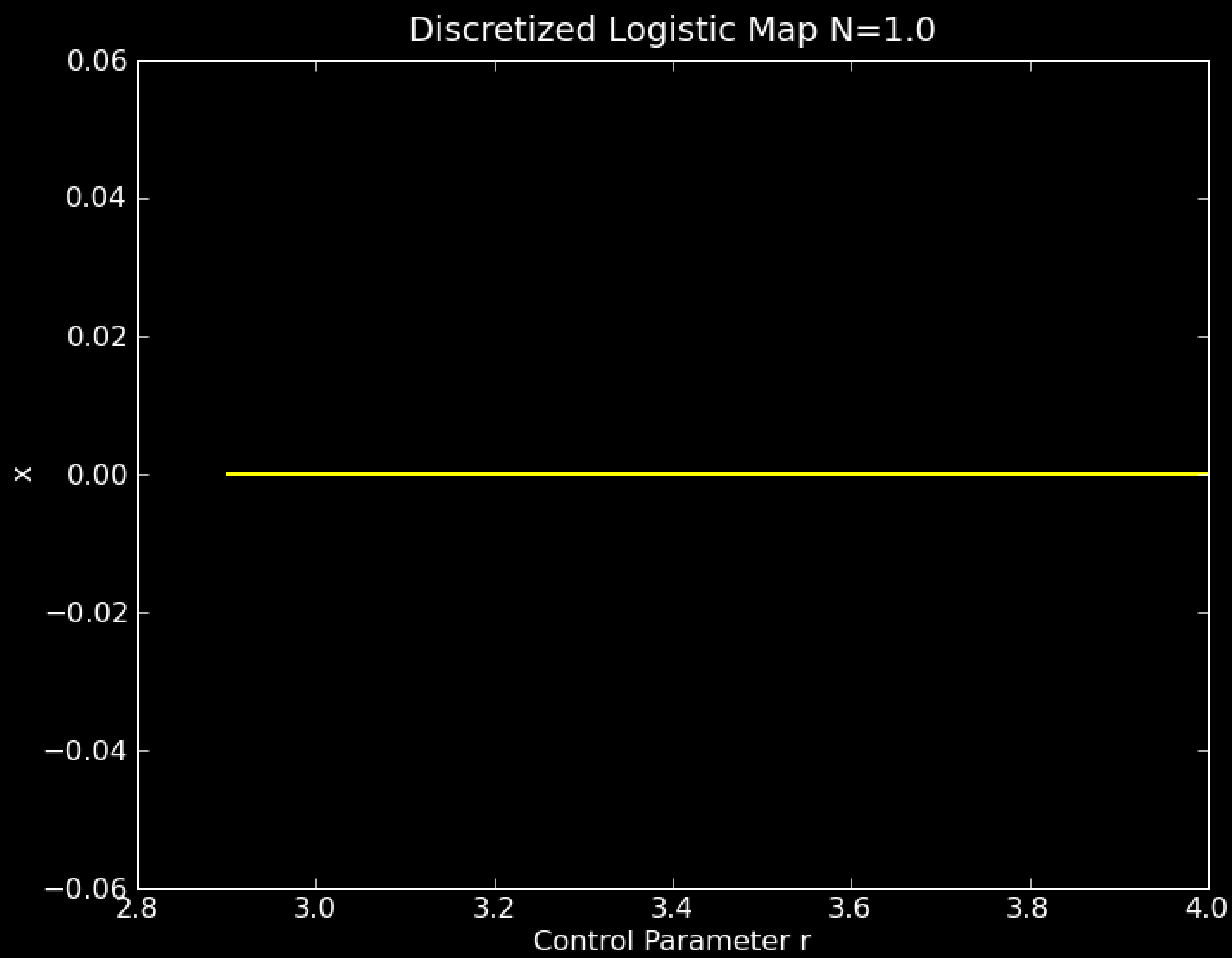
Scaling factor N



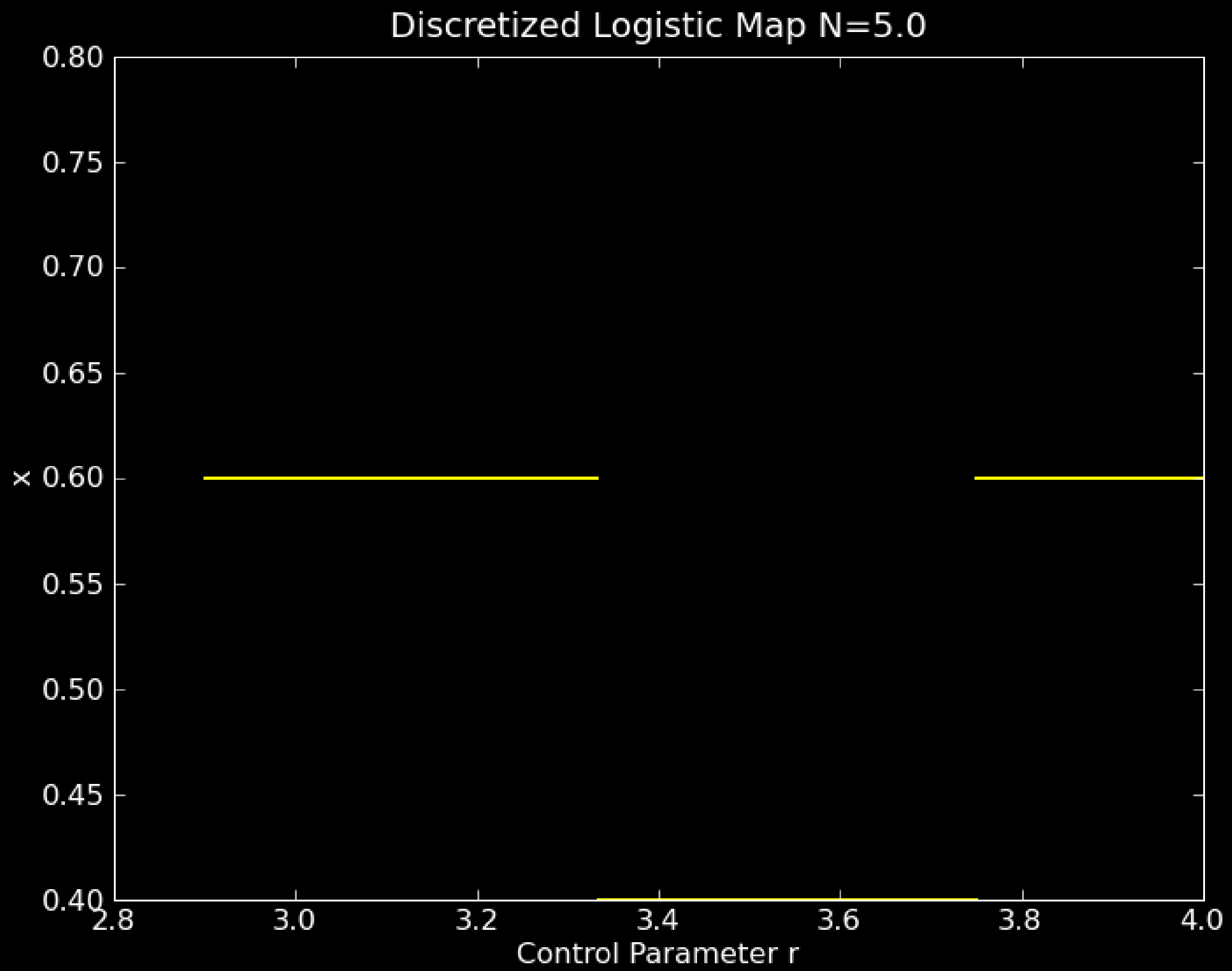
Scaling factor N



