Laser Rate Equations

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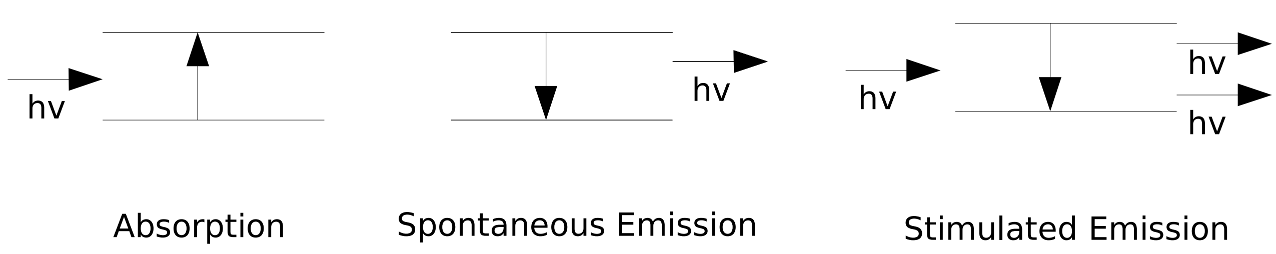
Abstract

The dynamical system that I will investigate involves lasers and their rate equations. The first objective will be to take their rate equations and search for any interesting behavior by classifying their fixed points and graphing out different diagrams. This will be implemented by an interactive GUI that will allow a user to input various conditions, while displaying any desired graphs.

Introduction-

It's a well known fact that Lasers have a wide range of basic and applied applications; which includes everything from Spectroscopy, Microscopy to Blu-ray players. I find this topic especially appealing since it's a real world example of a dynamical system. I also minored in optics as an undergraduate and this project will be an interesting way to explore a familiar topic.

The basic physical concept behind a laser involves some very basic atomic physics. In general, there are three different interactions that occur between a photon and an electron. These interactions include absorption, spontaneous emission, and stimulated emission. These are summarized in the diagram below:



Absorption occurs when an a photon excites an electron from a lower energy state to a higher energy state. Spontaneous emission occurs when an electron sits in a higher energy state and suddenly travels to a lower energy states.   This process unleashes a single photon with an energy equivalent to the difference between the two energy levels. The third interaction (there are others, but these are the important ones) is stimulated emission, which is like a combination between absorption and spontaneous emission. This unique interaction involves an electron sitting in a higher energy state while a photon with the exact transition energy disturbs the system. The system responds by outputting two photons that are identical to the initial photon.   It's really important to note that stimulated emission is the sole physical principle that causes lasing; one photon coming into the atom leads to twice the number of photons with the same frequency coming out. It should be obvious that this process could lead to an avalanche of photons, which is a desired laser property.

Dynamical System-

The equations that describe the rate of change of laser photons is given below

$$\frac{dq}{dt} = Gnq - bq \qquad\qquad \frac{dn}{dt} = -Gnq - fn + p$$

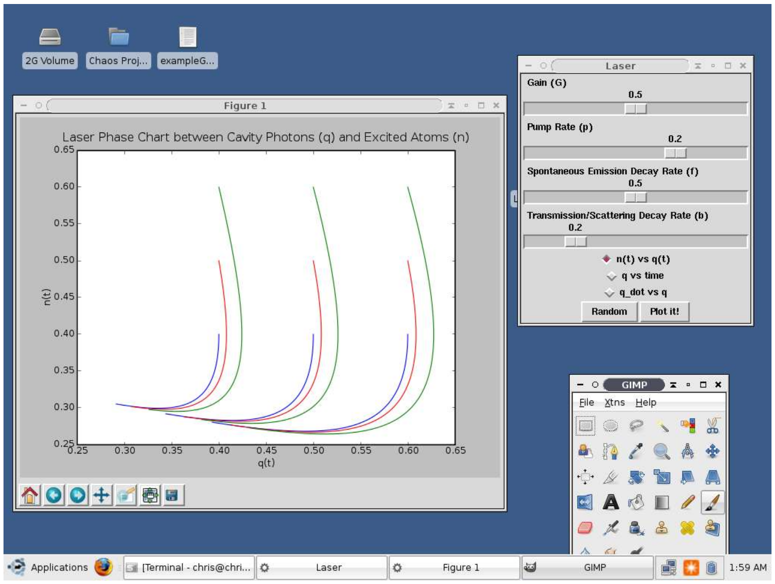
The variables in these equations are as follows: q= photons in the cavity, n=atoms in the excited state, G=gain coefficient, f=decay rate due to stimulated emission, b=decay rate due to transmission/scattering, and p=pump strength. The first rate equation describes the changing of laser photons as a function of time. The first term (Gnq) is the stimulated emission part of the equation. It is proportion to n and q since more excited atoms/cavity photons would obviously increase the rate of stimulated emission. The second expression (bq) is the sole loss term since the only way to lose photons is for them to actually leave the cavity by scattering or transmission. The second equation describes the change of excited photons as a function of time. The only gain term is the pumping term , which increases the number of excited atoms by some external sources. Stimulated emission and spontaneous emission are responsible for the two loss terms in the equation. Spontaneous emissions is a loss term since it a random process that doesn't produce a net increase of photons in the cavity. If it is still difficult to realize the “Gnq” term as the stimulated emission part of these rate equations, let's picture the follow circumstance. Let's imagine the pumping rate and all the loss coefficients are zero:

$$\frac{dq}{dt} = Gnq \qquad \frac{dn}{dt} = -Gnq$$

This set of equations imply that a decrease in the number of excited atoms directly relates to an increase in the number of laser photons (adding photons is the only way for the excited atoms to change in this case) This is exactly the correct physical procedure that is need to describe stimulated emission.

Methods-

In order to investigate this system I created a tool using python and Tk that allows a user to change the parameters in the laser rate equation, while plotting various graphs depending on those parameters. The screenshot of this tool is given below:



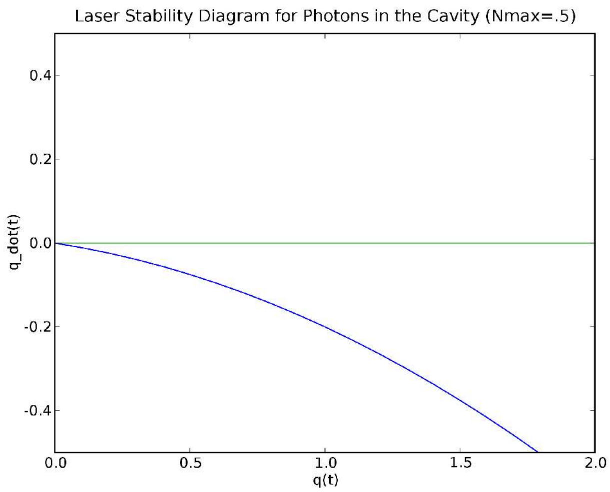
This tool allows a user to alter the parameters in the laser rate equations, while plotting three unique graphs. These Graphs are: 1) The number of excited photons versus the number of photons in the cavity (n vs q) 2) The number of photons versus time (q vs t) 3) The change in the cavity photons versus the the number of cavity photons (q_dot vs q).

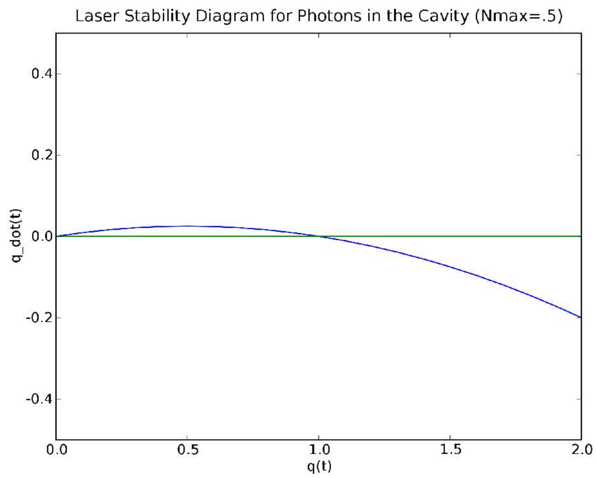
Results-

The graph of the change in cavity photons versus the the number of cavity photons (q_dot vs q) allows a user to look at the stability of the system. When q_dot is set to zero in the first equation and solve for n, the following condition arises:

$$\frac{dq}{dt}=Gnq-bq \quad \text{-->} \quad 0=Gnq-bq \quad \text{-->} \quad n=\frac{b}{G}$$

This implies that if the value of G is set to one and value of n is set to one half, there should be some sort of transition when the value of b is adjusted. The diagrams below show two such instances when b=.4 (the first graph) and b=.6 (the second graph).





From these two plots it's easy to point out the such a transition. With the first graph, the curve only crosses the q-axis at zero, while the other graph crosses twice. This implies that if the value of “b” over “G” is greater than “n”, then we have a stable situation in the laser, where lasing occurs.

In order to better observe the exact nature of this stable situation, let's plot the number of laser photons versus time (q vs t). Like the previous case, it's useful to take the other rate equation and set n_dot equal to zero.

$$\frac{dn}{dt}=-Gnq-fn+p \quad \text{-->} \quad 0=-Gnq-fn+p \quad \text{-->} \quad q=\frac{p}{b}-\frac{f}{G}$$

In order to focus of the stable point, let's set q=0 and solve for a pumping term. The pumping parameter will have a subscript “t” which will stand for a threshold value.

