

# Mitigating Large Cascades in Drossler Forest Fire Models

Mark Yoder  
UC Davis Physics and Geology  
June 2008

## Synopsis

We study variations on the traditional Drossler-Schwabl cellular automata Forest Fire Model (FFM) that show measurable change in the distribution of large events while maintaining scale invariance. We propose two models that demonstrate measurable deviation from the familiar FFM distribution slope (log-log) of -1. We compare these models to observed wild fire data in the Eastern US, Northern Territory Australia, southern California US (SCA), and Baja California Norte Mexico (BCA). We suggest that differences in flora, fauna; and perhaps more interestingly, land management policy account for the different distributions of large fires.

In particular, we focus our attention on the border region between SCA and BCA. These regions, because they are geographically contiguous, share nearly identical meteorological and topographical conditions. Nonetheless, they demonstrate substantially different wild fire size distributions and flora clustering patterns. We suggest this is largely a function of land management policy, which we suggest that our simulation emulates. We propose that these models, or perhaps enhanced versions of these models, might be employed to develop effective wild land management policies.

## Introduction

Having seen several devastating fire seasons in Southern California and throughout the greater western United States, policy makers and home owners have expressed concern and alarm. The traditional response to extreme wild fires has been increased fire suppression activity – more bulldozers, more bombers, more firefighters. While fire fighting is effective against small fires under a wide range of “normal” conditions, large fires and exacerbating circumstances, such as extended periods of hot, dry weather and strong winds, produce fires that are difficult if not impossible for humans to control or contain by any reasonable means. These especially large and ferocious firestorms are relatively rare but account for the vast majority of wild-fire related property damage and injuries<sub>(10)</sub>.

It is widely believed in the academic and wild-land management communities that our century or so long policy of fire suppression has actually increased our susceptibility to large, uncontrollable fires. By suppressing small fires, so the argument goes, we permit the forest to accumulate large volumes of combustible material and to form large contiguous clusters of trees, brush, and tall grasses. These regions, burn both hotter and, as per enhanced connectivity, more expansively than they would under natural circumstances. It is a logical extension of this argument to suggest that an expanded, more aggressive wildfire suppression policy further exacerbates this problem, perhaps mitigating damages from small to medium size fires but facilitating larger, more ferocious and destructive firestorms. We

address this argument in terms of clustering dynamics, percolation theory, and numerical cellular automata models.

Observations in North America and Australia suggest that real wild fires follow power law distributions in size and frequency, though the slope (exponent) of that distribution varies from one region to another. For example, fires in the eastern US follow a steeper (fewer large fires) distribution than fires in the western US<sup>(7)</sup>. This difference in slope could be a function of varying ecology, climate, or land management policies including fire suppression. A more striking and interesting phenomenon is observed in the chaparral near the southern California, US (SCA) and Baja California Norte, Mexico (BCA) border. Here, we observe distinct fire distributions and foliage clustering in two regions separated only by a political, human enforced border. Differences in wild fire dynamics near the border, we suggest, can be attributed almost entirely to distinct land management policies on either side of the border [10, 18].

We understand that real fire dynamics are complex. Observationally and anecdotally, we know that wild fire risk is affected by climate, geological topology (such as canyons, water ways, mountains, etc.), and meteorological factors - perhaps most importantly wind. We remind ourselves, as it so happens repeatedly throughout this process, that near this border region these factors differ only to the extent that political governors and bureaucrats mandate. We assume this factor to be small, even during an election year. We can use this region to study, in some isolation, the effects of land management policy on foliage clustering and ensuing wild fire dynamics. To this end, we employ variations on the Dorssel-Schwabl type cellular automata "Forest Fire Model" (FFM). We find the model's characteristic slope of approximately -1 surprisingly robust. We propose two variations that, in some agreement with findings by Yakovlev et-al<sup>(16)</sup>, produce convincing power law distributions with steeper slopes, by thinning and fragmenting large clusters.

### **Introduction to the FFM [3]**

The basic Forest Fire Model, a close cousin to dynamic sand-piles, is as follows<sup>(3)</sup>:

1. We establish an NxM site rectangular grid.
2. Like a dart in an English pub, we "throw" a tree at a random spot on the grid.
3. If that site is empty (grid value=0), we plant a tree there (set grid value=1).
4. If that site is occupied by a tree (grid value=1), we do nothing.
5. Every f steps (tree tosses), we throw a match, spark, or lightning strike at a grid element, again chosen at random.
6. If that site is occupied by a tree, it ignites.
7. Sites directly adjacent (nearest neighbor sites only, no diagonals) to any burning site burn. We iterate this step until the entire cluster is burned.
8. Sites "burned" in (7) return to "dirt" status (grid value=0), and we start over.

We run the model for either a fixed number of time steps or until we get some agreed upon number of events. A fixed number of fires some size  $k_0$  might be used to readily show slope differences, or a fixed total number of events might be used to simplify normalization.

Some common variations on the model include:

- **Discrete vs Continuous time distributions**

- **Discrete:**

- In a given “time-step” we attempt to plant one or more tree on a randomly chosen grid square.
- Every  $f$  “time-steps” (as per above), we throw a match at a randomly chosen square.

- **Continuous:**

- During a given time-step, each element in the grid spontaneously begets a tree with probability  $P_{\text{beget}}$ .
- During a given time-step, each tree-bearing element spontaneously combusts with probability  $P_{\text{combust}}$ .

- **Probability Distributions:**

- Typically, continuous distributions are used in both spatial and temporal random variables. There are strong arguments, however, to use Poisson distributions or some other scheme to add periodicity to the system.

- **Burning rates and properties:**

- **Instantaneous Fires:** Most models meant to simulate real wild fire conditions use instantaneous fires, in which once a fire starts, it burns to completion in a single time-step. The interpretive assumption, of course, is that a forest fire occurs over a relatively short timespan compared to the duration of a fire.
- **Continuous fires:** Fires can be propagated on the same scale as tree planting. These models, given the right parameters, do not require sparking; fires naturally burn continuously. In this scenario, for a given turn:
  - A tree is planted, maybe a match is thrown.
  - A tree next to a burning tree ignites.
  - Trees burning from the previous time-step return to the ground state.

