Chaotic Streamlines inside a Droplet

Radoslav Bozinoski*

University of California at Davis, Davis, California, 95616

The streamlines inside a neutrally buoyant droplet immersed in a linear flow are examined. It has been determined that the control parameters which lead to chaotic particle paths are the rate-of-strain tensors and the vorticity imposed on the sphere from the external flow.

Nomenclature

$\vec{E}$	rate-of-strain tensor
a	stress tensor ratio
$\vec{\omega}$	vorticity
$\vec{x}$	direction
t	time
r	droplet radius

Introduction

SIMULATION of "steady" and "unsteady" flow of aerodynamic bodies has matured a great deal over the past decade. Aerodynamic performance and flow structures can be predicted with acceptable accuracy except in the complex flow regions of mixing and at off-design conditions near stall. In these regions, complex flow structures with multiple eddies that mingle and mix are often not predicted well due to inadequate computational grid density and a breakdown of turbulence models.

The focus of the current effort has been to get a better understanding into the mechanisms that lead to chaotic behavior in fluid flows and more specifically in the flow of a neutrally buoyant droplet in a general linear external flow. It is the hope of the author that the examination of such systems and the development of visualization tools will help to get more insight into the current turbulence models and hopefully help to improve them.

As mentioned before it was determined that the parameters of interest that lead to chaotic behavior in the droplet were the vorticity, ω, and rate-of-strain tensor, E. More specifically, by varying the vorticity magnitude and its angel measured from the z-axis, chaotic steam paths were obtained that lead to some very interesting results.

*Graduate Student, Mechanical and Aeronautical Engineering Department, email - rbozinoski@ucdavis.edu.

Background

The flow of a neutrally buoyant fluid droplet suspended in a linear flow field is being considered and can be seen in Fig. 1. The axis is aligned with the principle rate-of-strain tensor ,E, with the coordinate system chosen so it is moving with the center of mass of the drop. The governing equations can be seen in the next section and the reader is referred to the paper by Stone[1] for a more detailed description of the flow field.

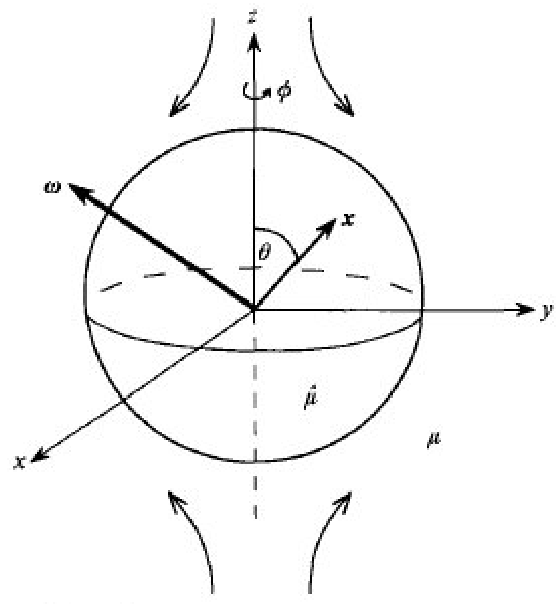


Figure 1: Drop in linear flow.

Dynamical System

The dynamical flow within the droplet with a radius of unity and moving in an external stream is given by the dimensionless form by,[1]

$$\frac{dx}{dt} = \frac{1}{2}\left[(5r^2-3)\frac{x}{1+a} - 2x\left(\frac{x^2}{1+a} + \frac{ay^2}{1+a} - z^2\right)\right] - \frac{1}{2}(\omega_y z - \omega_z y) \quad (1)$$

$$\frac{dy}{dt} = \frac{1}{2}\left[(5r^2-3)\frac{ay}{1+a} - 2y\left(\frac{x^2}{1+a} + \frac{ay^2}{1+a} - z^2\right)\right] - \frac{1}{2}(\omega_z x - \omega_x z) \quad (2)$$

$$\frac{dz}{dt} = \frac{1}{2}\left[(5r^2-3)z - 2z\left(\frac{x^2}{1+a} + \frac{ay^2}{1+a} - z^2\right)\right] - \frac{1}{2}(\omega_x y - \omega_y x) \quad (3)$$

Methods

To properly investigate the chaotic behavior inside the droplet a visualization tool called ChaoticDrop.py was developed using python with the aid of the 3D visualization library, VPython, as well as PyLab. There were nine total tools developed, both 2D and 3D, that were developed and are described below:

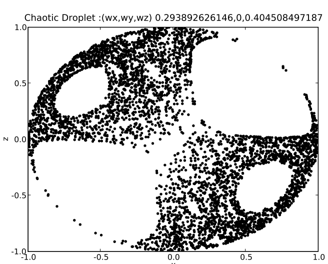
1− 2D Poincare Map generator
2− 2D Poincare Map generator using VPython
3− 3D streamlines using VPythons curve object
4− 3D streamlines using VPythons curve object + 2D Poincare Map generation
5− 3D particle tracking
6− 3D particle tracking with history (one particle)

