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A Traffic Analysis Using a Two Dimensional Cellular Automata Model

Some reasons for studying traffic and its effects on our population come from the purpose of finding a solution to the global problem that we currently face. With the demand for gasoline rising and the prices of a barrel of oil also rising we are at a point where the planners have a crucial role in figuring out how to reallocate traffic demand and orient people towards alternative modes in order to create more sustainable transportation networks.

In this paper I will present a cellular automata traffic model that was developed as a simplistic traffic model that has been contemplated by mathematicians and analysts for years. The Biham-Middleton-Levine (BML) has been around for quite some time now and has been referenced and used hundreds of times. I will propose an analysis of the BML model involving critical points and jam patterns on the lattice.

The BML model works as described: There are blue and red cars in the network. The blue cars travel south while the red cars travel east. In two time steps the cars will all update once, therefore each car gets updated at every other time step. The cars advance if there is nobody in the way, and they stay put if there is somebody in the place where the car desires to move.

The geometry of the system that we are using in this model is one of the parameters of the problem. We can evaluate different sized lattices based upon what kind of network we wish to look at and what type of network we would like to analyze.

Questions at Stake:

- What is the average velocity of the cars with respect to time?
- What are the phase space variables?
- When do the systems reach a limit cycle?
- Predictive power for what state space our system will reach?

Methods:

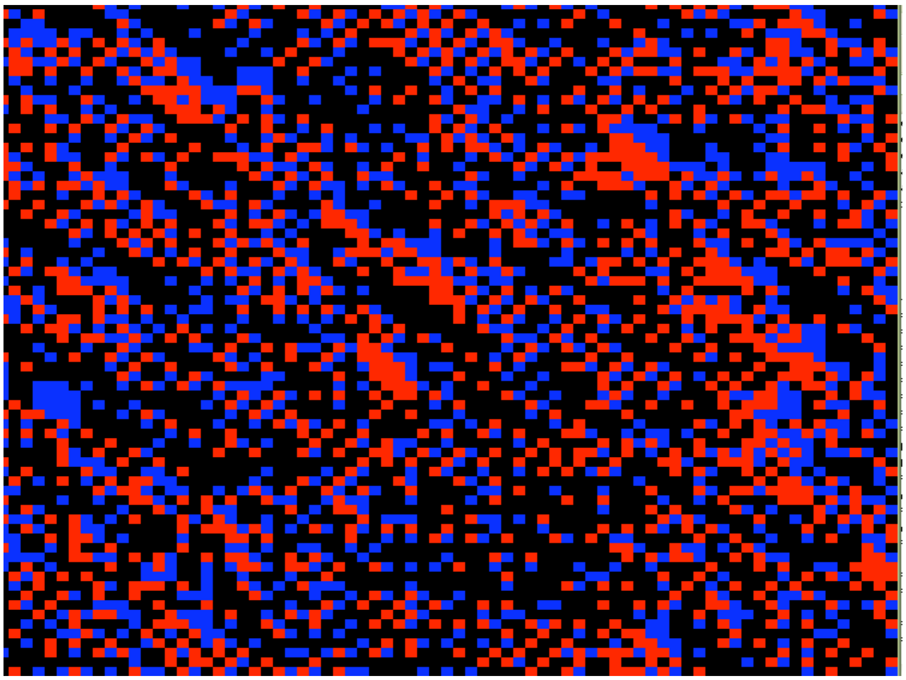
The methods used to analyze the traffic network on a lattice include the BML CA model described and an analysis of the difference of various sized lattices. First we wish to exam the characteristics of the network with random initial conditions. From these conditions we find that there are a variety of states that the network can enter and has sharp phase transitions. The square lattice seems to average at a linear fixed speed after a given set of iterations, while the Fibonacci lattice seems to vary even given similar initial conditions.

