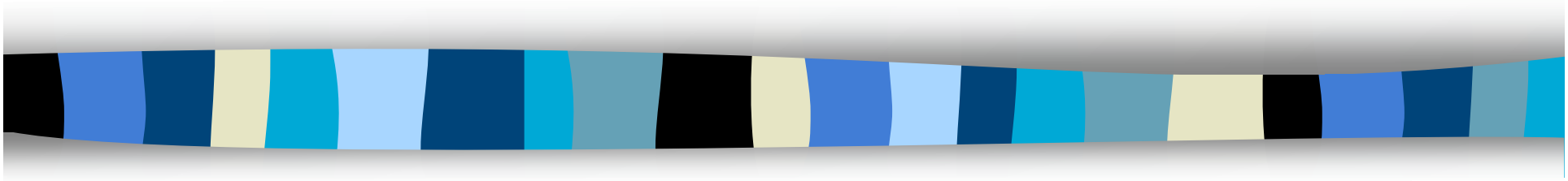
Bouncing Balls



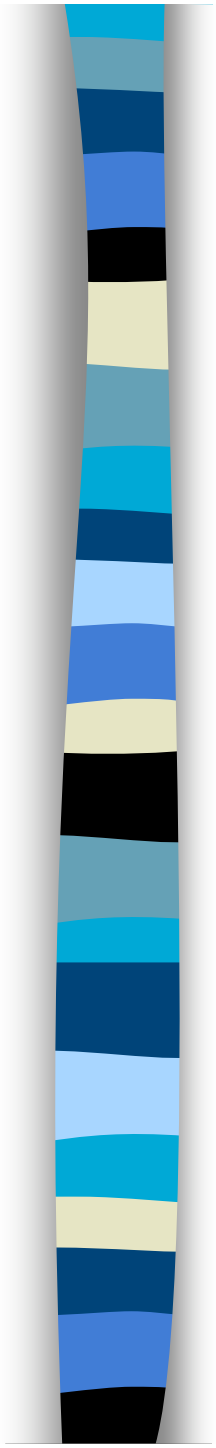
Alex Jing Wei Huang

Goals of this Project

- Analyze the ball collision rates under different conditions (parameters).
  - Different container shapes).
  - Different numbers of balls,
  - Different initial velocity ranges.
- Analyze the velocity distribution of these balls.

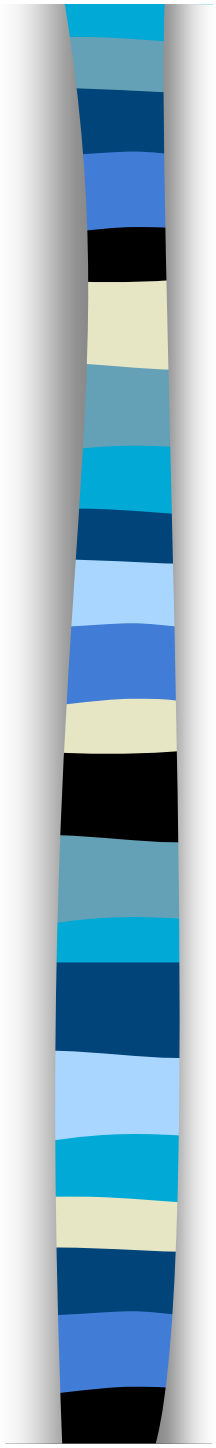
State Space of the System

- State space: 4 dimension.
  - Position (x and y coordinates)
  - Velocity (x and y directions)
- Other parameters of the system
  - Shape of the container
  - Radius of balls