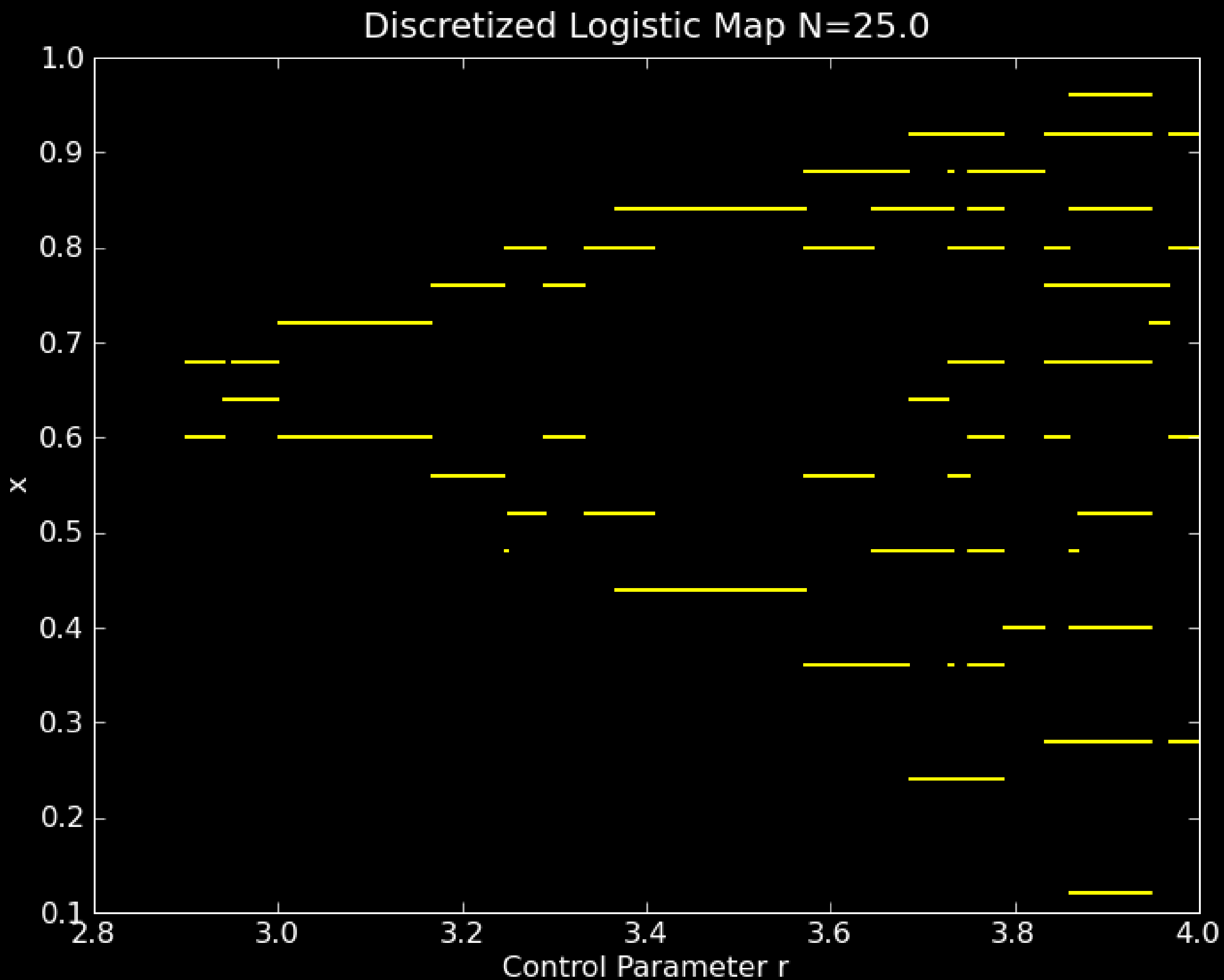
Bifurcation diagrams



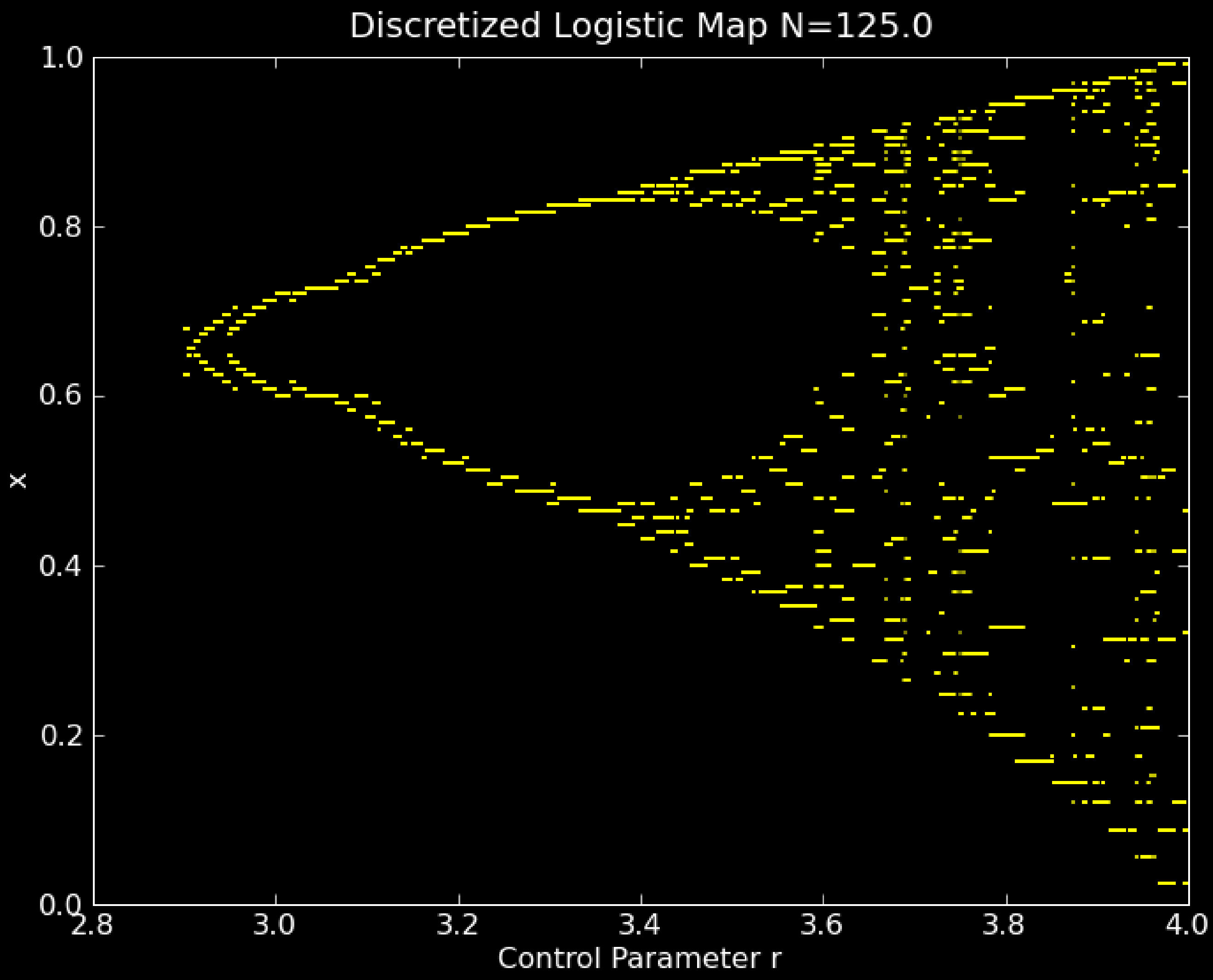
Bifurcation diagrams



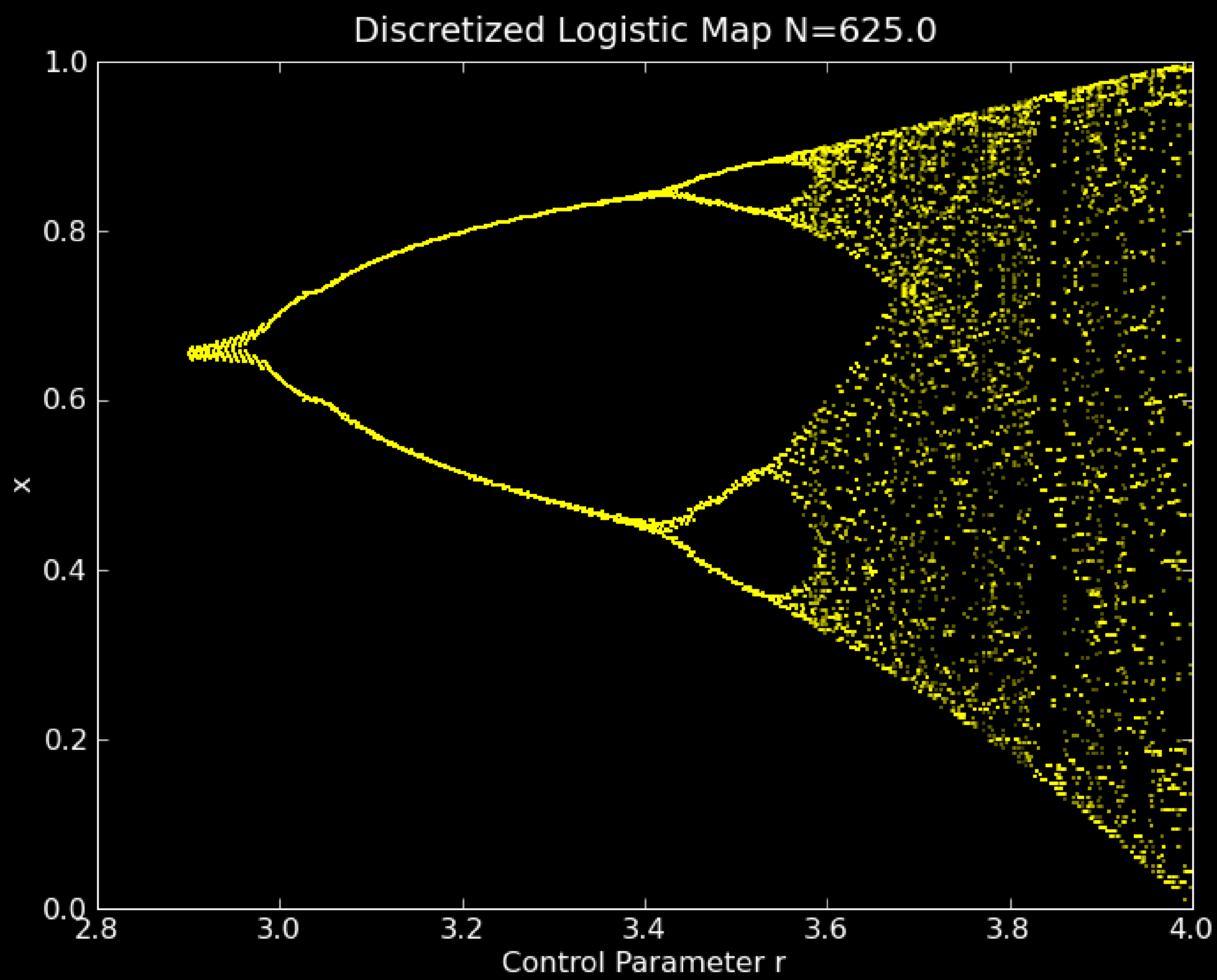
