SIRZ Model for Epidemics

And Zombies

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Abstract

The most common model for disease outbreak is the Kermack-McKendrick SIR model, comprised of a set of three differential equations. It divides the static population into three categories; those susceptible to the disease, those infected by it, and those who have recovered from it. The differentials of the population variables are described by constants for the infection rate and the recovery rate and the variables are time dependent. Because the time frame is short, the sum of the differential equations is zero, indicating the population is constant. This also implies that the disease is nonlethal, and this model is often used to describe diseases such as the common cold. These equations can be easily modified, however, to include any number of variables and factors. This project will analyze variations of the Kermack-McKendrick model to see the effects of additional factors such as a death rate, a birth rate, and zombies. The models were implemented in python, using the pylab function matplotlib for graphical analysis. Through implementation of these modified SIR models, we can predict the severity of several forms of disease outbreak based on estimation of the parameters. Most of the analysis concluded that, under reasonable circumstances, a zombie apocalypse is unlikely.

Background

March, 2009: The H1N1 flu virus infects its first victim in Mexico. Widely brushed off as media hype due to low ratings in sweeps month, the population panics for only a few days before turning their attention to discriminatory remarks made by a beauty pageant contestant on live television. Within weeks, the virus spreads to the United States, infecting a nine year old girl from Imperial County, California.

April 24: The World Health Organization (WHO?), issues its first notice on the outbreak, confirming the virus originated from conditions in pig farms in Mexico. Pig farmers object to the name “Swine Flu,” as re-panicked Americans stop eating bacon.

April 27: First confirmed case in Europe. First death confirmed in Mexico.

May 1: Unconfirmed reports of the living dead running wild in Mexican and American hospitals. Madagascar closes its ports.

May 7: Over 3,500 confirmed cases of H1N1 infection worldwide. WHO requests the U.N. locate undergraduate physics student and self-proclaimed “Zombologist” Daniel Spielman to model the outbreak.

Dynamic Model

The Kermack-McKendrick model for infectious disease is composed of three Ordinary Differential Equations:

$$\frac{dS}{dt} = -\beta I S$$

$$\frac{dI}{dt} = \beta I S - \nu I$$

$$\frac{dR}{dt} = \nu I$$

where S in the susceptible population, I is the infected population, and R is the recovered population. The values β and g are constants that represent the infection and recovery rates, respectively. The system is strongly nonlinear, and has no generic analytic solution, however, many important features can be derived. Firstly, note that:

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

Implying the population is static. This will have to be modified to model the fatal variant of swine flu. Also, a static population does not factor in a birth rate. However, because the timeline of the outbreak is so short (less then six months), this may not be an important factor. To include the lethality of a disease in the model, a death rate α is added.

$$\frac{dS}{dt} = -\beta I S$$

$$\frac{dI}{dt} = \beta I S - I(\alpha + \nu)$$

$$\frac{dR}{dt} = \nu I$$

Now the sum of the derivatives negative: the number of infected multiplied by the death rate. This can be describe as the differential of the number of casualties with respect to time. After spending three weeks in rural Mexico interviewing locals about their tales of animated corpses stalking the night, devouring livestock and human victims, I was able to develop the revolutionary SIRZ model for accurate prediction for zombie outbreaks of this type:

$$\frac{dS}{dt} = -\beta IS - \Phi\left(\frac{S}{R+S}\right)Z$$

$$\frac{dI}{dt} = \beta IS - I(\alpha + \nu)$$

$$\frac{dR}{dt} = \nu I - \Phi\left(\frac{R}{R+S}\right)Z$$

$$\frac{dZ}{dt} = \alpha I - (\phi_1 Z)(1 - \phi_2)$$

This completes the model for what we in the field refer to as "Swine Zombies." They are no longer infectious after death, but exist only as a shell of their former humanity, consumed by a hunger for your friends and family. The model is expanded to four dimensions and adds two new constant parameters to represent the encounter frequency and danger posed by the zombie threat.