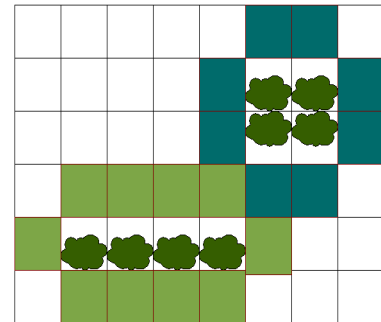
Clearly, these models do not simulate real wild fires. However, they often produce spectacular images and they might, be useful to model other phenomenon like galaxy spiral arms or other steady state variations in pressure and density.

- **Immunity:**

- Immunity to fire can be parameterized to simulate differences in foliage type, meteorological conditions, or fire suppression activities. We suggest two basic types of immunity:
  - **Mean Field Immunity:** Mean field immunity is suggested from the basis that large fires are difficult to put out. As the fire propagates, then, we include a

probability related to the size of the fire that either the entire fire will spontaneously quench itself or that the fire will spread to connected elements, in the normal way. As implied, this probability diminishes to zero as the fire becomes large.

- **Local Immunity:** Local immunity attempts to address the issue of immunity directly between elements. We assign, a finite probability that a tree element adjacent to a burning tree element does not catch fire. A highly connected tree, with several burning nearest neighbors for example in a dense cluster, is less likely to survive by chance than elements in a narrow, sparse filament. This approach, in principle, might emulate the observation that large fires are more difficult to suppress than their smaller cousins and does not require geographically disparate elements of the fire to share information.



*Illustration 1: Cluster circumference is a function of shape.*

### FFM Analytics

Consider the simple observation that, for a discrete FFM step:

1.  $P_{anyfire} = f \cdot \rho$

where  $f$  is the firing frequency and  $\rho$  is the mean density of the grid. Similarly, we identify a single fixed point in the density. We can estimate the mean steady state grid density as follows:

2.  $\frac{dn}{dt} \equiv \dot{n} = (1 - \rho) - f \cdot \rho \cdot \sum_k p_k \cdot k$

3.  $\sum_k p_k \cdot k = \langle \Delta k \rangle = \langle \text{fire size} \rangle$

4.  $\dot{n} = 0$

5.  $\rho = \frac{1}{1 + f \cdot \langle \Delta k \rangle} \Rightarrow \frac{1}{2} - \epsilon(f)$

6.  $\rho_{experimental} = .400 \pm .019$

In (5), we recognize that the average fire size per sparking interval, in the limit of well tuned parameters and steady state, is nearly equal to the firing frequency such that the average number of trees removed per sparking interval equals the average number that arrive. Experimentally, we find an average density of .400 +/- .019 over a modest sample, which is consistent with findings by Grassberger et-al [1]. The average difference between the density before and after a fire was on the order of .0001.

Clustering dynamics are a bit more complicated. Employing some elements from percolation theory, we can write the balance equations for new arriving nodes:

7.  $n_{k,t+1} = n_{k,t} + C_{k-1}n_{k-1} - C_k n_k$
8.  $n_{(1,t+1)} = n_{(1,t)} + C_0 n_0 - C_1 n_1$
9.  $n_{(1,t+1)} = n_{(1,t)} + (1-\rho)^5 - 4 \cdot n_1$
10.  $n_1 = N \cdot \rho \cdot (1-\rho)^4$
11.  $P_{attach,1} \equiv C_1 = n_1 \cdot 4$
12.  $n_0 = (1-\rho)$
13.  $C_0 = 1 \cdot n_0 = (1-\rho)$

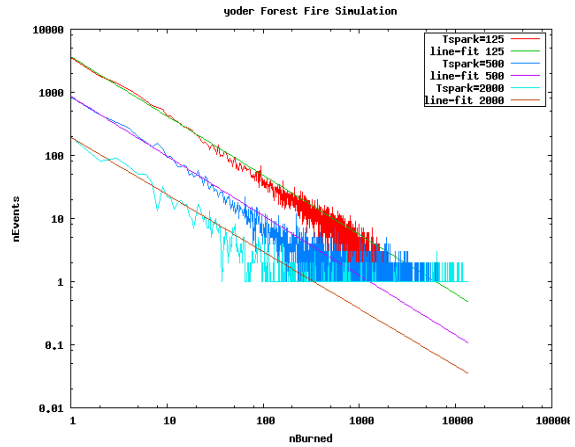
Cluster coalescence can be written as:

14. 
$$dP_{k,t+1} = \sum_{k_i+k_j=k; i>j} \widetilde{C}_{k_i} \cdot n_{k_i} \cdot \widetilde{C}_{k_j} \cdot n_{k_j} - n_{k-1} \cdot \widetilde{C}_{k-1} \cdot \sum_i n_{k_i} \cdot \widetilde{C}_{k_i} - n_k \cdot \widetilde{C}_k \cdot \sum_j n_{k_j} \cdot \widetilde{C}_{k_j}$$
15.  $\widetilde{C}_k = C_k - \text{loops}$

Here,  $C_k$  are cluster circumferences which, as we indicate in (5), are equivalent to the probability that a new node attaches to a cluster of size  $k$ . In the third equation, we observe that new nodes form size one clusters if the new site and the four adjacent sites are empty; a size one cluster becomes a size two cluster if a newly arrived node lands on any of the four sites adjacent to it. If we limit cluster growth to newly arriving elements, ignoring the coalescence of existing clusters, we could construct a recursion relationship similar to the Barabasi-Reka preferential attachment network [15]. Cluster coalescence, however, is integral to FFM dynamics, and as (14) shows, is quite complicated for large clusters. Accordingly, we employ our discrete, numerical model as our primary tool to study FFM dynamics.

The FFM is often touted as a classic degree-scale invariant, self organizing critical (SOC) model. Strictly speaking, however it is neither scale invariant, nor is it truly critical. As Grassberger et-al discuss quite thoroughly<sup>(1)</sup>, scale variance emerges for small fires and in the limit of very large or small firing frequency. Over a broad range of parameters (grid size and sparking frequency), however, the model demonstrates convincing power law behavior over a wide range of fire sizes. We focus our attention on this region and consider the small and large fire ends of the distribution prone to artifacts of the model.

Unlike a truly critical system, the FFM cannot be triggered by simply loading the grid. It is more correctly described as “critical susceptible.” The system evolves to a state in which it is, in conjunction with an a stochastic process, increasingly susceptible to some sort of cascading catastrophe. In a nutshell, as large clusters form, the system becomes susceptible to large fires, but a spark is required to actually trigger an event [1].