Bifurcation diagrams



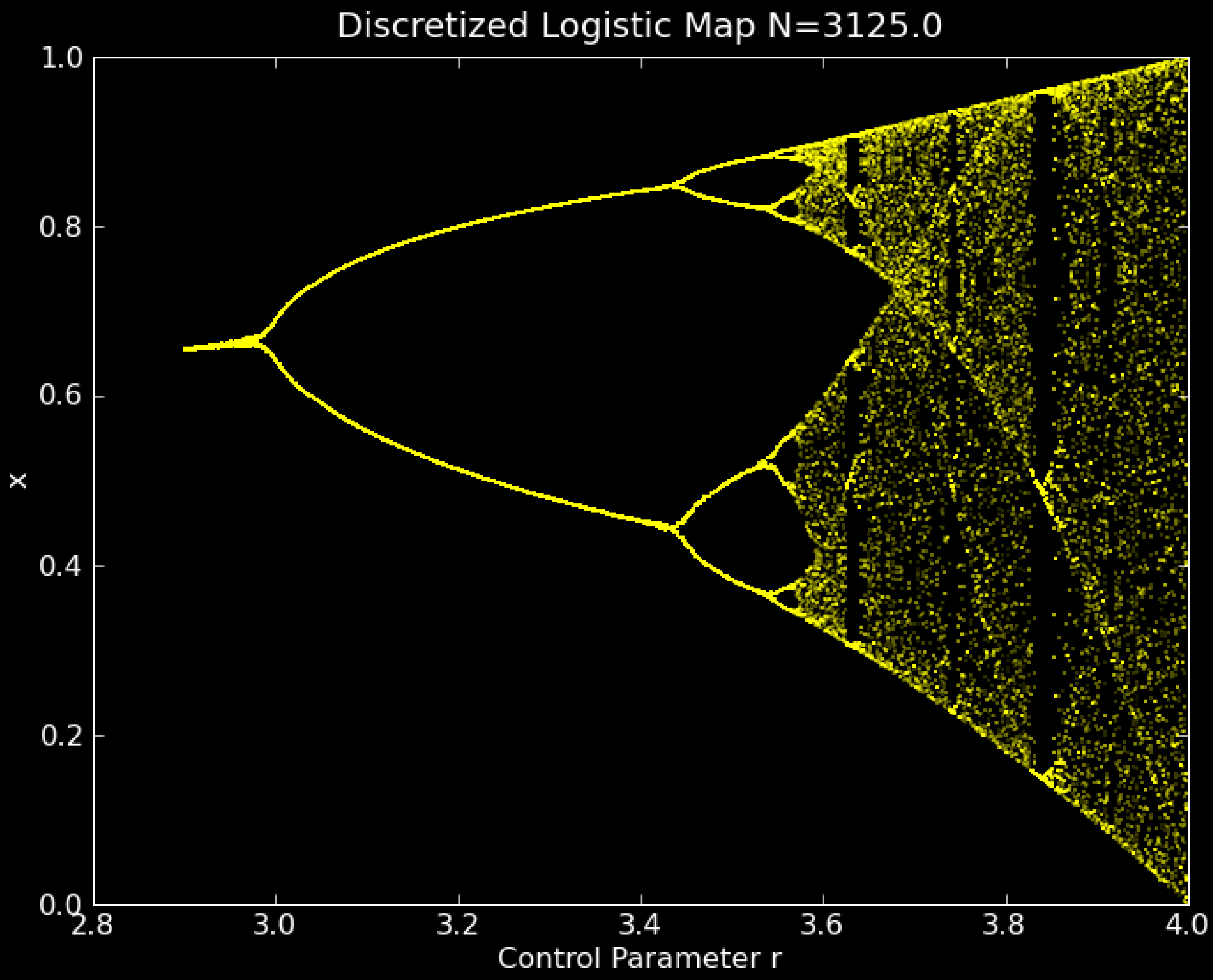
Bifurcation diagrams



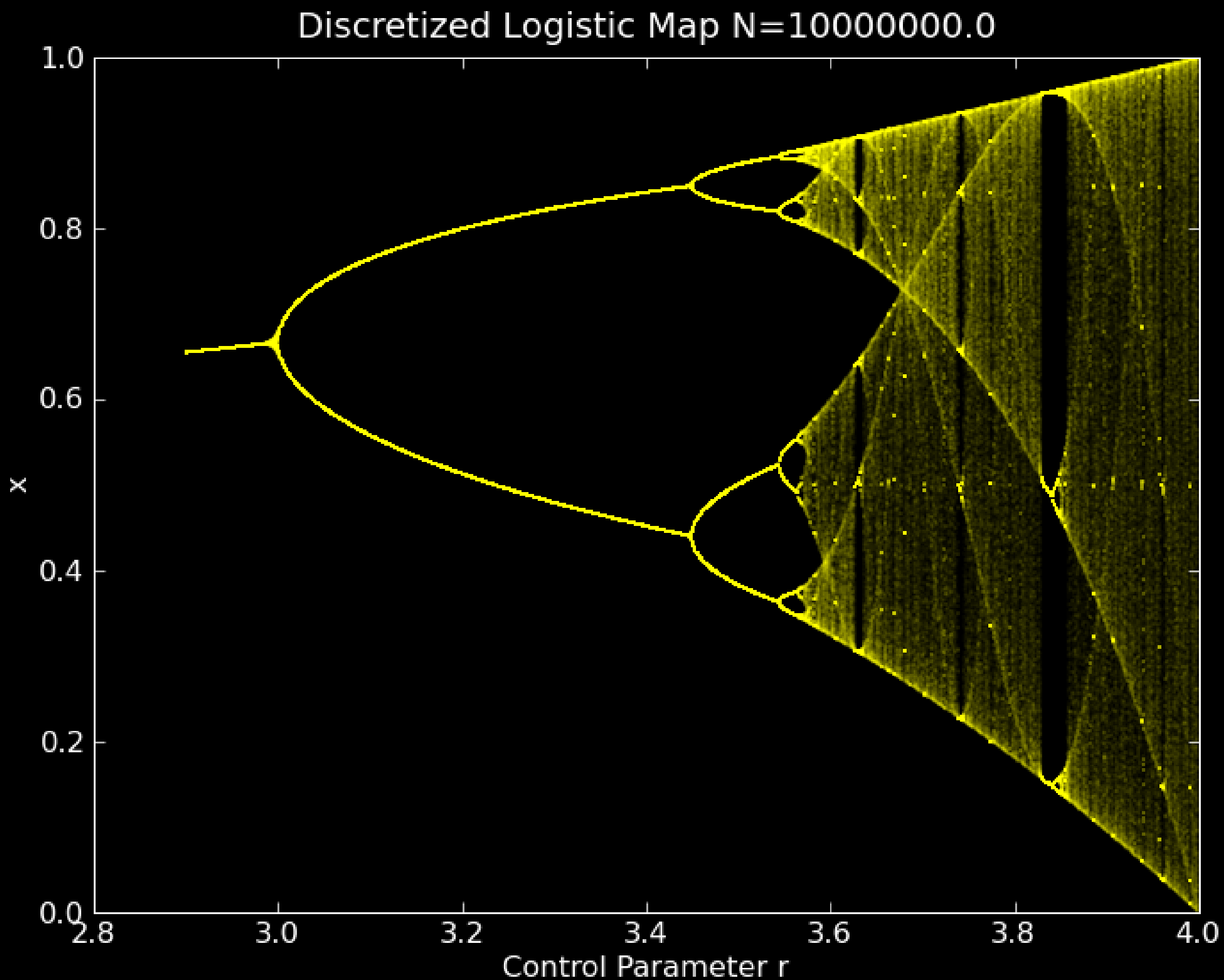
