The Big, Big Picture

Reading for this lecture:

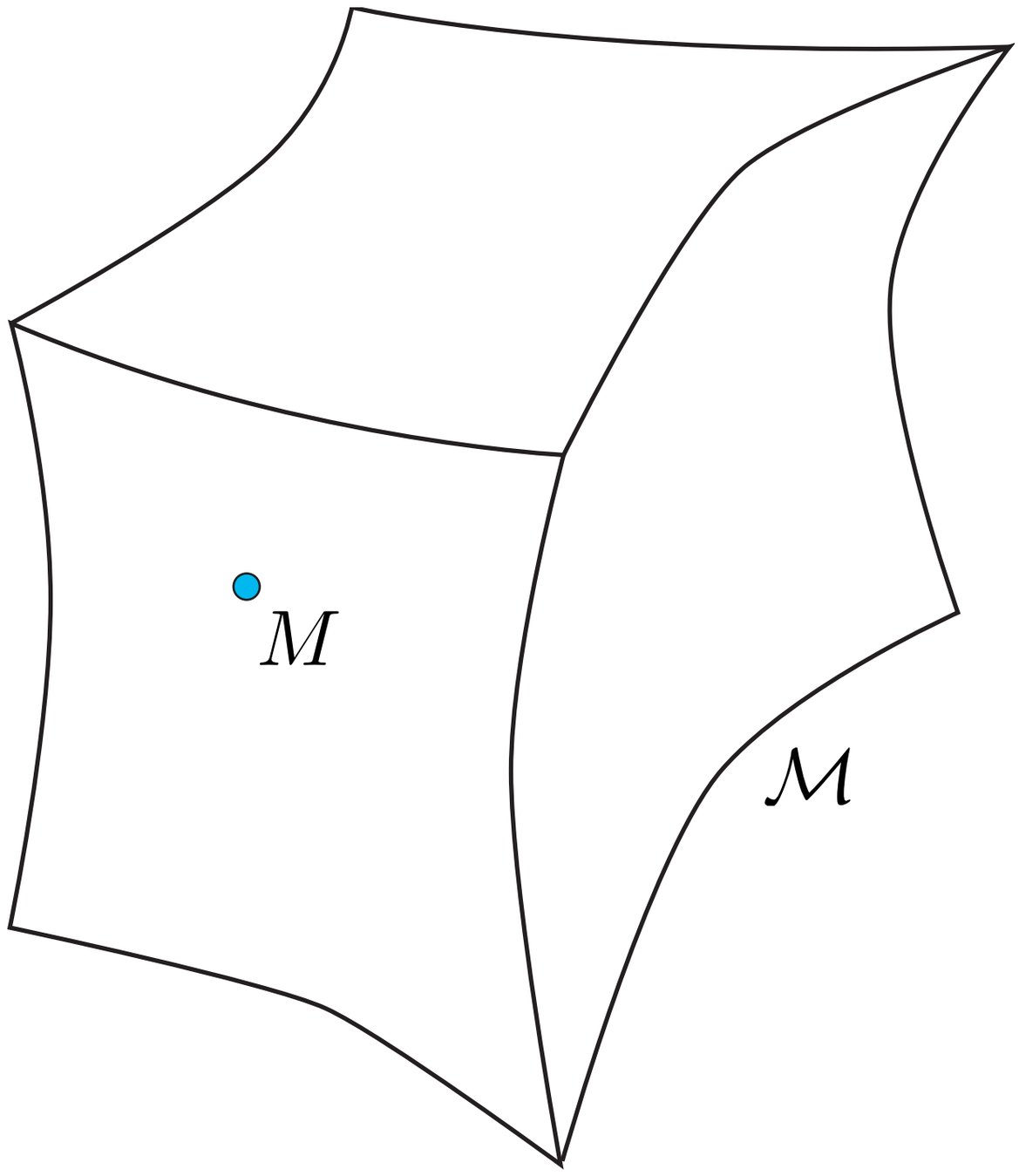
NDAC, Chapter 3

Projects!

The Big, Big Picture (Bifurcations I) ...