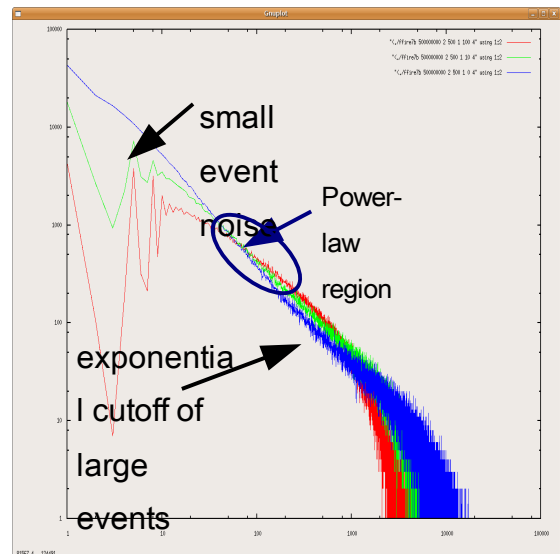


*Illustration 2: Distribution of standard FFM events (nEvents vs nBurned) fit to lines, slope~-1. red: f=125, blue: f=500, cyan: f=2000*

### Interpreting the FFM Results and Observed Wildfire Data

The classic FFM produces a power law distribution of events as a function of the event size. This slope appears to be invariant over a wide range of grid size and firing frequency. Often, the power law behavior does not persist over the full range of the resulting distribution; very large and very small events may fall off the power law distribution. Specifically, the range of small events can become noisy, and we see an exponential cutoff of large events where clusters are removed aggressively, for example by an rapid sparking frequency. Also, we see high counts of very large events, often referred to as the “finite volume effect,” when grid density is permitted to exceed the critical percolation, permitting clusters to grow very large between sparking events.

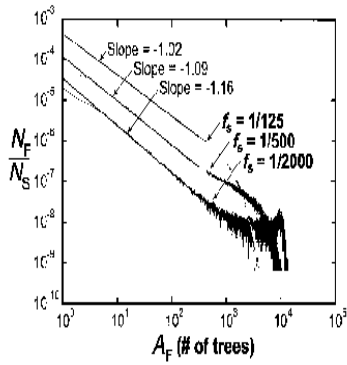
We focus our interest on the power law region, and somewhat freely discard anomalies at the large and small event ends of the distribution, for two reasons. Firstly, we do not wish, nor do we feel we are qualified, to define a characteristic length for the model. In terms of real wild fires, we assume our grid



*Illustration 3: Interpreting FFM data: We for some variations and parameterizations of the FFM, we expect small and large events to deviate from the power-law distribution.*

elements represent regions ranging from perhaps dozens of trees to a few hectares, to several square kilometers. Our objective is to develop a model capable of describing phenomenon over large areas of wild-land that we can run on a desktop workstation, which is to say using a relatively small number of grid elements. Secondly, and perhaps more importantly, observed wildfire data convincingly fit power law distributions with exponent ranging between roughly -2 and -1.

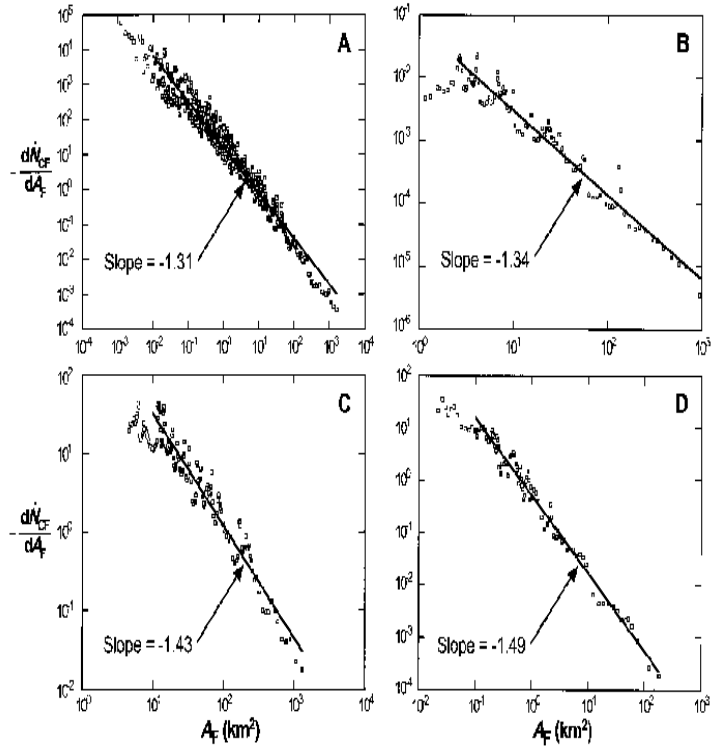
REPORTS



**Fig. 1.** Noncumulative frequency-area distributions of model forest fires for a grid size of 128 by 128 squares at three sparking frequencies,  $f_s = 1/125$ ,  $1/500$ , and  $1/2000$ . The number of fires per time step ( $N_F/N_S$ ) with area ( $A_F$ ) is given as a function of  $A_F$ , the number of trees that were burned in each fire. For each sparking frequency, the model is run for  $N_S = 1.638 \times 10^9$  time steps. The small and medium fires correlate well with the power-law relation (Eq. 1) with  $\alpha = 1.02$  to  $1.18$ ;  $-\alpha$  is the slope of the best-fit line in log-log space and is shown for each sparking frequency. The finite grid-size effect can be seen at the smallest sparking frequency,  $f_s = 1/2000$ . At about  $A_F = 2000$ , fires begin to span the entire grid.

four data sets from the United States and Australia. The data sets are from a variety of geographic regions with different vegetation types

*Drawing 1: Observed wild fire distributions as reported by Turcotte, Morein, Malamud 1998 [2]*



**Fig. 2.** Noncumulative frequency-area distributions for actual forest fires and wildfires in the United States and Australia: (A) 4284 fires on U.S. Fish and Wildlife Service lands (1986–1995) (9), (B) 120 fires in the western United States (1150–1960) (10), (C) 164 fires in Alaskan boreal forests (1990–1991) (11), and (D) 298 fires in the ACT (1926–1991) (12). For each data set, the noncumulative number of fires per year ( $-dN_F/dA_F$ ) with area ( $A_F$ ) is given as a function of  $A_F$  (13). In each case, a reasonably good correlation over many decades of  $A_F$  is obtained by using the power-law relation (Eq. 1) with  $\alpha = 1.31$  to  $1.49$ ;  $-\alpha$  is the slope of the best-fit line in log-log space and is shown for each data set.