Bifurcation diagrams



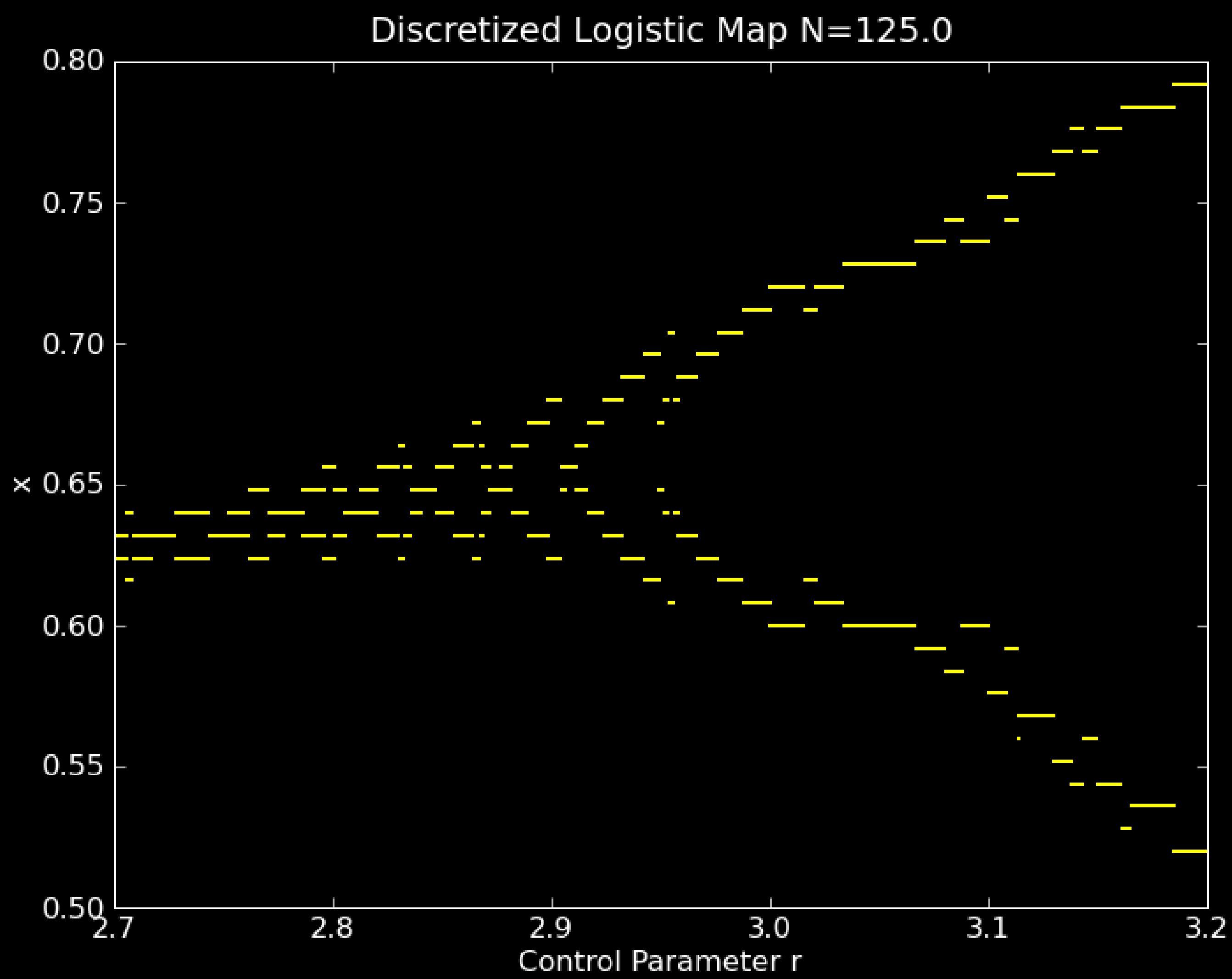
Bifurcation diagrams



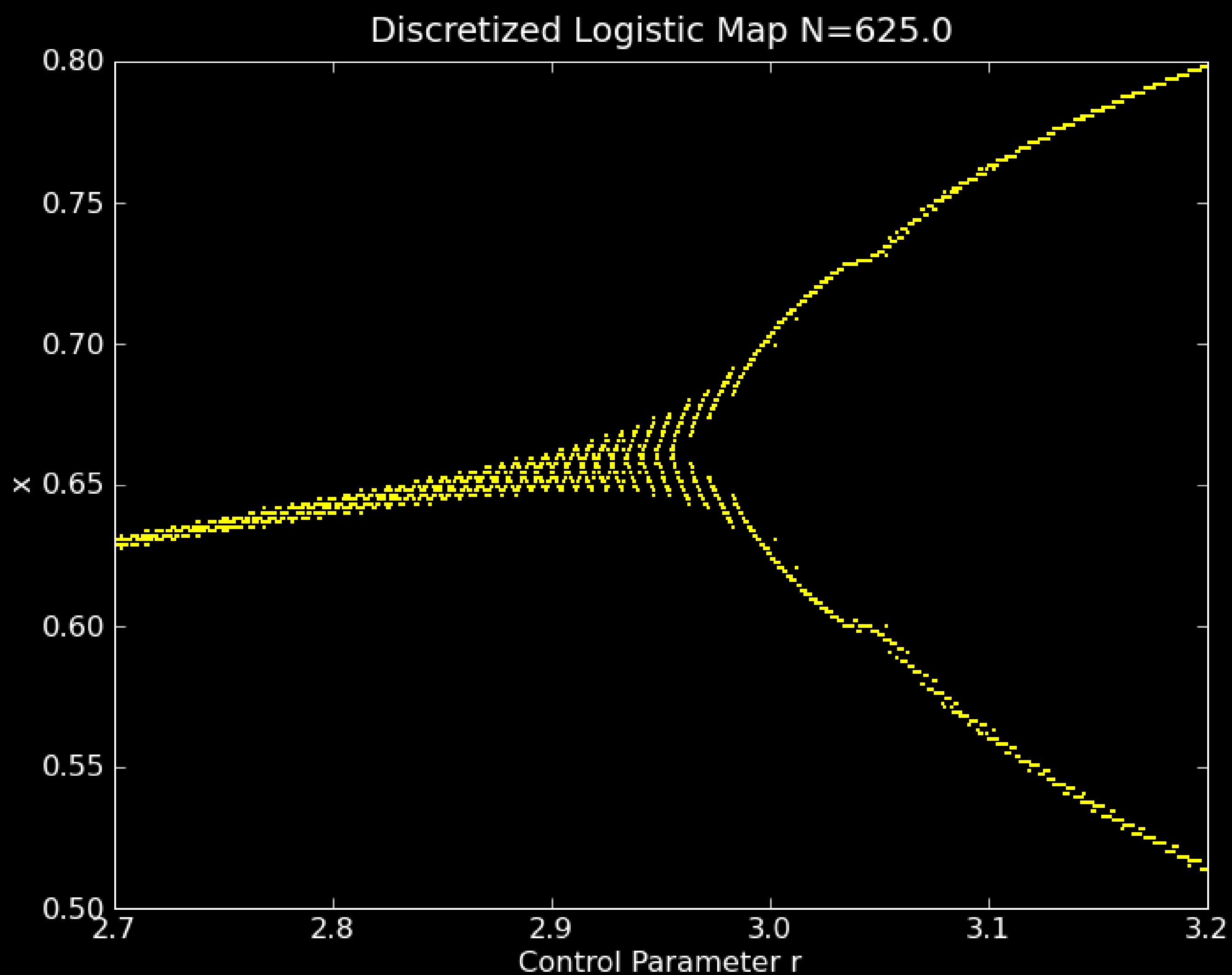