Methods

With a solid model in place, accurate estimation of the parameters is garnered through data from past epidemics and the initial data from the swine flu outbreak. The values for φ_1 and φ_2 have no historical context and must be estimated through analysis of firsthand accounts from survivors. Due to the rural setting, the value for φ_1 will be set very low, while debate still continues over the accurate estimation of φ_2.

Once the parameters are estimated, a computer simulation will run a time series analysis of the model. A four dimensional Runge-Kutta integration method will approximate the population dynamics for the two-way predator-prey model by day. For visual simplification, the susceptible, recovered and infected portions of the population will be summed into a total population variable, to be graphed against the number of zombies on the loose.

For statistical integrity, a second set of visual representations will be constructed through computer simulation of the final population parameters across the standard deviations of individual variables.

Results

Simulations of the SIR model accurately depicted past flu outbreaks, and were used to generate realistic expectations for the parameters values included. These models have been well documented, and always converge to a stable fixed point. After the addition of a death rate, the model became much more sensitive to initial conditions. For β within a reasonable range (about 0.5), the population usually recovered. As the death and infection rate increased, the death toll mounted. While the sum of α and γ was kept relatively constant, the ratio of the two was varied widely. As the ratio approached zero, the model converged to the standard SIR Model. The largest realistic ratio was about 2:1, which gave a simulation of the Bubonic Plague, killing off more than a third of the initial population. A fatality to recovery ratio of one is unrealistic for any disease. With β as large as 2.0, the model began to reach apocalyptic proportions, killing off all, or nearly all of the population. These models are very unrealistic and serve only to satisfy our inner mad scientist.

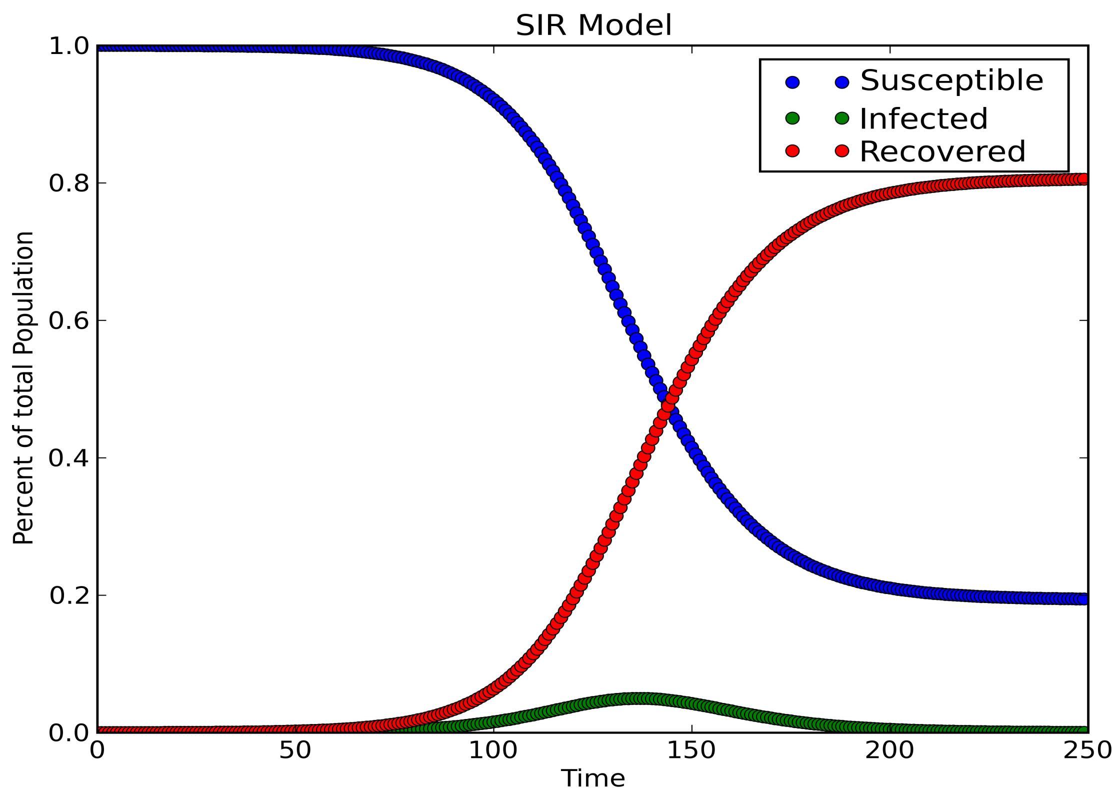
The introduction of the Z term consistently decreased the final population in the simulations. Under the most catastrophic reasonable conditions upwards of 55% of the initial population was killed in less then a year. For the conditions estimated for swine flu, moderate infectivity, but a low death rate, the death toll was much less devastating, less than 15% of the initial population.