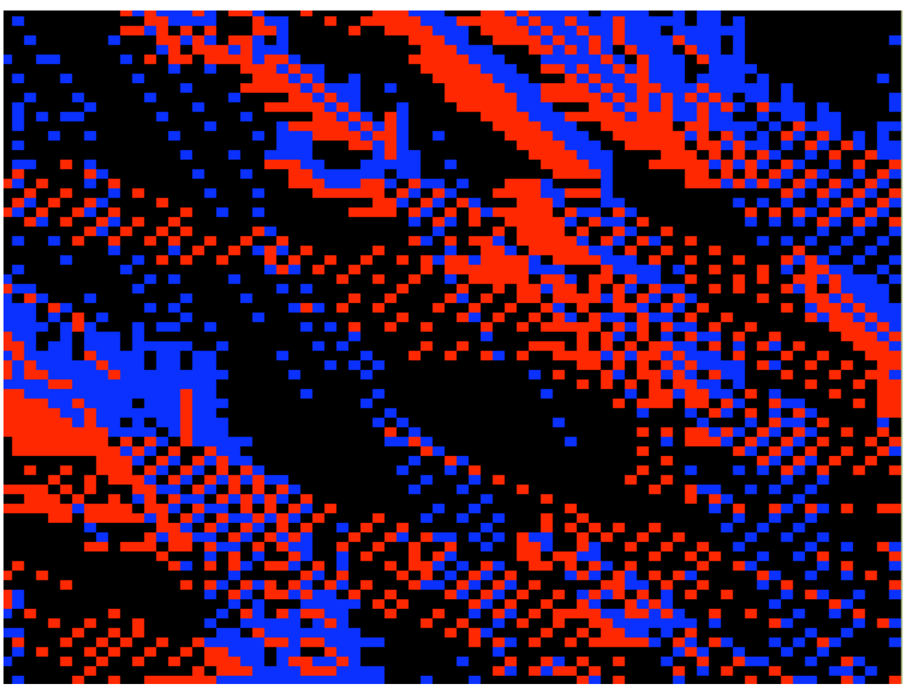
Observations:

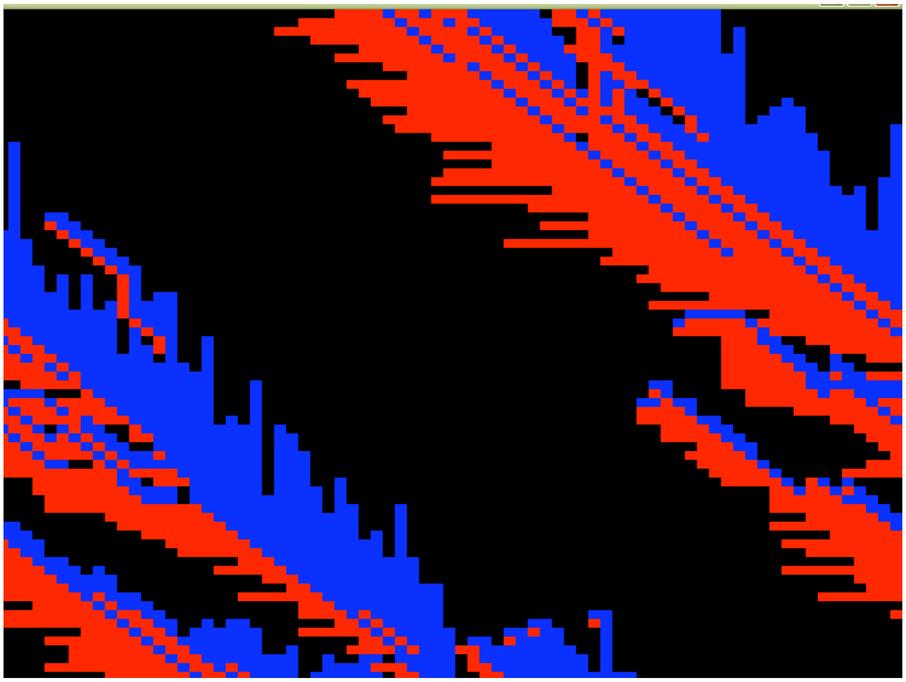
Characteristics of this network that were observed include the phase transition time, differences between square and Fibonacci lattices, differences in similar but slightly perturbed initial states, and differences between different random lattices of the same dimension.

Let us take a look at how the simulation of the BML model runs in the python module.