Bifurcation diagrams



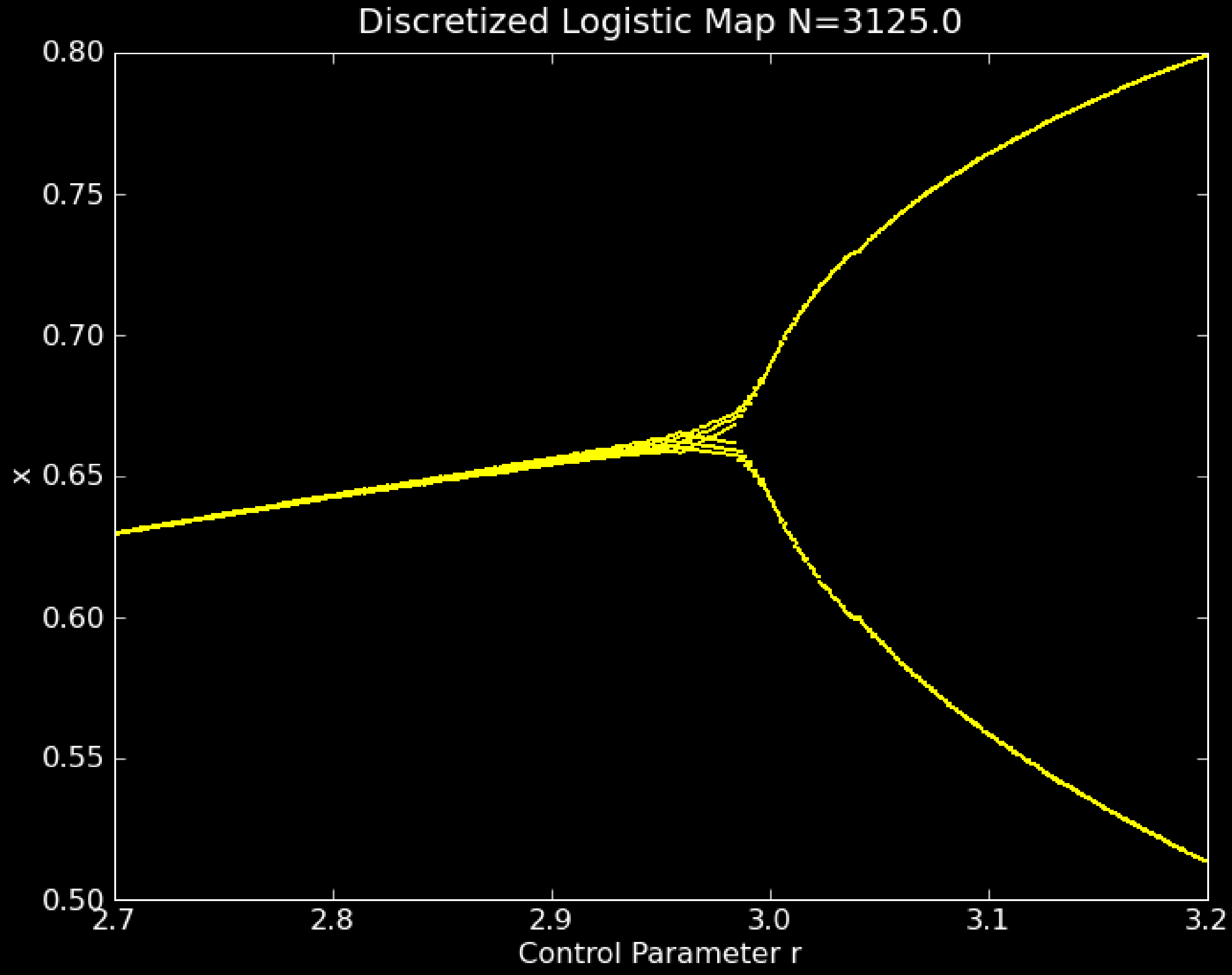
First period doubling



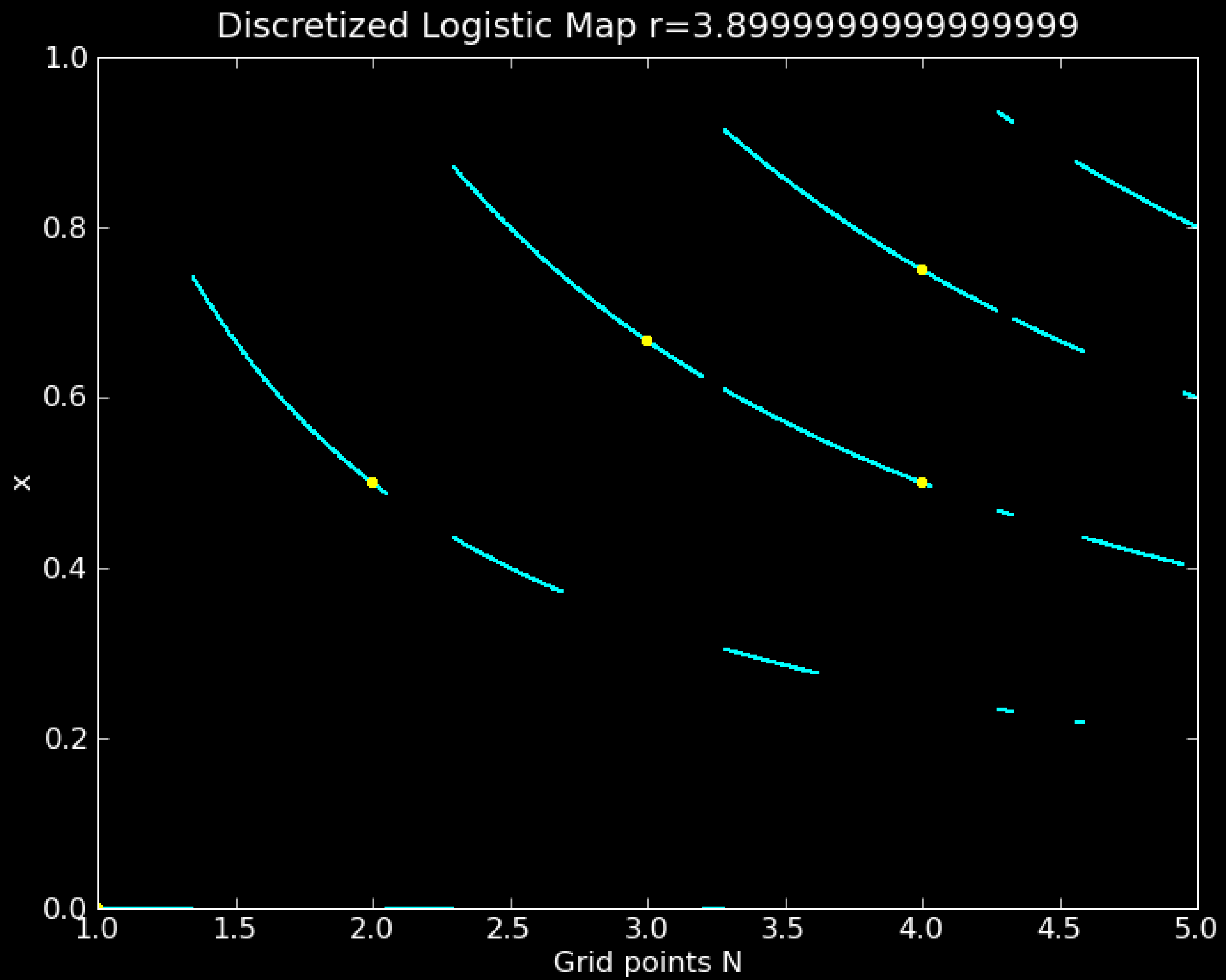
First period doubling