In all of the models, the number of infected individuals showed a similar pattern. The number would quickly rise to a peak proportional to the infection rate, then drop to zero just as quickly. After the number of infected fell to zero, the survival of humanity was determined by the value of φ_2, as the humans and zombies fought desperately to kill each other off. φ_2 values of less than 0.5 showed the scales were in humanity's favor, while those greater than 0.5 favored the zombies. Just as influential, however, were the population levels after the infection died off. Even if φ_2= 0.9, if the number of zombies never reached a critical mass, humanity would win the day.

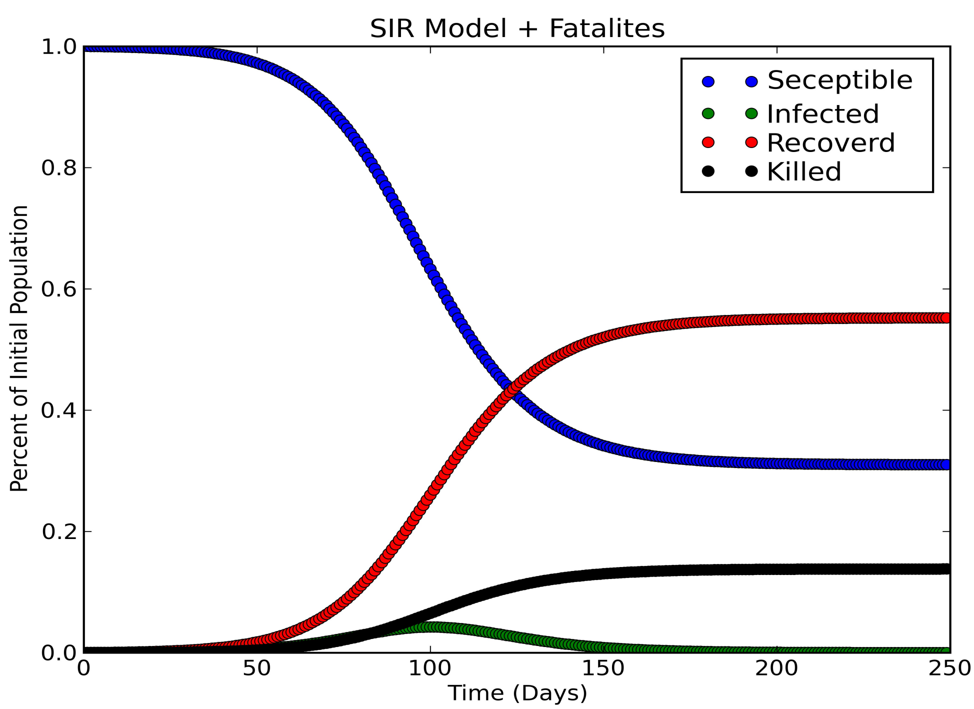
Conclusions

While a zombie pandemic is one of the most substantial threats to human existence, the form in which it comes is critical. As soon as an outbreak is detected, it must be quickly and accurately assessed in order to determine the danger it poses to humanity. There is a strict correlation between the deadliness and infectivity of the disease and the damage of the outbreak. In this regard, highly infectious diseases such as colds and flues are less likely to threaten the existence of humanity than deadly plagues on a biblical scale.