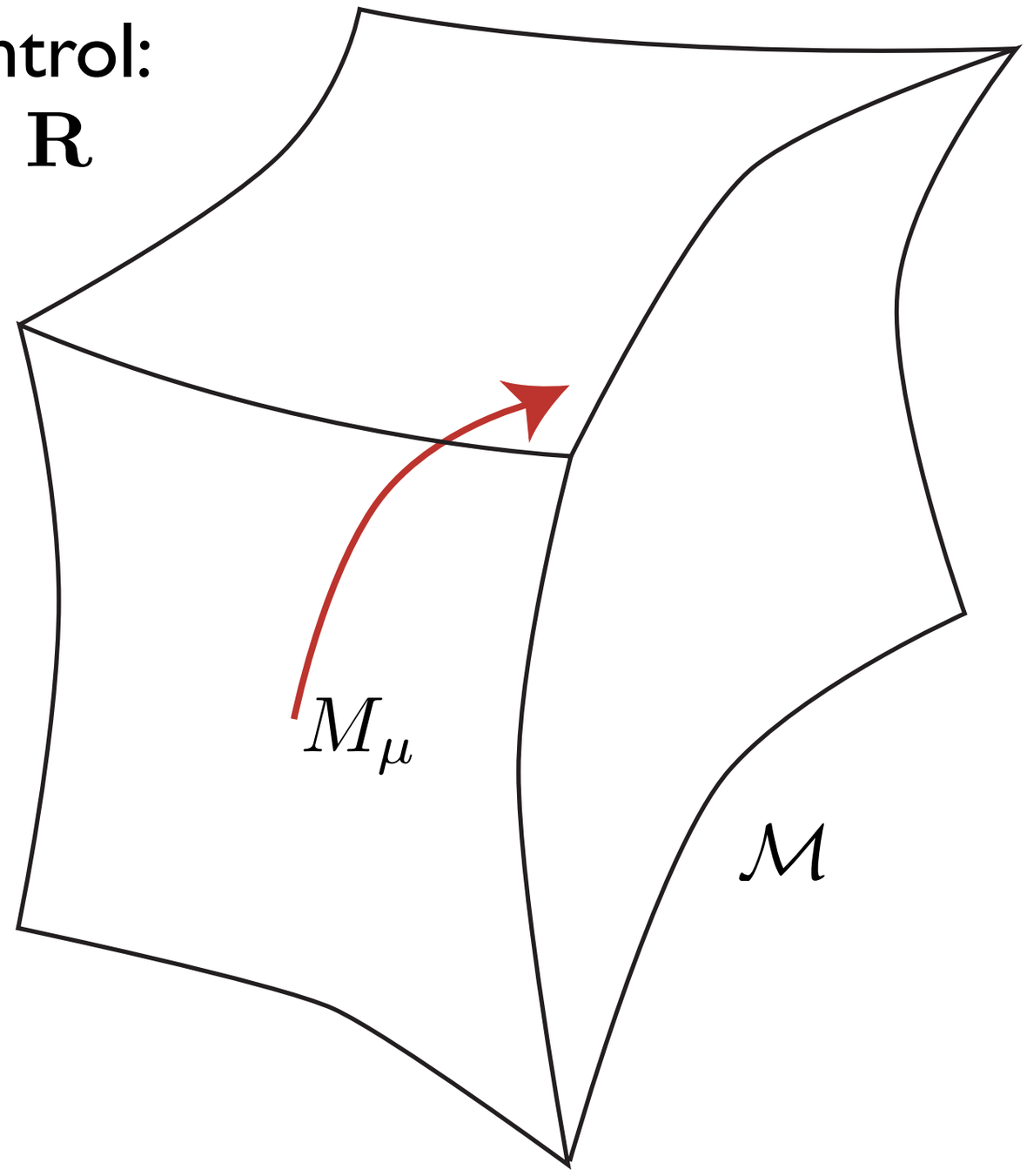
Space of all dynamical systems: $\mathcal{M}$

a particular dynamical system: $M \in \mathcal{M}$



The Big, Big Picture (Bifurcations I) ...

Model of experimental control:
an arc in $\mathcal{M}$: $M_\mu,\ \mu \in \mathbf{R}$