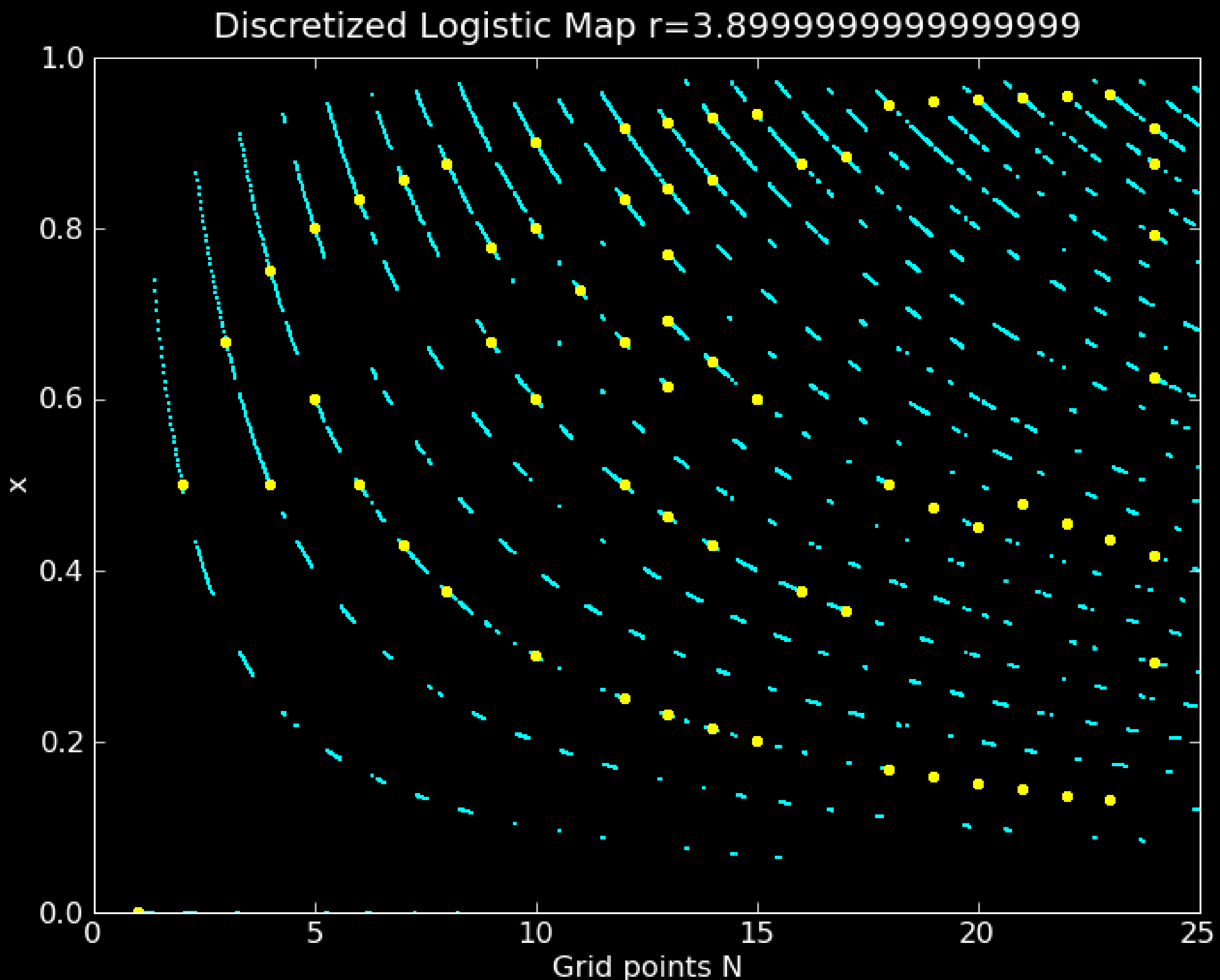
First period doubling



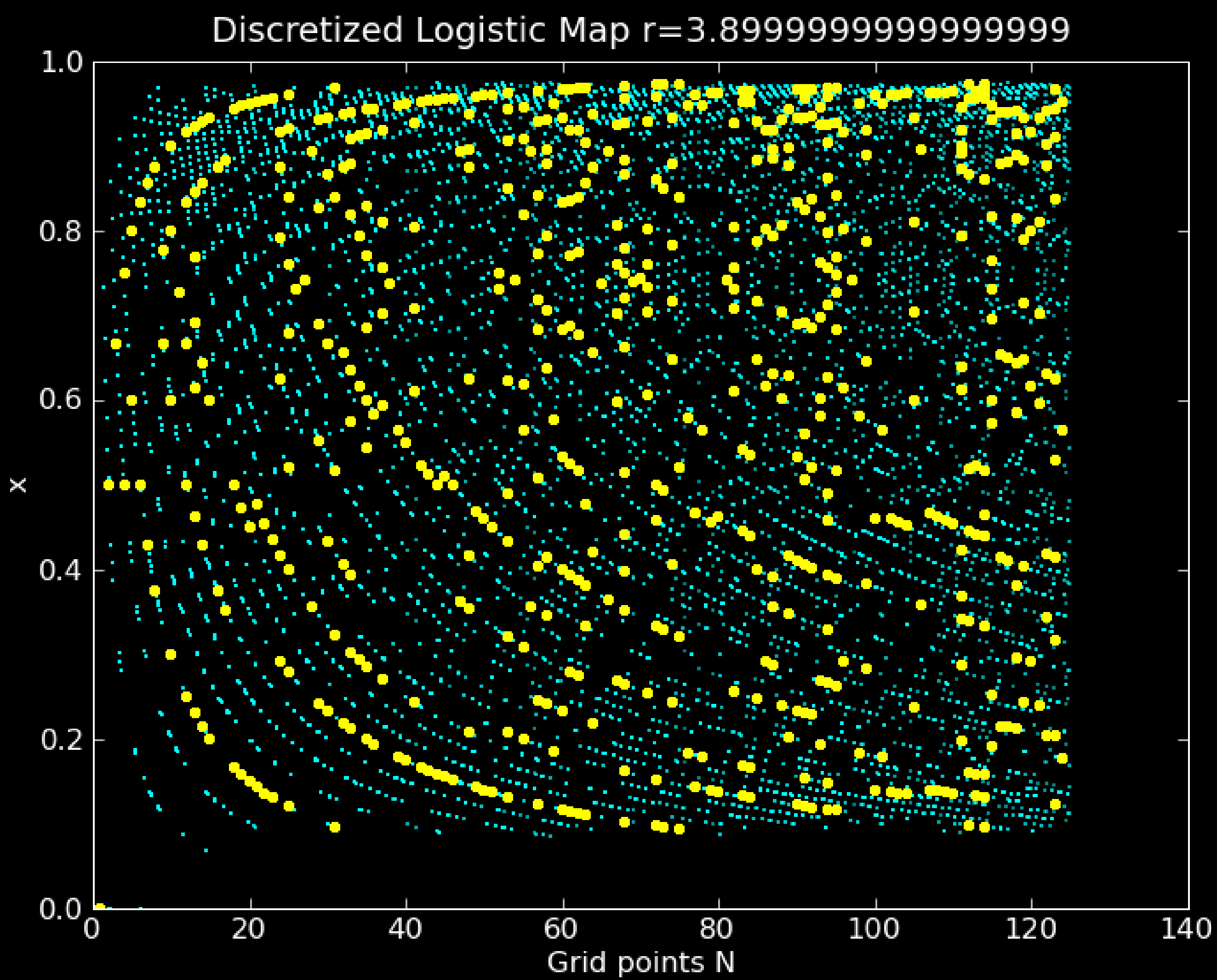
Bifurcation diagrams (N)