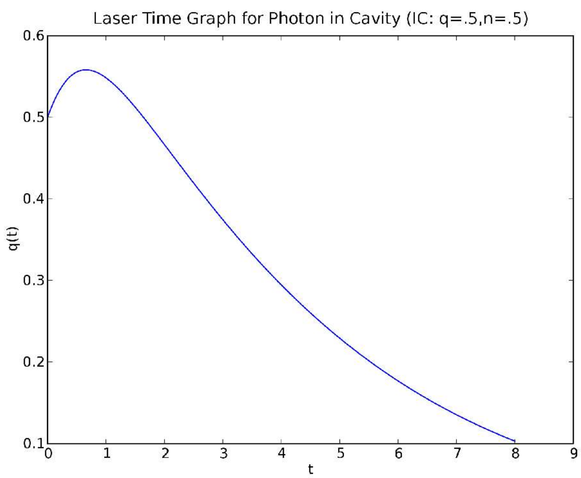
$$q=\frac{p}{b}-\frac{f}{G} \quad \text{-->} \quad 0=\frac{p}{b}-\frac{f}{G} \quad \text{-->} \quad p_t=\frac{f*b}{G}$$

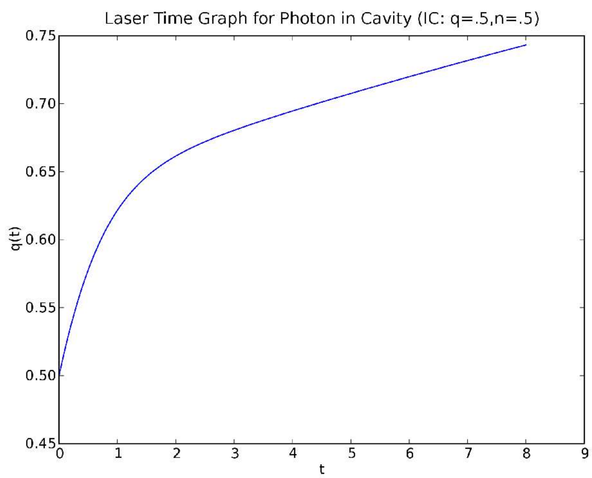
Therefore, it would be helpful to set up a situation where the pumping threshold is higher and lower then this threshold term and watch for any consequences. The two graphs below set up a scenario where the first graph has

$$p < \frac{f * b}{G}$$

while the condition on the second graph is:

$$p > \frac{f * b}{G}$$





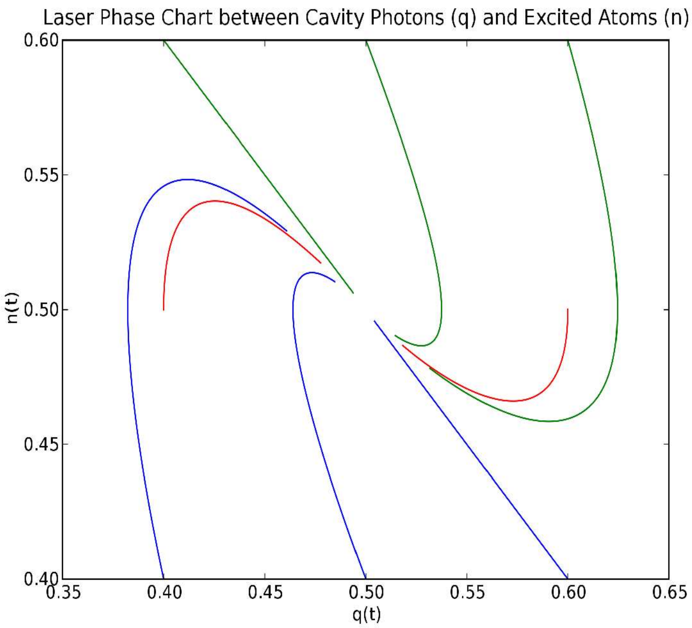
These two plots show the explicit nature of this pumping threshold. If the pumping rate is less than then the pumping threshold, then the number of laser photons in the cavity will eventually go to zero. In other words, the pumping from the ground state to the excited state cannot keep up with the all the losses. On the other hand, if the pumping rate is greater than the threshold rate, then there will a increasing number of photons over time.

It's also interesting to note that if we call

$$n_t = \frac{b}{G} \quad \text{-->} \quad p_t = f * n_t$$

This reiterates the fact the there is only a steady state situation if the pumping rate equals the stimulated emission decay rate of the net total of atoms at threshold.

The final graph my tool plots is the phase portrait for a given combination of parameters. These graphs plot the number of atoms in the excited state (n) versus the number laser photons in the cavity (q) and they are useful to pinpoint a few interesting characteristics in the laser system when it is accompanied by the other graphs. For example, the plot below show the existence of a stable point at .5 and an eigen-direction along the diagonal.



Conclusion-

This project was a success in terms a taking a familiar dynamical system and building a tool that confirms the well known results from classical laser theory. From this tool, I was able to determine the pumping threshold rate and graphically show various situations when a user moves around this threshold. Nevertheless, my main objective was to build a GUI that allows a user to change the various laser parameters, while plotting the resulting diagrams and that has been accomplished.

Bibliography-

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