The three images that proceed are those of a square lattice of size 70x70 with a density p=0.44.

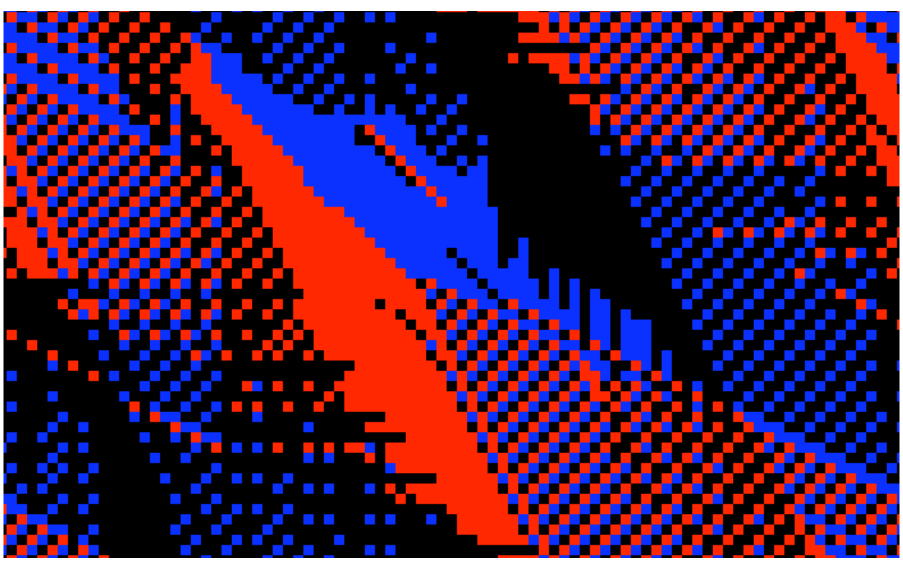


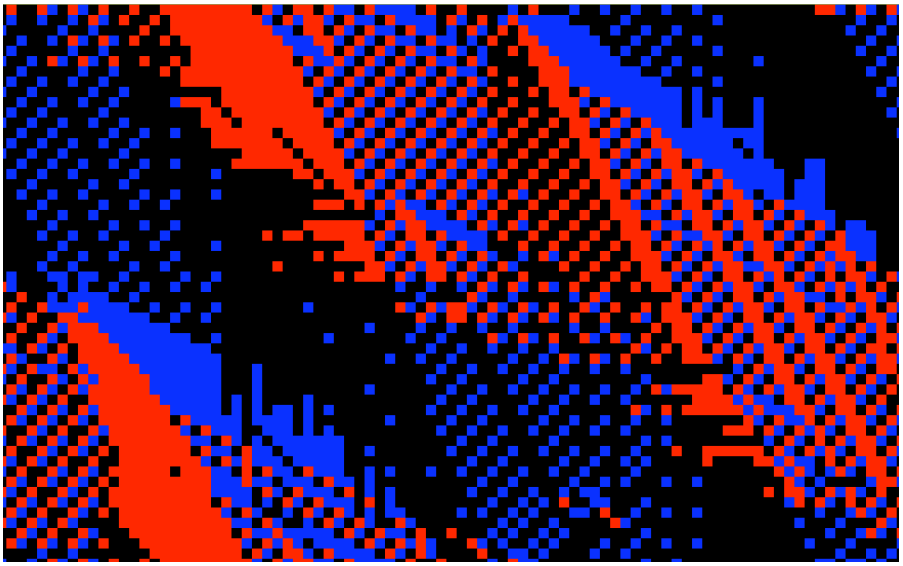




This lattice in the final state has jammed in a state of multiple bands.