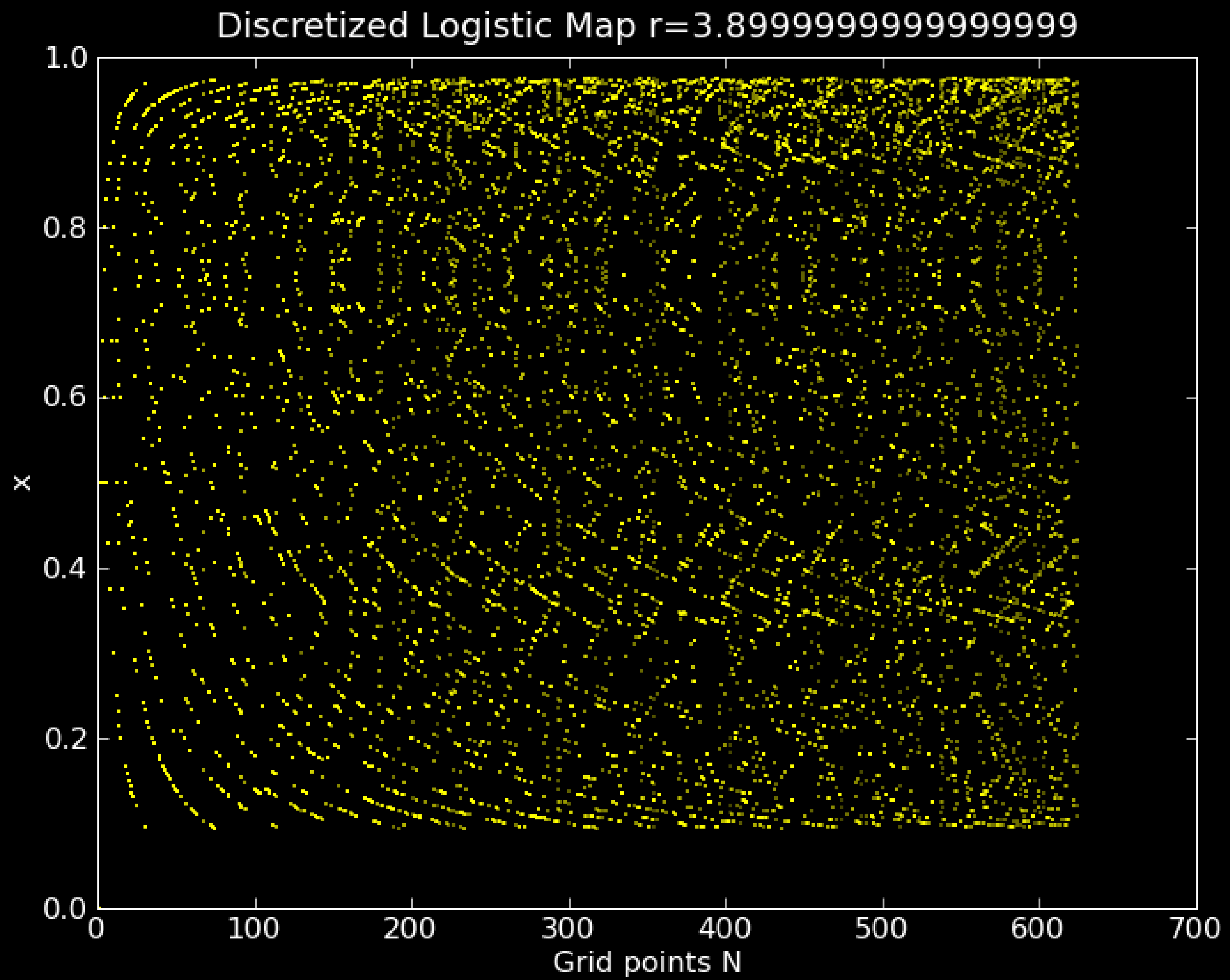
Bifurcation diagrams (N)



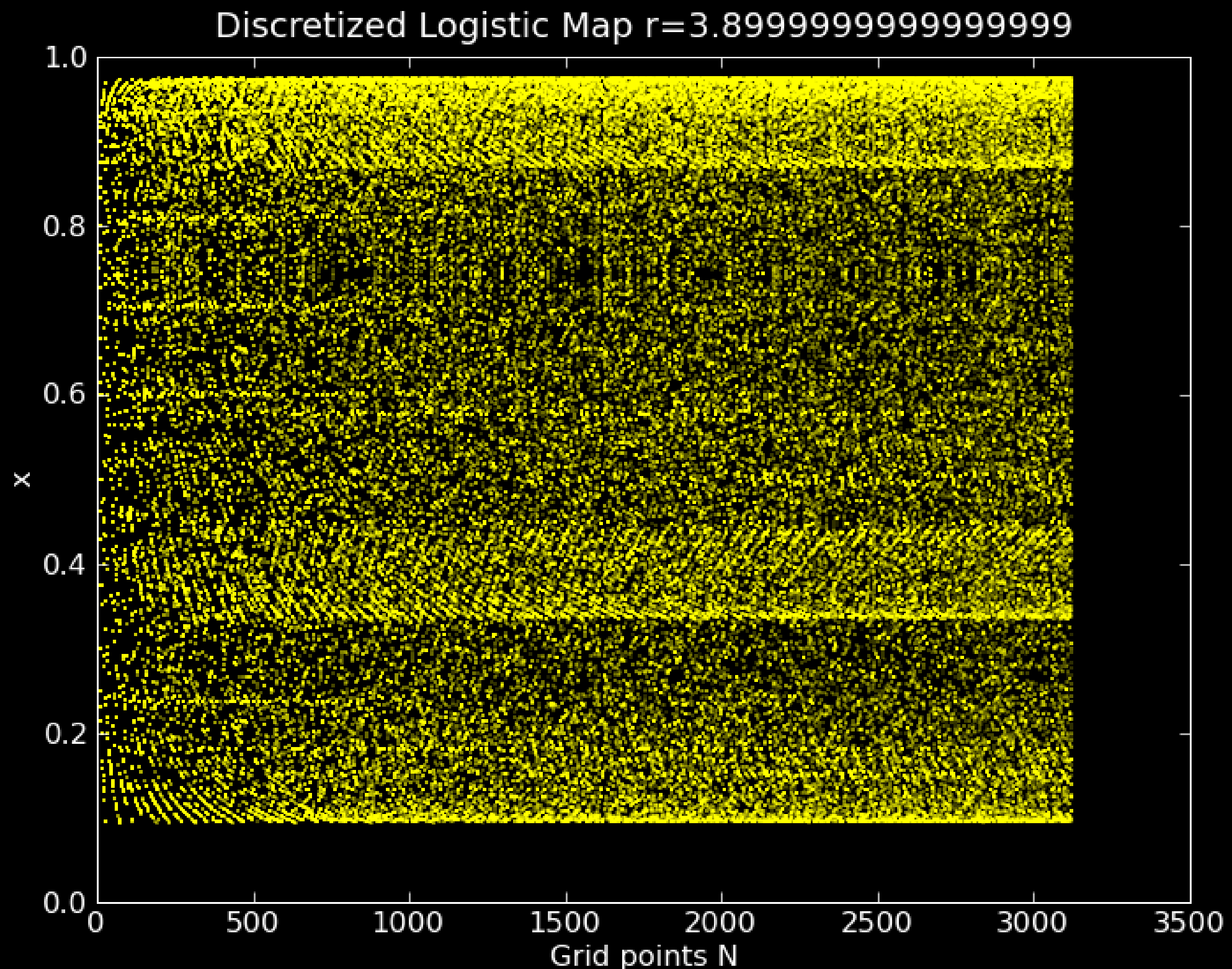
Bifurcation diagrams (N)



Bifurcation diagrams (N)