Simulation Tools

- Python
  - Pygame pakage
  - Visual pakage



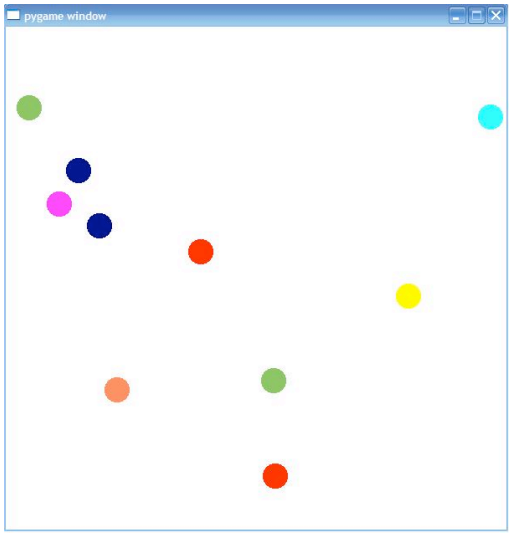
Properties of the System

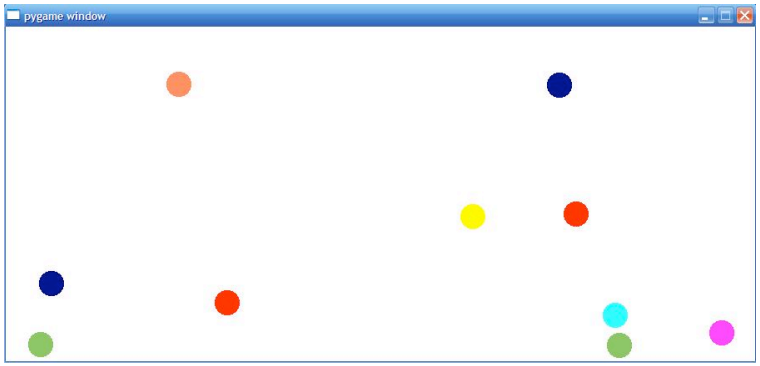
- Balls are moving in 2D space.
- Balls have random initial velocities.
- All balls have same radius and mass.
- Balls can collide with walls and other balls.
- Energy is conserved during all time.

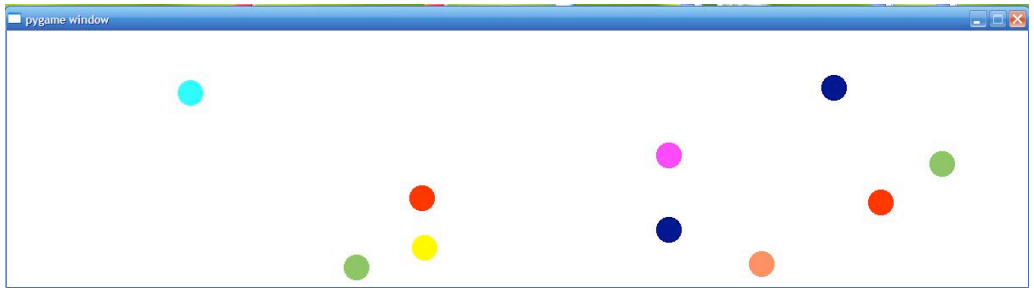
Experiment 1: The collision rates with respect to different container shapes

- 10 balls (with random initial velocities between –5 to 5) are put in three containers with same area and different shape.
  - 600 x 600 (pixel x pixel)
  - 400 x 900
  - 300 x 1200
- We calculate the number of collisions after moving for a day ($24*60^2$ seconds).
  - Note by “second”, I mean iteration.

Experiment 1 (cont.)







Experiment 1 (cont.)

- 600 x 600 square window

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	8373	8565	7114	7471	5755	**7455,6**
Ball to ball	9673	6905	7186	5357	4387	**6721.6**
Total						**14177.2**

Experiment 1 (cont.)

- 900 x 400 rectangular window

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	9032	7903	7539	9442	6929	**8169**
Ball to ball	6543	5165	8268	6768	4457	**6240.2**
Total						**14409.2**

Experiment 1 (cont.)

- 1200 x 1200 rectangular window

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	10409	10688	9323	9696	10400	**10103.2**
Ball to ball	5840	6205	4710	6264	5914	**5786.6**
Total						**15889.8**

Experiment 1 (cont.)

- Summary of data

	600 x 600	900 x 400	1200 x 300
Ball to wall	7455,6	8169	10103.2
Ball to ball	6721.6	6240.2	5786.6
Total	14177.2	14409.2	15889.8

Experiment 1 (cont.)

- Conclusion
  - We can minimize the number of ball-to-wall collisions by putting them in a square container.