7− 3D particle tracking with history (two particle)
8− Calculate Lyapunov characteristic exponents
9− Automated Poincare map generation

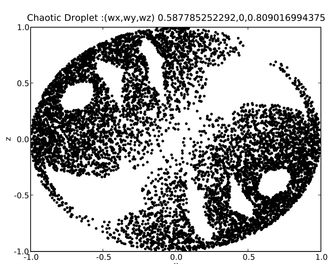
Tool option 1 generates a Poincare map in the xz-plane using the Matplot library that is part of Pylab. Option 2 performs a similar function but instead of generating the map using the Pylab module it uses the VPython module and updates the map at each iteration. The third tool developed generates streamlines using the curves object in the VPython module. This tool contains the entire history of the fluid particle. The next tool combines option 2 and 3 and is very helpful to see the chaotic regions and stable regions in the sphere. Besides tool 1 tools 5 through 7 are the most useful. They allow for particle tracking with and without history of closely grouped initial conditions. These modes allow the user to see exactly how chaotic the system can be depending on the initial conditions. Tool 7 is especially usefully for this. It traces two particles with a 1% difference in initial conditions, which is far less than the error in any modern wind-tunnel measurement tool at UCD, and shows how chaotic the system really can be. The last tool is a specialized function which calls tool 1 and generates a whole series of Poincare maps. The first set of maps have a constant orientation from the z-axis and vary the vorticity magnitudes, the second set of maps holds the vorticity at 0.1 and varies the orientation and the third set also hold vorticity magnitude but at a higher value of 2.0. It should be noted that all the tools developed require the same input, mainly the following: α, ω_x, ω_y, ω_z, x_0, y_0, z_0, dt, number of iterations

Results

Figure 2 shows the Poincare maps for $\theta = 0.1\pi$ and varying ω's. Here we can see as ω is increased the stable regions, or islands, start to colloid and eventually go away and leave a completely chaotic domain in the sphere. The values for ω are range from 0.5 to 1.4. After 1.4, ω is increased to 8.0 and we see the stable regions return until we get a mostly stable sphere domain except for the edge of the sphere and the sphere equators.



(a)



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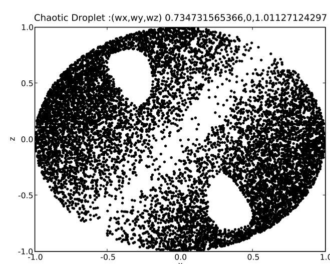

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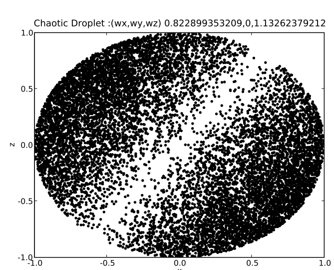

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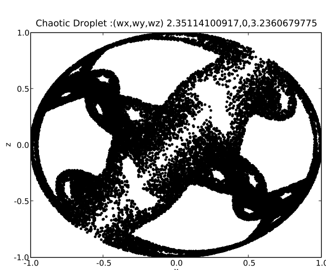

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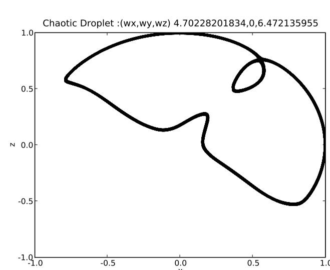
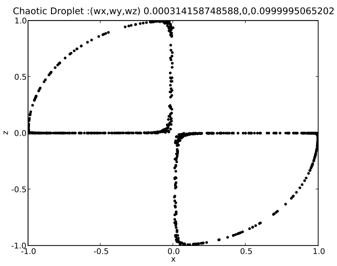


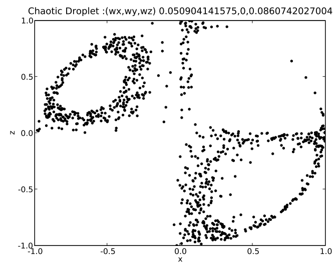
(f)

Figure 2: $\theta = 0.2\pi$ at with = (a) ω=0.5, (b) ω=1.0, (c) ω=1.25, (d) ω=1.4., (e) ω=4.0, and (f) ω=8.0.

In Fig. 3 we see the result for keeping ω constant at 0.1 but varying θ from 0.0001π to 0.4π. Here we can see a similar trend except the sphere starts out with most of the chaotic region near the edge of the domain and near the equators. From there as θ is varied the chaotic regions grow in size and eventually fill in most of the sphere's domain.