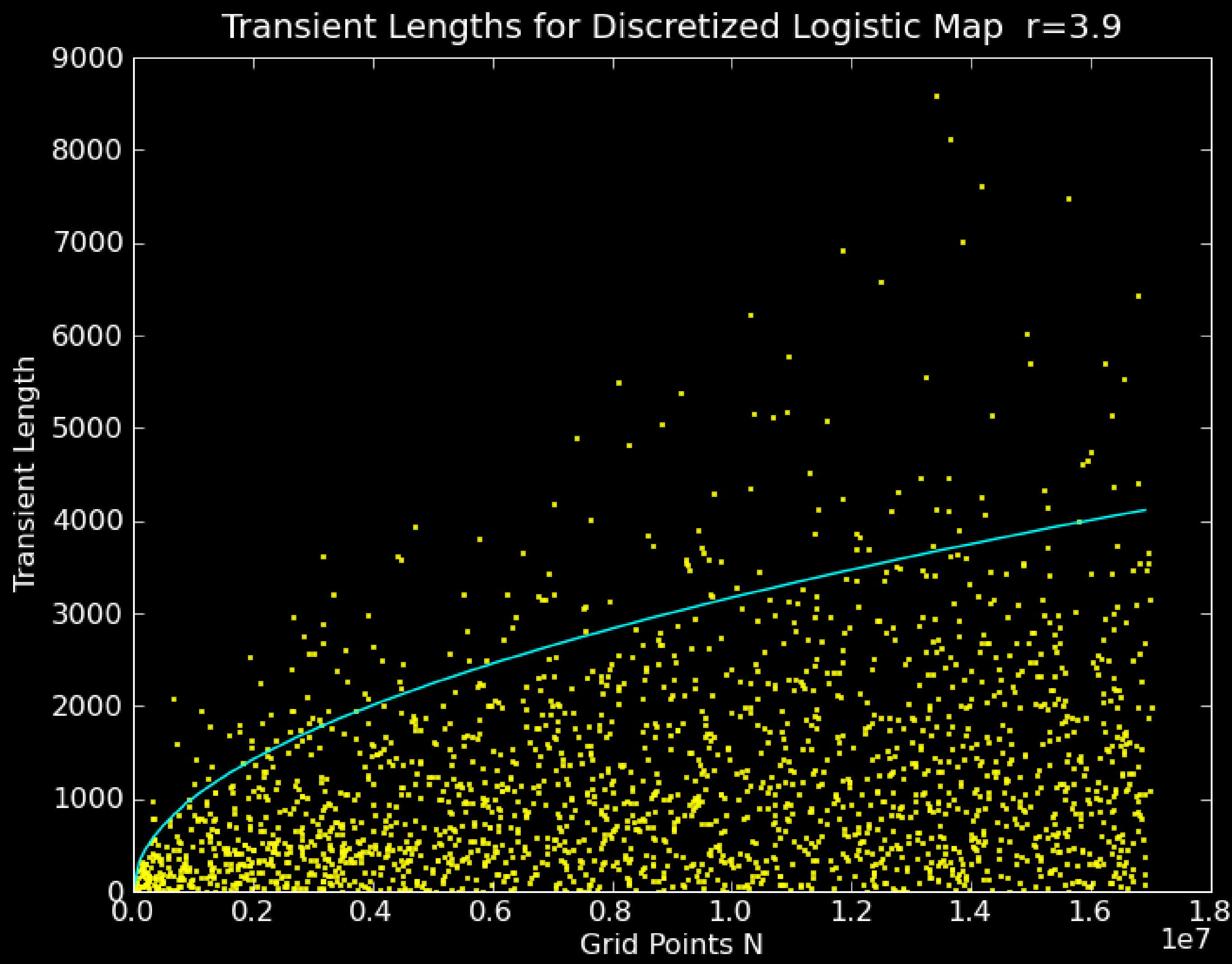
Bifurcation diagrams (N)



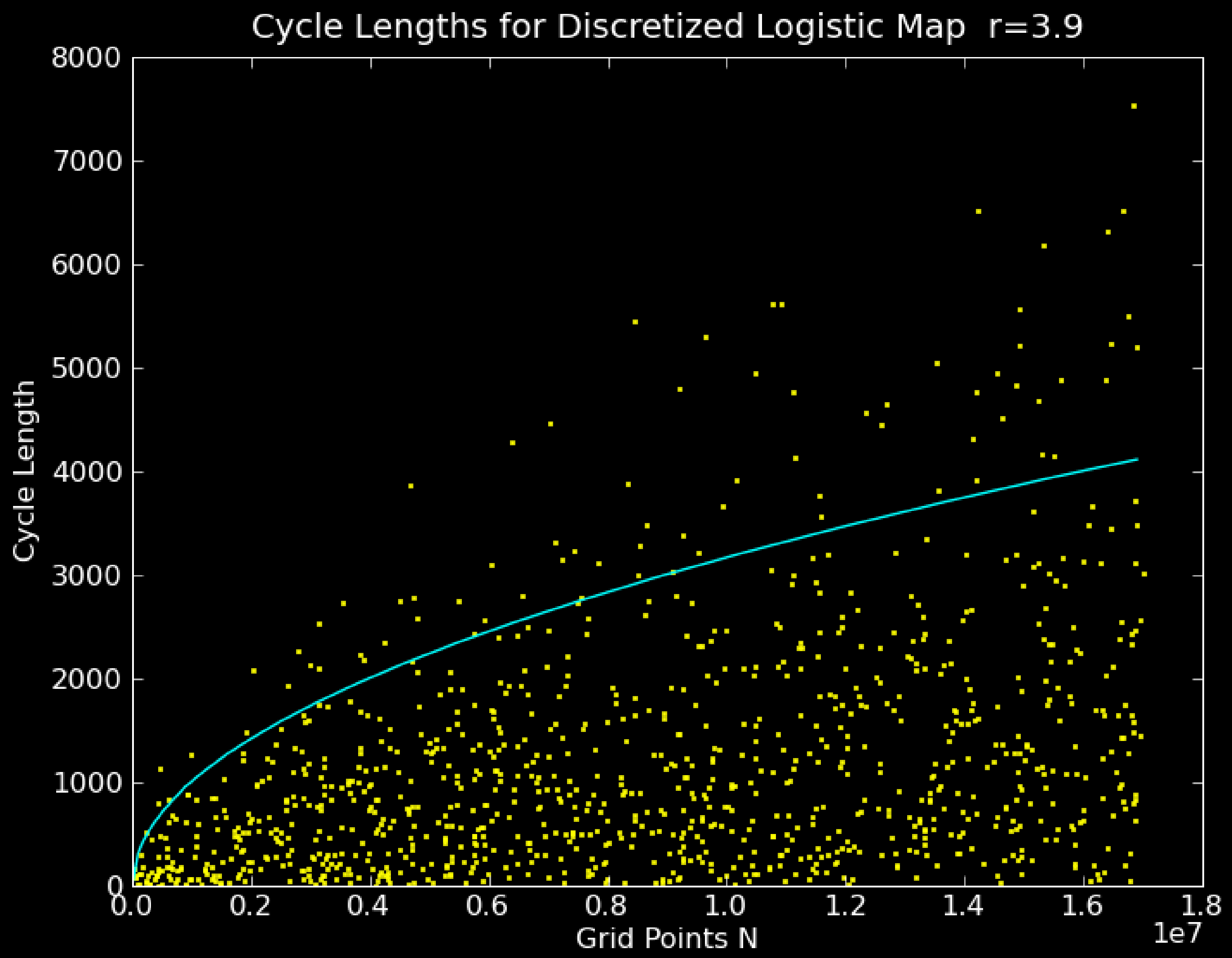
Transient/cycle lengths

- Both appear to scale like the power law N^k
  - k depends on r
  - $k \approx ½$ in chaotic regions
- We can interpret k in terms of entropy (see report for details):
  - Result:
    - Entropy of map := H_{map}
    - Entropy of IID random variable := H_{IID}
    - Then $k \approx H_{map} / H_{IID}$

Transient/cycle lengths



Transient/cycle lengths



In my report:

- Calculating k (power law) for various r
- More on entropy
- Comparison of Lyapunov exponents of continuous and discrete Logistic map
- The tent and cusp maps
- ...

Thanks!