## Failed Models

It is worth taking a moment to discuss models that yielded null results, both to appreciate the robustness of the FFM and to illustrate the governing dynamics of the model – cluster density. We examined three distinct approaches to changing the fire size distribution. First, we attempted to steer the fire dynamics using information in the system nodes – the trees or individual grid elements. In particular, we employed parameters that emulated the age of the tree or stand of trees. Second, we addressed the overall density of the grid, and thirdly we study the effect of connectivity between grid elements. These three approaches yielded null results.

### Node Level Information

The original approach was to directly correlate stand age with propensity for large fires. Obviously, a rule such that young trees do not burn is equivalent to those elements arriving to the grid at some later time, so we look for another way to parameterize age or otherwise place information into the nodes of the system. We experimented, then, with a rule that young trees would burn but not spread to adjacent elements; older trees burn and ignite adjacent cells as per the standard model. The thinking behind this model was that the young trees would form small fire breaks. Elements would be removed without triggering larger fires. Ultimately, however, we find that we get two populations with similar clustering properties, and aside from a little bit of noise in the small-fire end of the distribution, we end up with the standard FFM. We further modified this model to preferentially plant new trees on the “outside” of clusters, but again observed a null result.

### Connectivity

In a second group of models, we address the connectivity of individual grid elements. In the first model, we dilute the grid with some density of “rocky” sites where trees could not be planted. We suggest the possibility that the probability with which a fire continues to propagate might follow a relationship of the form:

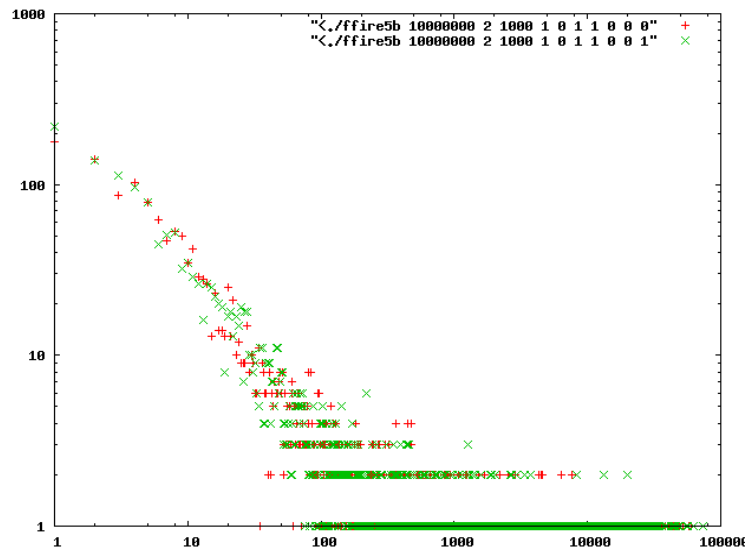
$$16. \quad P(t) \propto \prod_{0 \leq k \leq 4} \frac{k_{i,t}}{k_{max}} \rightarrow \prod_{0 \leq k \leq 4} \frac{\langle k \rangle_t}{k_{max}} \rightarrow \left( \frac{\langle k \rangle_{burning}}{k_{max}} \right)^{t-t_0}$$

where  $k$ , in this case, represents the connectivity of individual grid elements. That is to say that burning grid elements ignite adjacent grid elements with some finite probability; at each time step of fire propagation, there is some finite probability that the fire will continue to burn. If we reduce the average connectivity of the grid, we potentially impede a fire's capacity to propagate through a large number of time steps.

Specifically, we employed an even-random distribution of “rocky” sites. For low to moderate densities of

these barren sites, we observed no measurable change in the distribution. For densities above the critical percolation threshold, not surprisingly, we observed an exponential cutoff for large fires, but otherwise the standard FFM power law distribution with exponent -1 persisted. It may be possible to reduce the number of large clusters by using more targeted arrangements of these rocky sites, for example single width vertical or horizontal structures, but to do so without imposing a characteristic length, and so breaking scale invariance, is tricky. A linear structure of rocky sites, for example, has little effect on a cluster whose extent in the same direction as the line is much larger. Put another way, as clusters become larger, linear structures begin to appear point-like, and we are back to our original random distribution of sites. It might also be possible to produce a measurable signal from this model in conjunction with some form of local immunity parameterized such that (16) converges.

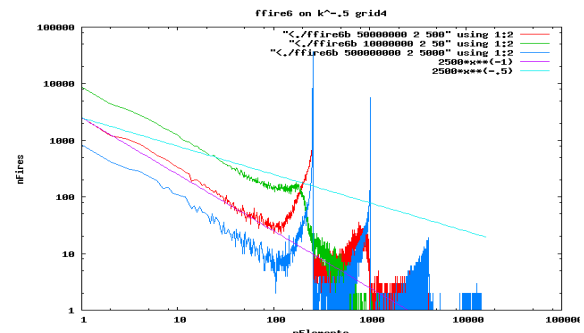
We also address grid element connectivity directly by permitting next-nearest neighbor (NNN) fire propagation. Obviously, a given set of points distributed randomly onto the grid form “larger” clusters when NNN connections are permitted. The distribution of events, however was unchanged. We further modified the model by including only older NNN elements in fires and varying sparking frequency, to emulate increased combustibility of older stands. Again, the resulting distribution was indistinguishable from the standard FFM with slope of -1.



*Illustration 4: red +: Standard FFM, green x: FFM with fire propagating to all NNN.*

## Fractal Planting Substrate

We extended this idea to plant the grid on a Sierpinski like fractal structure such that  $n_k \sim k^{-5}$ . We lay down a fractal substrate of fertile and, again, “rocky” sites where trees cannot be planted. When we run the model, we plant trees at random on the grid as usual. The grid, however, is subdivided into sub-grids that burn independently of one another; fires do not spread from one sub-grid to another. When we drop a match, we it will land on a grid element which is part of a sub-grid of some size according to our chosen fractal distribution. At low firing frequency, where each cell was unlikely to saturate, we expect and observe the standard FFM distribution. When we run the model with a a very low firing frequency, such that the grid will likely saturate beyond its critical percolation limit, the ensuing fire will burn a large percentage of the cells in that sub-grid. We expect then, to see the fractal substrate in the fire size distribution. For moderate saturation, we see peaks around our fractal sub-grid size values and a sawtooth pattern with slope -1 between eigenvalues. Dissatisfied, we retired this model while it was still quite young. A more convincing variation of the model might be to use a more continuous distribution of sub-grid sizes. For example, to create a distribution on a large but finite grid with a distribution slope -1/2, we use the following distribution of sub-grid sizes:



*Illustration 5: Fire distribution of a FFM planted on a Sierpinski gasket for varying firing frequencies: green: 50, red: 500, blue: 5000. Plots for slope -1 (magenta) and -.5 (cyan) are shown for reference.*

| cluster substrate for $P(k)=a \cdot k^{-.5}$ : |                                |       |             |        |
|--|--------------------------------|-------|-------------|--------|
| N<br>(number of sub-grids)                     | k<br>(number of grid elements) | P(k)  | P(k)/Ntotal | L side |
| 1  | 1024                           | 1024  | 0.0156      | 32     |
| 8  | 256                            | 2048  | 0.0313      | 16     |
| 64   | 64                             | 4096  | .0625       | 8      |
| 512  | 16                             | 8192  | .125        | 4      |
| 4096   | 4                              | 16384 | .25         | 2      |
| 32768  | 1                              | 32768 | .5          | 1      |

The resulting distribution of fires is heavily peaked around the k values; the space between is dominated by a slope of -1. Further statistical analysis of the peaks might yield a different distribution, but preliminary analysis indicates the peaks also demonstrate a slope of -1. It might be possible to construct a smoother distribution of events by selecting a more continuous set of k-size sub-grids from the distribution, being careful to maintain consistency with the prescribed distribution.

The scientific justification behind this model was to emulate natural fractal topologies. It is widely accepted, for example, that drainage basins, landslides, and other natural planting substrates follow fractal distributions. Though we retired this model quite quickly, the preliminary results are characterized better by the standard FFM distribution of  $k^{-1}$  than by the imposed fractal distribution of  $k^{-1/2}$ . The model also problematically requires a very large grid, to contain a suitable number of sub-grids, to the extent that it arguably exceeds the scope of desktop workstation science. In short, this approach seems contrived, unpromising, and computationally burdensome though further investigation would be necessary to say so conclusively.

**Models that Work:  
Clustering and Cluster Density**

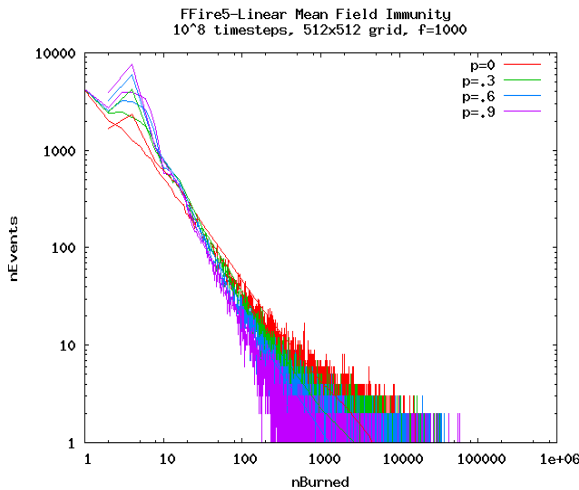
Having investigated models exploiting nodal information, connectivity, mean density, and environmental topology, we turned our attention to clustering. Work by Yakovlev et-al<sup>(16)</sup> suggest that cluster density is fundamental to characterizing cluster, or fire, size distribution. We look for models that directly address cluster density of modify fire propagation with respect to cluster size or number of elements involved in a fire. We present two models that yielded measurable changes in fire distribution slope. These models are designed to either directly modify cluster density or fragment existing large clusters into smaller pieces.

**Mean Field Immunity**

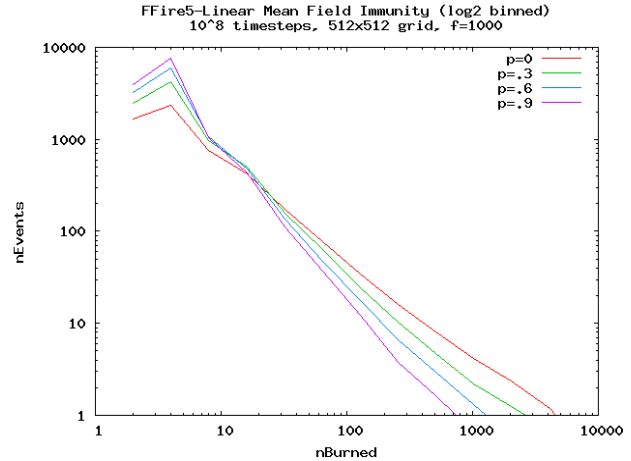
Starting with the standard FFM, we introduce a “quenching” function. Fires propagate via a breadth-first algorithm. As the fire propagates outward, we test a the function:

$$17. \quad (\text{random number}) < \frac{\text{immunityParameter}}{(1 + \frac{n\text{Burning}}{5})}$$

where immunityParameter is provided at run-time, nBurning is the number of trees burning in the current fire, and the denominator 5 was chosen by trial and error to provide sufficient modulation. If this condition is true, we quench the fire. Note that as fires become large, the right side of the inequality becomes very small. As the the plot shows, this model provides measurable changes in the slope of the distribution. Because this model permits large fires to burn freely, we interpret the results to suggest that the smaller fires tend to fragment large clusters, resulting in a steeper distribution of events. This type of immunity might account for natural differences in the distribution of fires in regions with distinct fauna



*Illustration 6: Fire distributions from a Mean Field Immunity (MFI) model: 512x512 grid, 10 million time steps, immunity factors, red: 0, green: .3, blue: .6, violate: .9.*



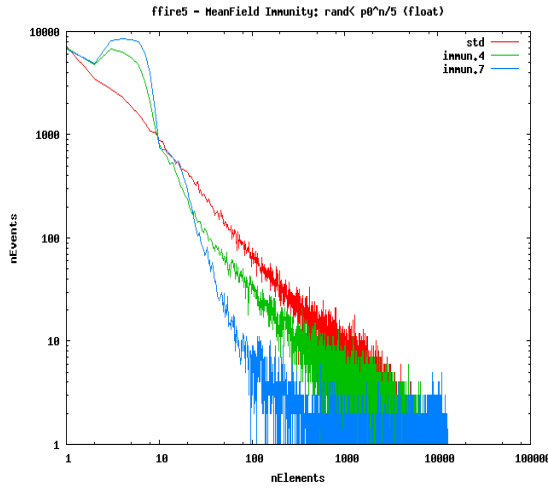
*Illustration 7: The same MFI model data binned logarithmically.*

and climatological conditions, for example the lush eastern US compared to the drier chaparral regions in southern California and northern Mexico.