Experiment 2: Collision rates with respect to different numbers of balls

- Window size: 600 x 600 .
- Number of balls
  - 10
  - 15
  - 20

Experiment 2 (cont.)

- 10 balls (same data from Experiment 1)

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	8373	8565	7114	7471	5755	**7455,6**
Ball to ball	9673	6905	7186	5357	4387	**6721.6**
Total						**14177.2**

Experiment 2 (cont.)

- 15 balls

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	10709	11188	12681	11876	11753	**11641.4**
Ball to ball	13004	13756	14361	13929	13834	**13776.8**
Total						**25418.2**

Experiment 2 (cont.)

- 20 balls

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	15023	17600	15865	18095	13209	**15958.4**
Ball to ball	22627	29090	25182	29167	22140	**25641.2**
Total						**41599.6**

Experiment 2 (cont.)

- Summary of data

	10 balls	15 balls	20 balls
Ball to wall	7455,6	11641.4	15958.4
Ball to ball	6721.6	13776.8	25641.2
Total	14177.2	25418.2	41599.6

- Conclusion
  - Both ball-to-wall and ball-to-ball collisions increase as the number of balls increases.

Experiment 3: Collision rates with respect to different initial velocity ranges

- Window size: 600 x 600 .
- Number of balls : 10
- Initial velocity ranges
  - [-5, 5] in each x and y direction
  - [-10, 10]
  - [-15, 15]

Experiment 3 (cont.)

- Initial velocity range : [-5, 5]

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	8373	8565	7114	7471	5755	**7455,6**
Ball to ball	9673	6905	7186	5357	4387	**6721.6**
Total						**14177.2**

Experiment 3 (cont.)

- Initial velocity range : [-10, 10]

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	12593	13801	13514	14209	16859	**14195.2**
Ball to ball	11418	11176	10218	11809	12878	**11499.8**
Total						**25695.0**

Experiment 3 (cont.)

- Initial velocity range : [-15, 15]

	Data 1	Data 2	Data 3	Data 4	Data 5	Average
Ball to wall	17701	23766	21139	17312	22281	**20439.8**
Ball to ball	13301	17173	14962	13830	16048	**15062.8**
Total						**35502.6**

Experiment 3 (cont.)

- Summary of data

	[-5, 5]	[-10, 10]	[-15, 15]
Ball to wall	7455,6	14195.2	20439.8
Ball to ball	6721.6	11499.8	15062.8
Total	14177.2	25695.0	35502.6

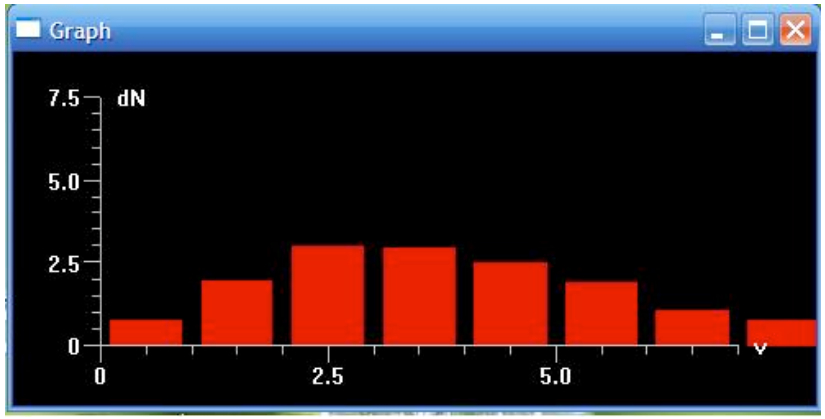
- Conclusion
  - Both ball-to-wall and ball-to-ball collisions increase as velocity range increases.

Velocity Distribution

- Balls' velocities are changing during the experiment (due to collisions with each other).
- We analyze the velocity change by plotting the histogram of these balls' velocities at each time step.

Velocity Distribution (cont.)

- The simulation verifies that, after certain period of time, the balls' velocities will follow the Boltzmann distribution.



- For proof, please look up Wikipedia

Future Exploration

- How the collision rate is affected
  - if the balls are moving in 3D?
  - if the container is a triangle or a circle?
  - if balls have different radius and mass?