The Fibonacci lattice over time looks like this:

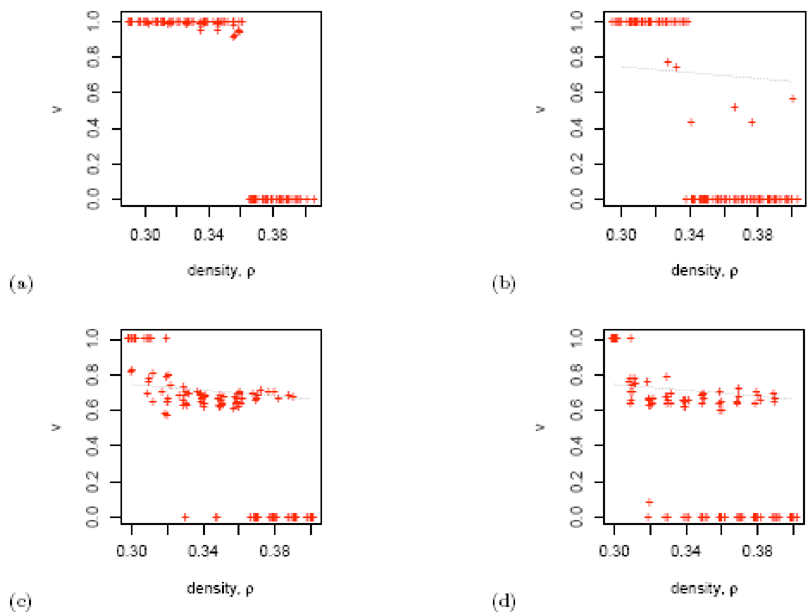




The Fibonacci lattice has interesting characteristics and develops in a unique manner. Given the same initial conditions we notice different characteristics of the Fibonacci lattice. The lattice has sharp phase transitions and converges upon one of two formations, although we are not exactly sure why it goes to either one or the other.

Analysis:

In the process of evaluating the BML model and its implications there were several results that came about. The model yields a phase transition from a free flow traffic network to a jammed traffic network. In between there are various states of fluctuation between free flow and jammed states. We notice this phase transition in the following graphs. The progression from top left to bottom right shows the phase transitions as they occur in a square lattice in increasing sizes. The velocities reported are those that are found as an average of the system after the traffic has converged upon its final state of free flow, fluctuating, or jammed state.



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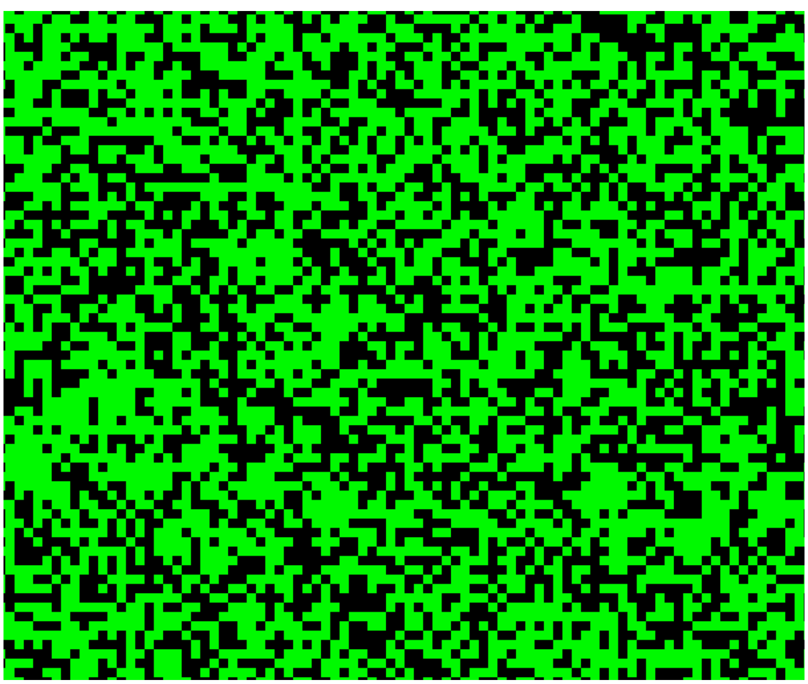
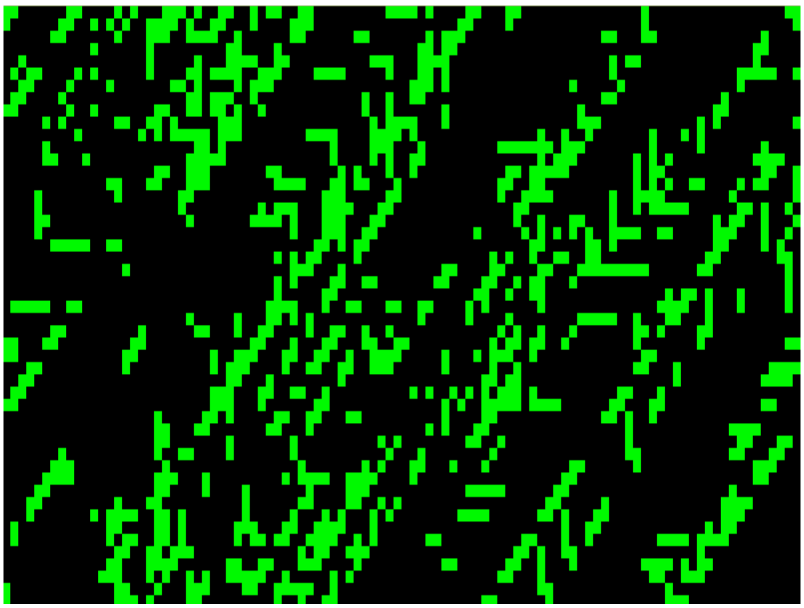
While analyzing why certain things happen the way they do we find value in looking at the difference diagram of the same density and sized lattice, but with different randomly generated initial states. Here is a snapshot of the difference diagram:



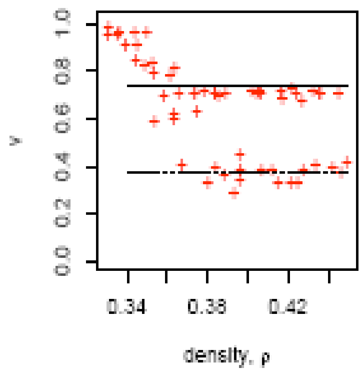
As the two systems iterate, we start to see patterns emerge in the difference diagrams. It appears as though the initial random states being completely different end up converging on a similar fixed state.

The next diagram shows the difference of two lattices that start out at almost the same initial states with only one perturbed cell:



The diagram above shows the bands that emerge after the differences of the two diagrams seems to converge. The initial lattices only differed by one cell, but after the systems reach their fixed states they have multiple bands of different cells. The concept of a Lyapunov exponent is illustrated here as we start with a small perturbation and then observe the difference diagram after a period of time.

We notice about the Fibonacci lattice that it seems to vary in its convergence upon an average velocity given the same initial densities. We can see in the following graph that given random initial densities of p there is a bifurcation in the sense that some lattices seem to converge upon a final average velocity around .7 where some go to .4. This bifurcation is on that has only been observed in the Fibonacci lattice.



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Solutions to the traffic problem:

The analysis of the BML model gives us some insight in ways to organize traffic on a grid. When we evaluate the patterns of jams/free flow states we can understand how to organize traffic in a manner that leads to the highest efficiency on a city grid. Here is an idea from Philip Ball:

> One of the major triggers of jams in dense traffic is fluctuations. Drivers lose concentration, get too close to the vehicle in front, and then slow abruptly. If random perturbations like this could be smoothed out, the threat of jams would be much reduced. One way would be to design traffic regulations which have the effect of forcing drivers to pay more attention in the areas containing potentially jam-forming obstructions such as bottlenecks. Another possibility would be to implement traffic control measures at on rams that adapt to changes in traffic density.[iii]

With the BML model at stake, what we can determine as being useful from this model is the importance of timing on the system of traffic lights. With the right timing on city grids we can organize our traffic networks in these free flowing bands (theoretically). There will always be some kind of deviation from the theoretical model, but it is from these models that we can use to determine how to better organize current and plan for future traffic networks.

References:

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[i] D'Souza, Raissa. BML revisited: Statistical physics, computer simulation and probability.
[ii] D'Souza, Raissa. BML revisited: Statistical physics, computer simulation and probability
[iii] Ball, Philip. Critical Mass: How One Thing Leads to Another.