(a)



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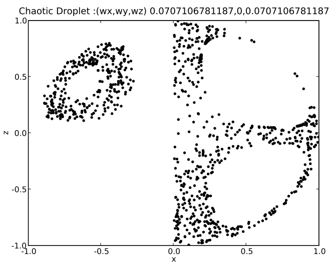

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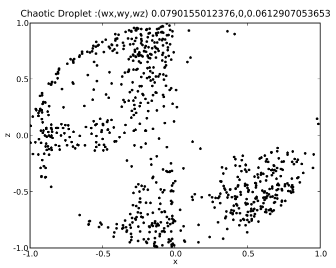

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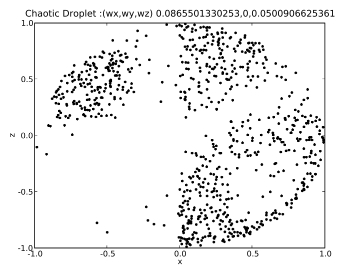

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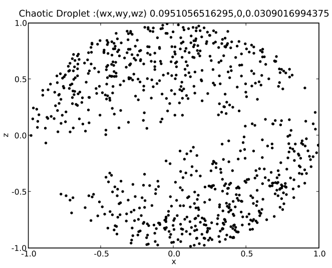
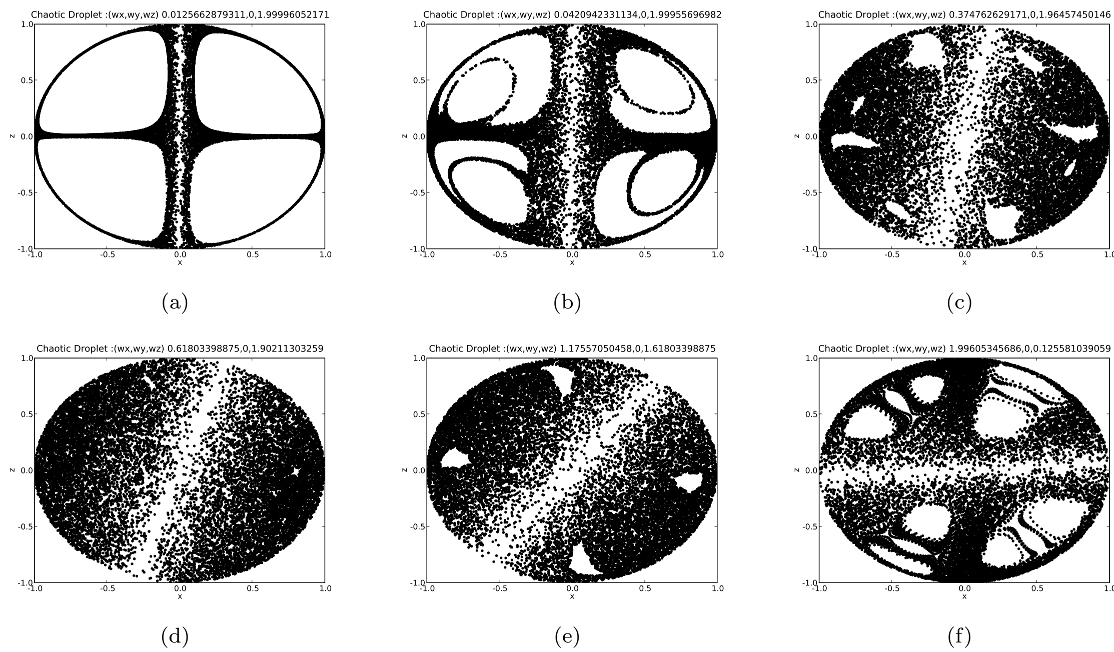


(f)

Figure 3: $\omega = 0.1$ **at with = (a)** θ=0.001π, **(b)** θ=0.17π, **(c)** θ=0.25π, **(d)** θ=0.29π, **(e)** θ=0.333π, **and (f)** θ=0.4π.

The last figure, Fig. 4, shows what happens when once again ω is held constant at 2.0 and θ is varied. Here we see that as we vary θ bifurcations form in the stable regions which eventually increase the chaotic region and leads to very interesting flow structures.



(a) (b) (c)

(d) (e) (f)

Figure 4: ω = 2.0 at with = (a) θ=0.002π, (b) θ=0.0067π (c) θ=0.06π (d) θ=0.10π (e) θ=0.20π, and (f) θ=0.48π.

Conclusion

We have shown that even in a simple system where a neutrally buoyant droplet in a generally linear flow can behave every chaoticly. The particle paths chaotic behavior inside the sphere has been show to be dependant on four parameters, a, which is the strain rate ratio, the magnitude of ω and the orientation, θ, from the z-axis.

References

[1]H.A. Stone, Ali Nadim, and Steven H. Strogatz. Chaotic streamlines inside drops immersed in steady stokes flows. *Journal of Fluid Mechanics*, 232:629–646, 1991.