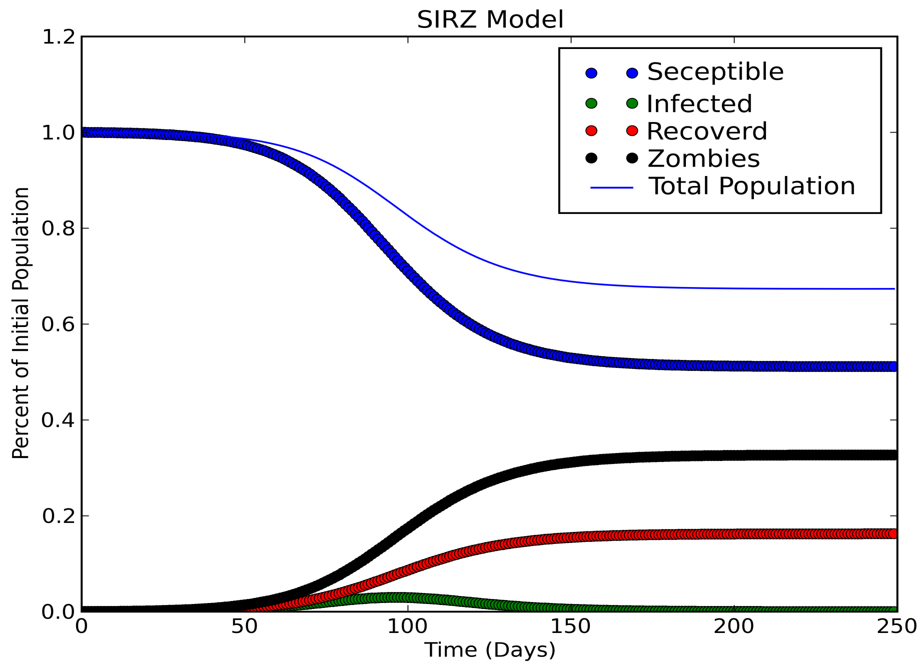
The recent development of Swine Zombies must be addressed immediately, but due to the nature of their proliferation, computer simulations have ascertained that they pose little threat in the long run. My advice to the U.N. would be to contain the flu outbreak geographically, and then nuke it from space.



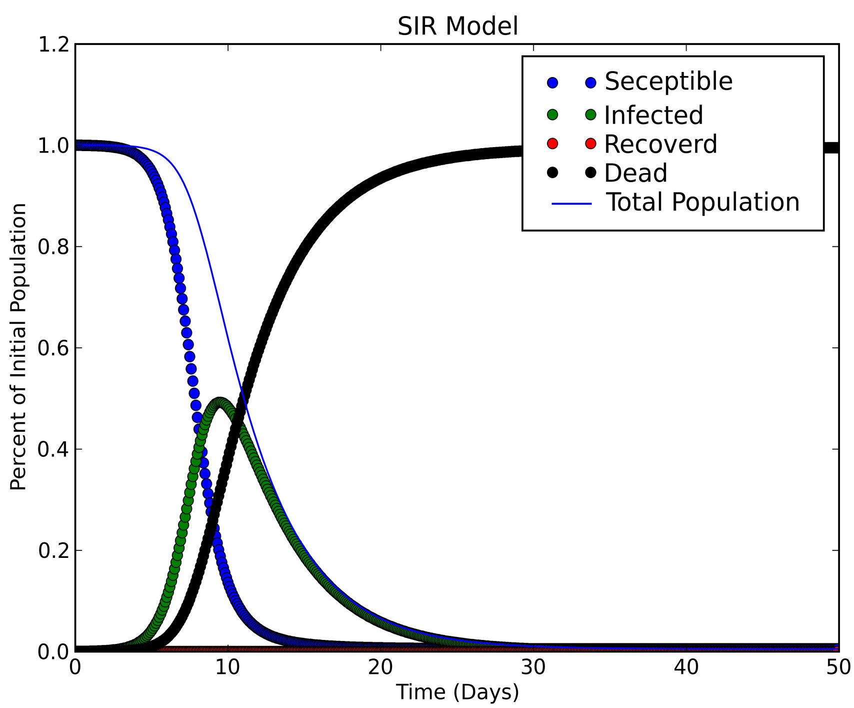
Sample SIR model for H1N1 (non-lethal variant)