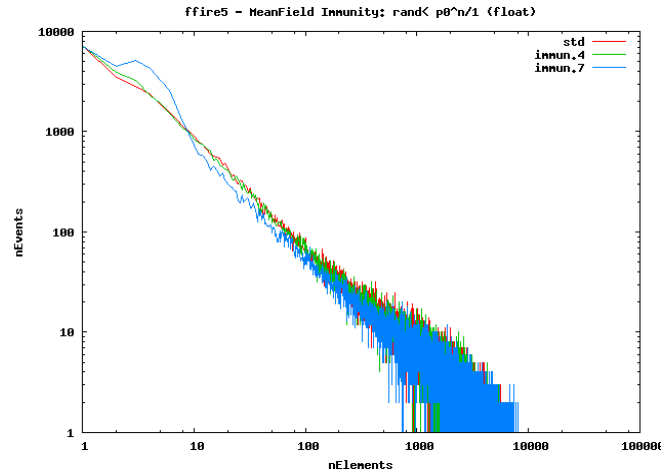
It seems, however that not all immunity models are created equal. Initial results from a model where immunity fell off exponentially,

$$1. \quad P_{quench} \propto e^{(-a \cdot k)}$$

showed promising initial results but did not hold up over a large number of time-steps. The results of these two models may yield suggestions regarding proactive wild land management and wild fire suppression policies. Excessive suppression of very small fires, for example, may prevent larger, but still quite small and non-destructive, fires from fragmenting large clusters.



*Illustration 9: Mean Field Immunity with exponential decay over  $10^7$  time steps yields promising results.*



*Illustration 8: Mean Field Immunity with exponential decay over  $10^8$  time steps. Note the slopes converge to nearly -1 for medium to large fires*

## Trapped Sheep

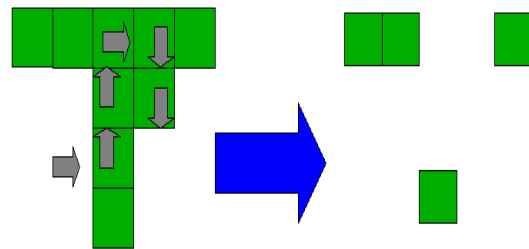
Another model that yields measurable results incorporates virtual sheep. Basic sheep behave as follows:

1.  $N$  herds of sheep are placed randomly on the grid.
2. During each time step, each sheep (heard) moves with probability  $P$ :
  - If the square they occupy is also occupied by a tree, they eat it
  - If the square they occupy is empty, they look to adjacent squares. If one or more of these squares has a tree, they move to one of those squares
3. To sheep, the grid is toroidal.

Note that sheep seek out clusters fundamentally differently than the firing mechanism. Specifically, a cluster's cross section to the fire starting mechanism is exactly its area, or number of contiguous elements. To sheep, however, a cluster's cross section is proportional to its circumference (empty grid elements adjacent to occupied elements) minus any loops. Also observe that sheep often do not eat an entire cluster and may, in fact, fragment a cluster into several smaller pieces. In these respects, sheep address clusters very differently than do forest fires.

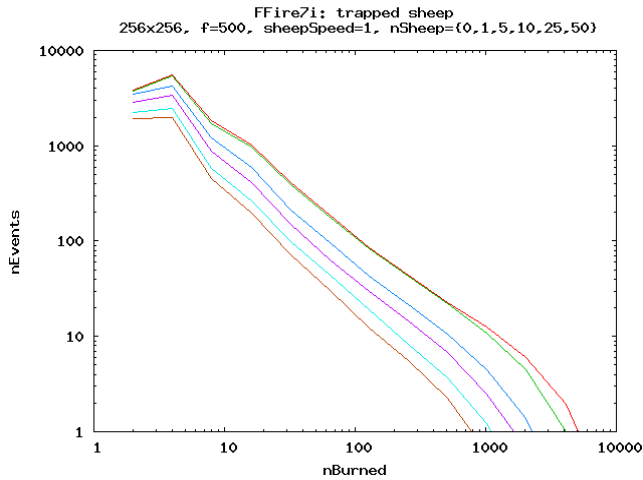
By themselves, sheep do not appear to yield a scale invariant result, probably because they are not dynamic with respect to the grid density or cluster numbers; there are either too many, and we see an exponential cutoff or there are too few and they have little affect. The system seems to suggest some form of logistic map whereby well fed sheep beget sheep, and sheep who have not eaten for some time are painlessly and humanely removed from they system.

This involves a level of programmatic and computational complexity that we had hoped to avoid. By accident, however, we introduced an inexpensive dynamic with similar consequences. By removing the rule that transitions sheep from the far right side of the grid around to the left side created a trapping region along the far right side of the grid. In the absence of vegetation, sheep random-walking in this area are weakly attracted, albeit not strictly confined, to the edge of the grid. As clusters form in this region, they follow them, eating as they go, deeper into the grid. Thus on, grids made sparse either by very active sheep or large fires, sheep tend to retire; as grid density increases or extended clusters are formed, sheep become more active and are funneled into the grid. It is worth noting that trapped sheep, as described above, are not ergodic. A single fast sheep tends to clear the trapping region very efficiently, and so rarely ventures very far into the grid. Many slower sheep, however, follow and remove clusters on a time scale more comparable to the rate of cluster formation, and so spend more time deeper in the grid.

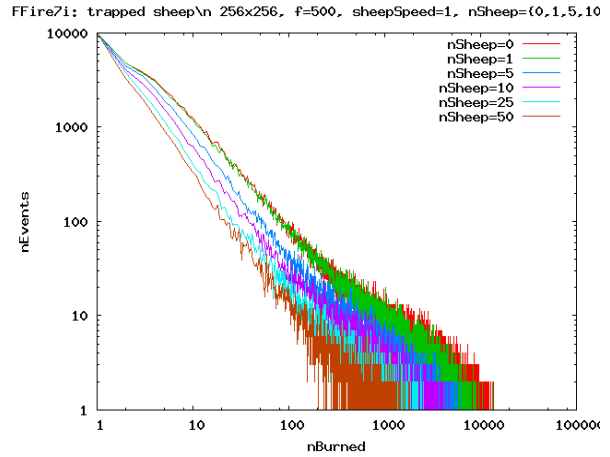


*Illustration 10: Sheep will fragment a T-shape cluster.*

We acknowledge that the slope change under the current model appears to be modest, but it does suggest a proof of concept. A more explicit dynamic, by which sheep are activated by high grid density or some more explicit measurement of clustering might yield improved results. The “sheep” concept might be modified to emulate insects, disease, or some abstract circumference seeking phenomenon.



