Lorenz Composer

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A Lorenz chaotic attractor is used to generate a sequence of notes, creating various musical melodies. The chaotic nature of the attractor is evident in the resulting music, allowing one to analyze the nature of the chaotic attractor in a new and different way.

Introduction:

The intriguing nature of the Lorenz chaotic attractor is most represented visually. Other manners of representation needed to be explored. The famous Wolfgang Amadeus Mozart would compose melodies using dice and a set of governing rules. With this method, Mozart was able to derive musical melodies from chaos. The intention of the Lorenz Composer is to accomplish this in a similar manner, using the Lorenz attractor as the dice.

Music often is represented by a set of patterns. Repetition often allows the music to be consonant and pleasing. With the Lorenz composer, many of the resulting melodies exhibit random behavior, creating an interesting musical effect.

The Lorenz composer simulates a Lorenz chaotic attractor for a given set of parameters. The x-positions of the resulting trajectory manifested themselves in the form of notes.

The generated melodies were found to exhibit the sweeping chaotic behavior. It was clear when the notes were revolving around fixed points. The melodies would sweep from high to low pitches and vise versa. This back and forth motion was observed mostly when the parameter R, greatly exceeded the Hopf bifurcation value of 24.74. These melodies specifically conveyed the range of possible notes. The chaotic nature of the Lorenz attractor was surely evident.

Background:

     A quick description of the general rules of Western music
will greatly aid in the understanding of the composer. In the
tempered scale of western music there are twelve pitches that
constitute an octave. These pitches are:
     C, C#, D, D#, E, F, F#, G, G#, A, A#, and B.
After B, the sequence repeats itself with the C from the octave
above.
     The tones in music can be understood as sine waves of
varying frequencies. The frequency of a pitch is twice that of
the pitch in the octave below.
     The chromatic scale of music uses all 12 pitches, the major
scale on the other hand, leaves out the sharps (#) in order to
create more pleasing and consonant melodies.
     The Lorenz composer was limited to a use of three octaves
such that the nature of the Lorenz attractor could be more
easily seen.

Dynamical System:

     The Lorenz chaotic attractor was first discovered when Ed
Lorenz wished to analyze atmospheric convection rolls. His
observations boiled down to a set of differential equations:

     $dx/dt = \sigma (y - x)$
     $dy/dt = Rx - y - xz$
     $dz/dt = xy - bz$

with σ, r, and b > 0.

     The most important parameter from the equations would be R,
known as the Rayleigh number. When r > 1, two fixed points
result at   (sqrt(b(r-1)), sqrt(b(r-1)), r-1)
and   (-sqrt(b(r-1)), -sqrt(b(r-1)), r-1).
     Symmetric about the y-axis the stability of the fixed
points change with increasing r.

Methods:

    By using the two non-zero fixed points of the attractor as
a reference point, the x-axis was divided into equally spaced
intervals: 36 for the chromatic scale, 21 for the major scale.
Each interval corresponds to a note. The pitches range from low
to high starting from the negative side of the x-axis to the
positive. The Runge-Kutta method of numerical integration was
utilized. At certain time steps, the position of the trajectory
over the x-axis was translated to a note and appended to the
melody. The melody was then later converted to a sound file.

    A python synthesizer written by Travis Eaves was found and
modified to be used as a basis for the note to sound conversion.
The original code can be found at:
http://mail.python.org/pipermail/python-list/2000-August/049968.html

Results:

    The resulting melodies were found to have the chaotic
nature of the attractor evident in the choice of notes. As R
increased, the sweeping and back and forth nature of the
attractor from low to high pitches could clearly be
distinguished.

Conclusion:

    It is very much possible for a Lorenz chaotic attractor to
be represented in a non-visual way. Musical melodies can be
generated from the attractor, often resulting in pleasing music.

Bibliograhy:

Strogatz, Steven:
    Nonlinear Dynamics and Chaos, Perseus Books Publishing,
        1994