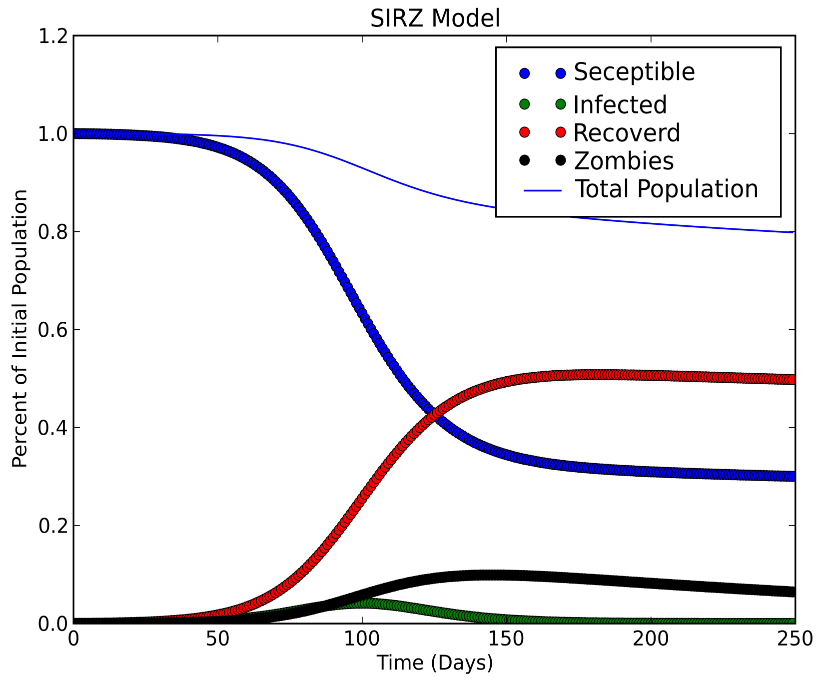
Sample SIR model for H1N1 (lethal variant)



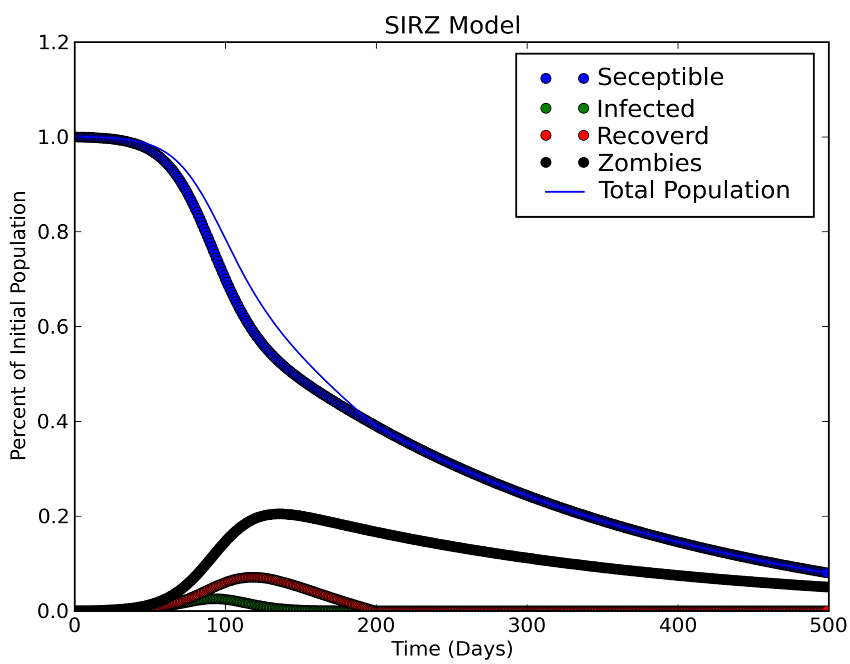
Modified SIR model for Bubonic Plague



Modified SIR model for Apocalyptic Plague



SIRZ Model for Swine Flu



SIRZ Model for Bubonic Plague + Extra Strong Zombies

Final Population Ratios by Initial Condition