Exploration of Fractal Dynamics

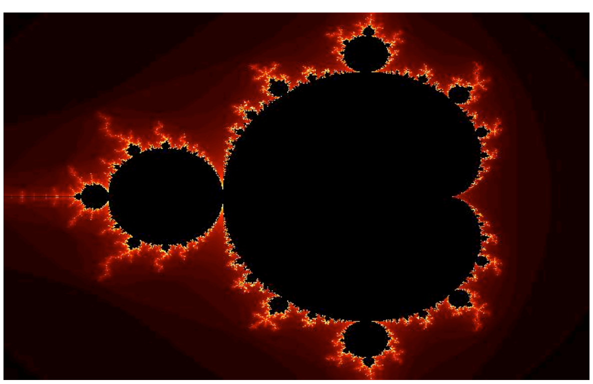
Ian Nickles
Dept. of Applied Science
ian.nickles@gmail.com

Abstract: The Mandelbrot and Julia sets can be said to be the most common fractals. While there exist plenty of fractal browsers that let us explore the extreme details and intricacies of these by being able to zoom in on desired regions, more of an understanding can be obtained about these fractals with the tool described here that explores the dynamics of the fractal parameters. This tool is not capable of the zoom feature, but allows the user to adjust the parameters of the Mandelbrot and Julia sets in attempt to get a better understanding of the dynamics of these fractals.

My first exposure to chaos was the Mandelbrot set, from which I was immediately intrigued. The idea of something having infinite complexity coming from such a simple looking equation was strange to me. At the time I hadn't even known calculus, so the idea of infinity itself was an adventure to think about. Even having a better understanding of infinity as I do today, fractals and self symmetry are still of great interest. I've fiddled with a number of fractal browsers, zooming in on different locations of numerous fractals hoping that if I keep zooming maybe I'll start to understand why there is self symmetry in these. I don't think this approach will get me very far in my quest for understanding, but the mystery is a persistent driving force to keep looking.

I have considered a new way to try to better understand these fractals. This is a demonstration of a new technique, which will hopefully lead to a more complete program, one that will allow the user explore the dynamics of fractals and experience first hand how the geometry of fractals is affected as parameters are changed. There exist programs that explore different sets with different parameters, but none I have seen that can be used to in real time witness the effects of slight parameter changes. As I was hoping to learn more about fractals with this project, I do not expect to. Sometimes seeing things from a different angle gives new insight, but this is not always the case. If anything we have a program and method for exploring these things, which can be fun just to look around. Might help with making fractal art.

The design of the project to to compute a fractal for every set of parameters across a set of values. The result is a large table of images, that the user sees one at a time, each image corresponding to the value set they have control of. The idea is that all fractals will be pre-generated with a fine enough parameter resolution to not miss out on too much of what is going on "between the lines". Unless on some supercomputer or cluster, regenerating each fractal for a change of parameters could take too long for the user to make any sort of real time comparison between two close sets of values. The design for the program will be expensive on disk space. The required disk space is proportional to the parameter resolution squared because there are two parameters (this is only for the model demonstrated, more elaborate ones may be conceived). The parameters for this demonstration are the real and complex part of the initial condition, for the Mandelbrot case, and the constant term in the Julia set case. If we had only one parameter value to tweak, a simple movie of stepping through this value should suffice for a user to see the dynamics, but a movie with two parameters does not the the user fully explore the system. A couple movies have been provided, stepping through small changes in parameter values to give the user a sample of what the final product could provide, a smooth continuum of images.