*Illustration 11: FFM distribution with "Trapped Sheep" logarithmically binned.*



*Illustration 12: FFM distribution with "Trapped Sheep" suggests a power law relationship as a function of the number of sheep. From right to left, nSheep=0, 1, 5, 10, 25, 50.*



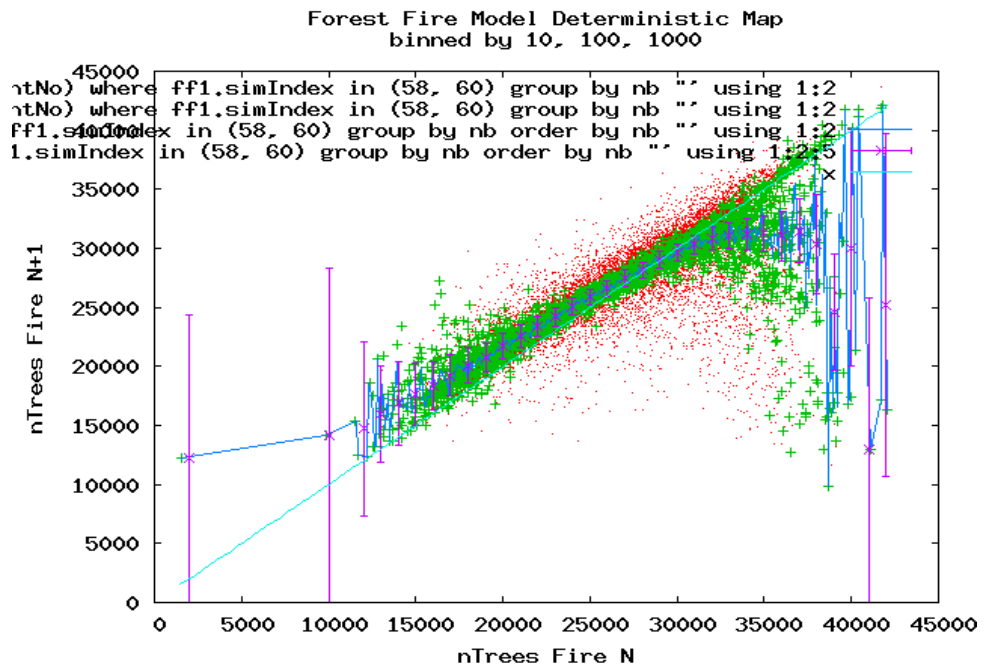
**Determinism?**  
**FFM Poincare Map**

In nature it may be possible to predict triggers, such as lightning, fireworks, or arson. Also, in nature, we can potentially observe clustering patterns via satellite or aircraft, and if we can establish a cluster map of a region, we can use standard percolation theory type methods to statistically predict the coalescence time of existing clusters into super-clusters. Accordingly, we could very likely produce useful risk assessments of large fires.

So far as the scope of this work is concerned, however, wildfires – both real and simulated, are not deterministic [10]. This is not terribly surprising since our model is based on evenly distributed random numbers and in our analytic analysis, we find just a single fixed point in density. Nonetheless, we construct a Poincare map of the grid density after a fire as a function of the grid density at the time of the previous fire.

$$18. \rho_{fire_{n+1}} = f(\rho_{fire_n})$$

We observe fairly tight clustering along a line with a slope slightly less than unity and a positive y intercept. At first glance, this appears almost deterministic. However, when we consider the distribution of fire events, we observe that most fires are small and so will result in a small change in density. Further, a sparse grid cannot have large fires. We expect a sparse grid to accumulate trees just as we expect a dense grid to shed trees. We observe that the scatter crosses unity at more or less  $\rho = .4$ , as expected. As the grid density increases beyond the fixed point density, we see larger fires. At densities above the critical percolation density ( $\sim .7$ ), we see very few points, presumably because above this density, where clusters become very large very quickly, huge fires occur very quickly. Here, we show results averaged by 1, 10, 100, and 1000 with error bars to show trends. The scatter plots suggest there might be multiple fixed points at higher density.



*Illustration 13: Poincare map (nTrees(fire\_n) vs nTrees(fire\_{n+1})) of a FFM, f=1000. nTrees(n+1) are averaged over 1 (red dots), 10 (green points), 100 (blue line), 1000 (purple line and errorbars). The cyan line is slope=1.*

## Conclusion and Discussion

We find the Dorsstel-Schwabl FFM to be very robust, producing a distribution of events with slope near -1. The model is, within a reasonable range of parameters, invariant with respect to grid size, and firing frequency. The model's dynamics also appear to be independent of node level information and grid element connectivity. The dynamics appear to be governed, perhaps exclusively, by clustering statistics. This conclusion is suggested by Yakolev et-al [16], and supported by the successes of our immunity and trapped-sheep models, in contrast with the failure of our node and connectivity based models.

Preliminary analysis indicate that our inverse linear immunity model achieves slopes between -1.5 and -1. These data suggest that large fires can be avoided by diminishing or fragmenting large clusters with smaller fires. The natural interpretation of this can vary. We might interpret the model, for example, to emulate wild fires in regions where fire propagation is hindered either by lush vegetation or by frequent rain storms. The immunity, then, can be interpreted as spatial or temporal (seasonal) variation in fuel moisture content, probability of precipitation, or possibly as fire fighting activity.

We recall, however, that our exponential immunity model model, converges to the standard FFM slope of -1 when iterated over sufficient time steps. We know anecdotally, and have some evidence to support the assertion, from wild fire fighters that small fires are easy to control, but large fires quickly become unmanageable. The question is, it would seem, how quickly? Does fire suppression potency fall off more like inverse linear or exponential?