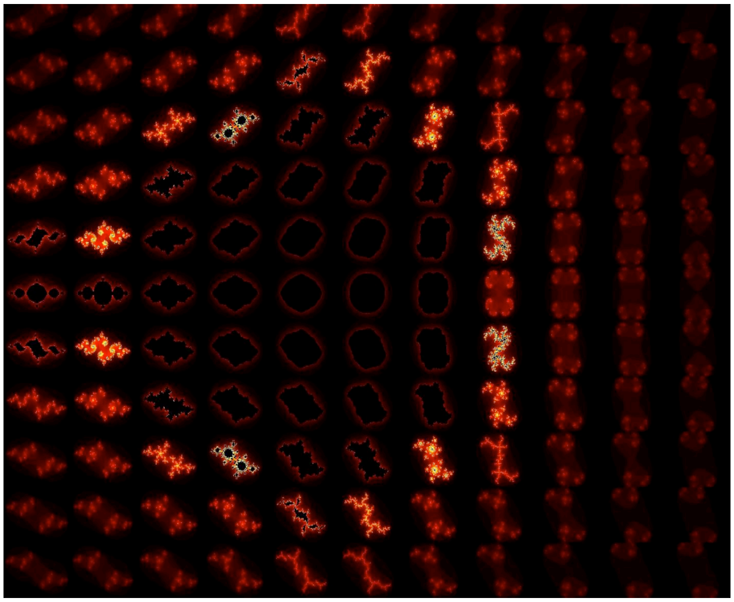


The Mandelbrot set was the first computer generated fractal. It comes from an iterative process

$$z_{n+1} = z_n^{\ 2} + c$$

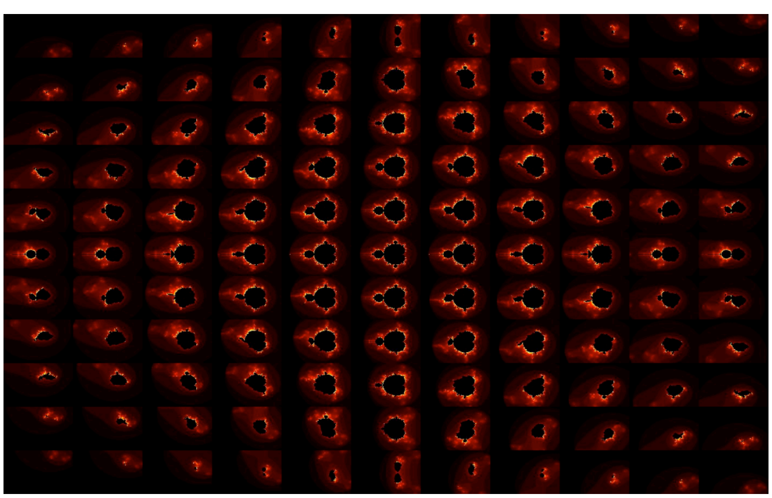
with initial condition $z_0 = 0 + 0j$.

Fig 1. is a rendering of the original Mandelbrot set. The colors correspond to how long that initial condition has survived in our attractor before we consider on a road to divergence. The brighter the point, the longer it has laster. The exception is the solid black figure in the middle. These points never left within our window. We color this black, the same color as those points that diverge very quickly only because it makes for a prettier picture.



The Julia set uses the same function of iteration, but keeps a constant value for c, and steps through different z_0 to generate the image. The figure above is a sample of what the table of images would comprised of. The images were organized in a such that the center of the image corresponds to the coordinates [0,0] and the x and y values range from [-1, 1]. Taking a step back one might notice how the brighter images form the main shape of the Mandelbrot set. There seems to be more activity for the Julia set on coordinates where there is activity on the Mandelbrot set. Even though this project wasn't to be laid out in way to see all images at once, it was already provided new insight into how these fractals behave. If we had taken a finer resolution between parameter space, which in this case it was $\Delta c = 0.2$ in both real and complex parts, we would perhaps see a shape more similar to the Mandelbrot set.

Due to time constraints, this program could not be completed and so we can not see first hand.



The figure to the left is the same operation as the one before performed on the Mandelbrot set. There does not seem to be any correlation between this map and a specific Julia set. The colors seem to have pyramid like slope figure to them, having four lines criss cross of decreasing slope, with a more steep slope in between.

I consider this project a success. I was able to create what I had envisioned, although not in full, I am still satisfied with the results. I find it very interesting to see how the geometry of these change. Others might find a program like this as interesting as the fractal browsers that are popular today, if even just for the sake of observing how nature alone can produce such beauty. On top of this, I personally now understand that the Julia is definitely much more interesting around the bright areas of the Mandelbrot set. This was not intuitive for myself.

Works Cited:

Mandelbrot set, http://en.wikipedia.org/wiki/Mandelbrot_set

History of the mandelbrot and julia fractals, http://www.icd.com/tsd/fractals/beginner1.htm

A Note on the History of Fractals, http://home.att.net/~fractalia/history.htm