The debate over proper, healthy management of wild lands in the last decade or so has evolved into a heated discussion between policy makers, scientists, union officials, and manufacturers of equipment like bulldozers and shovels. The very history of modern wild fire management policy is born of a history plagued by good intentions, ignorance, corruption, greed, and bureaucratic finger pointing. Some might argue that fire suppression policies were born of disastrous events like the Hinkley, Minnesota firestorm of 1894. Having witnessed several towns fully evaporated in an afternoon and hundreds of citizens burned to death and maimed in terrible ways, citizens demanded protection, lumber companies wanted their fiscal interests protected, and politicians seeking reelection leaped into action. A scientifically thorough or politically bold investigation of disasters like Hinkley might have suggested policies and practices by the lumber companies contributed heavily to these disasters. Lumber companies were notorious, for example, for quickly felling huge tracts of land and leaving behind the "slash," small branches, bark, and otherwise parts of the tree too small to mill, in huge piles to dry in the sun. These reckless, lazy policies of the lumber companies contributed to the conditions that facilitated many devastating fires [13]. It is ironic that the resulting modern fire suppression policies, by facilitating the growth of thick connected clusters, and suppressing the natural fragmenting processes, may actually mimic the lumber companies' negligence that contributed to their misguided institution in the first place.

Our work suggests that wild fire dynamics are governed strongly by clustering, and that the key to

successful, minimum impact wild fire management lies in finding ways to mitigate the growth of large contiguous clusters of foliage. We hope that policy makers will look to our work and work that follows it to formulate land management policies based on good science.

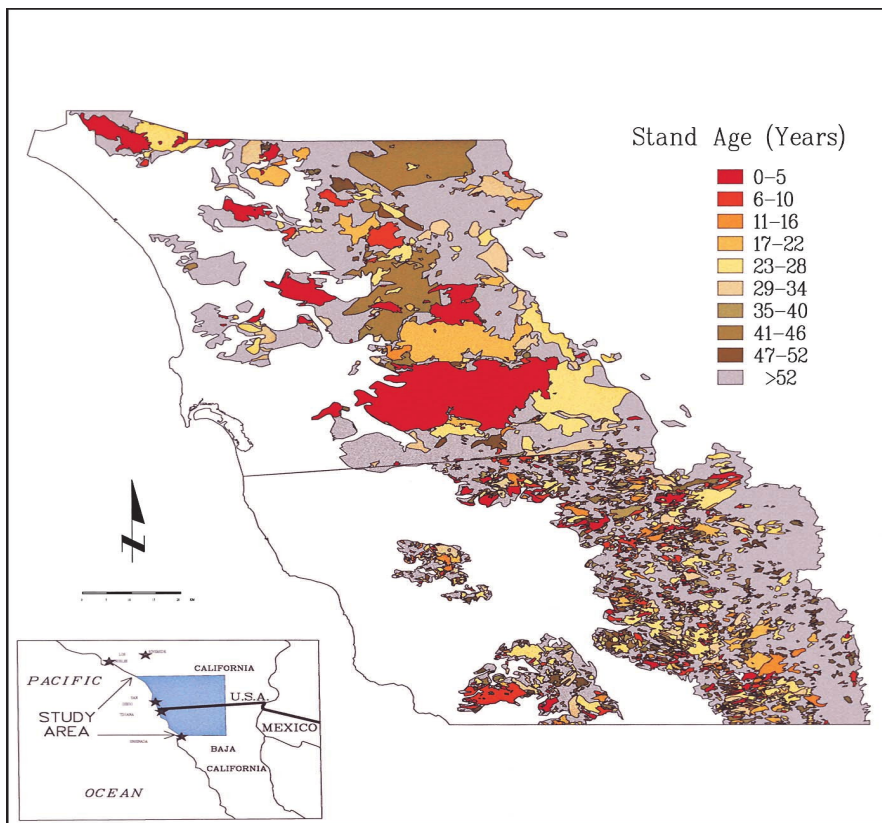


Illustration 10: Chaparral patch mosaic (time since fire) in 1971. Taken from Minnich 2001 [18]

## Bibliography

1. Grassberger, Peter: "Critical behavior of the Drossel-Schwabl forest fire model," **New Journal of Physics** 4 (2002) 12.1-17.5 (<http://www.iop.org/EJ/article/1367-2630/4/1/317/nj2117.html>)
2. Malamud, Bruce; Morein, Gelb; Turcotte, Donald: "Forest Fires: An Example of Self-Organized Critical Behavior," **Science**, 18 September, 1998 pp 1840-1841.
3. Turcotte, Donald L: "**Self Organized Criticality**," **Prog Physics**, 62 (1999)
4. "The Baja-Southern California Fire Model,"  
<http://www.californiachaparral.com/firescience.html>
5. Sole, Ricard; Manrubia, Susanna: "**Are Rainforests Self-organized in a Critical State?**" **Theoretical Biology** (1995) 173, 31-40.
6. Malamud, Bruce; Turcotte, Donald L: "**Cellular-Automata Models Applied to Natural Hazards**," **Computing in Science and Engineering**.
7. Malamud, Bruce; Morein, Gleb, Turcotte, Don L.: "**Log-periodic behavior in a forest-fire model**," **Nonlinear Processes in Geophysics** 12, 575-585, 2005.
8. <http://ejad.best.vwh.net/ujava/fractals/intro.shtml>
9. <http://en.wikipedia.org/wiki/Sierpinski-triangle>
10. Personal Communication: Donald Turcotte, UC Davis Geology
11. Personal Communication: John Rundle, UC Davis Physics
12. Personal Communication: Gleb Morein, UC Davis Physics
13. Brown, Daniel James: **Under a Flaming Sky: The Great Hinkley Firestorm of 1894**, Copyright 2006, The Lyons Press
14. Personal Communication: William Newman, UCLA Planetary Physics, Astronomy, Mathematics
15. Barabasi, Albert-Laszlo; Albert, Reka, "**Emergence of Scaling in Random Networks**," **Science** Vol 286 15 October 1999.
16. Yakovlev, Gleb; Newman, William; Turcotte, Donald; Gabrielov, Andrei, "**An inverse cascade model for self-organized complexity and natural hazards**," **Geophysics Journal Int** (2005) 163, 433-442.
17. Minnich, Richard; Chou, Yue Hong, "**Wildland Fire Patch Dynamics in the Chaparral of Southern California and Northern Baja California**," **International Journal of Wildland Fire** 7(3): 221-248, 1997
18. Minnich, Richard, "**An Integrated Model of Two Fire Regimes**," **Conservation Biology**, Vol 15 No 6, December 2001.
19. Stauffer, Dietrich; Aharony, Amnon: **Introduction to Percolation Theory, 2<sup>nd</sup> Edition**, CRC Press, (c) 1991, 1994.
20. Drossel, B; Schwabl, F.; "**Self-Organized Critical Forest-Fire Model**", **Physical Review Letters**, 14 September 1992.
21. Personal Communication: Raissa D'Souza, UC Davis Department of Mechanical And Aeronautical Engineering