What happens as μ is varied?
The arena of Bifurcation Theory

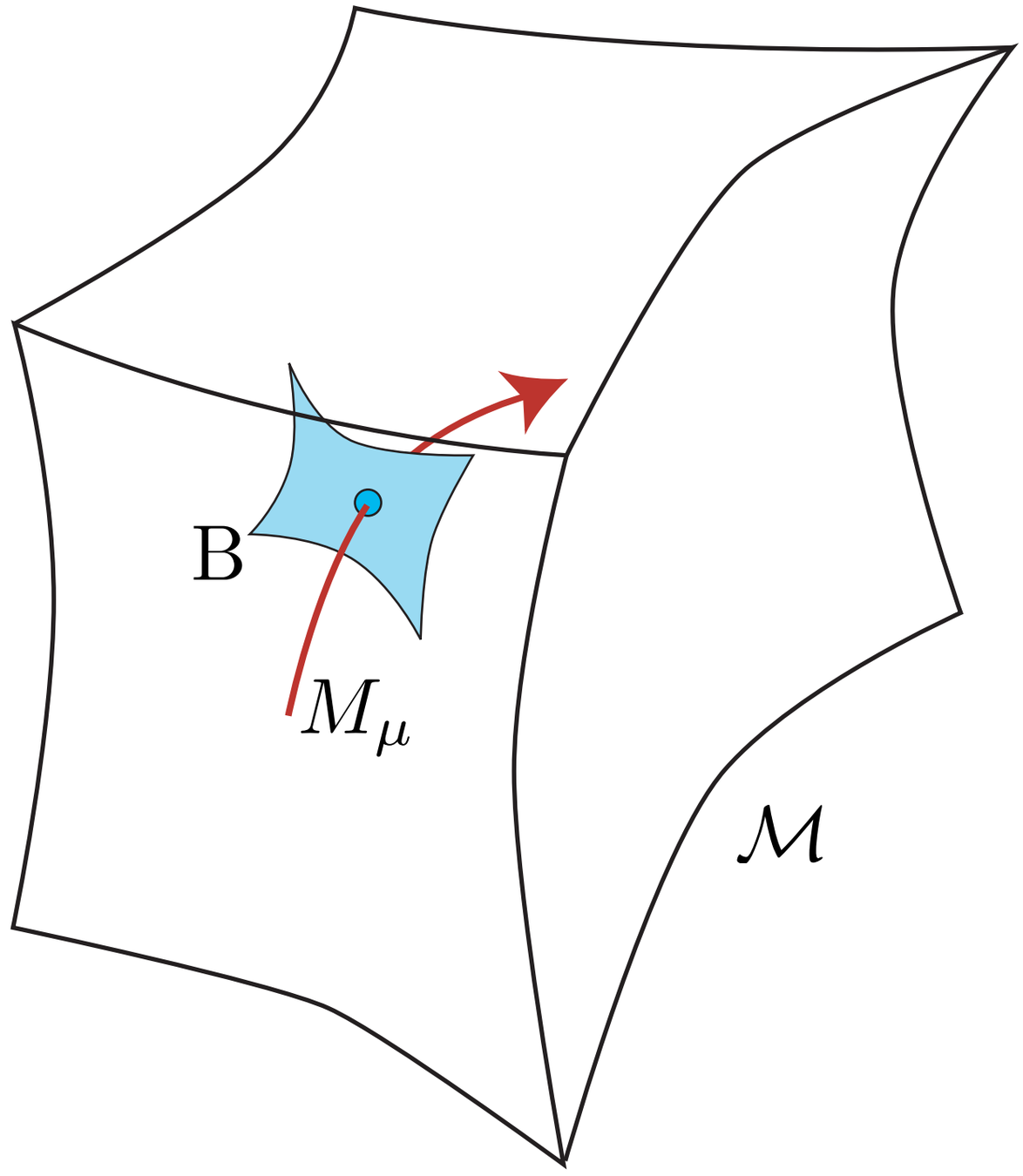
The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory:

Bifurcation: Qualitative change in behavior
as parameter is (slowly) varied

Bifurcation surface: B

"Change"
= Topologically
distinct behaviors

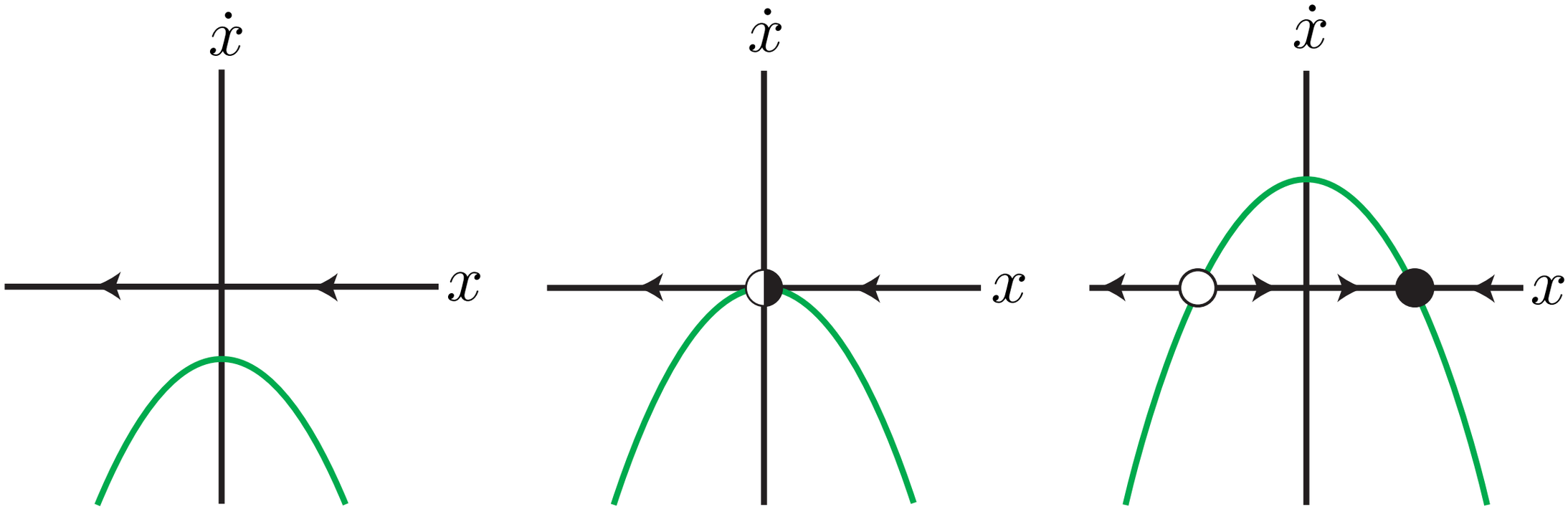


The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of 1D Flows:

Saddle node bifurcation: Blue-Sky Catastrophe

Normal Form: $\dot{x} = r - x^2$



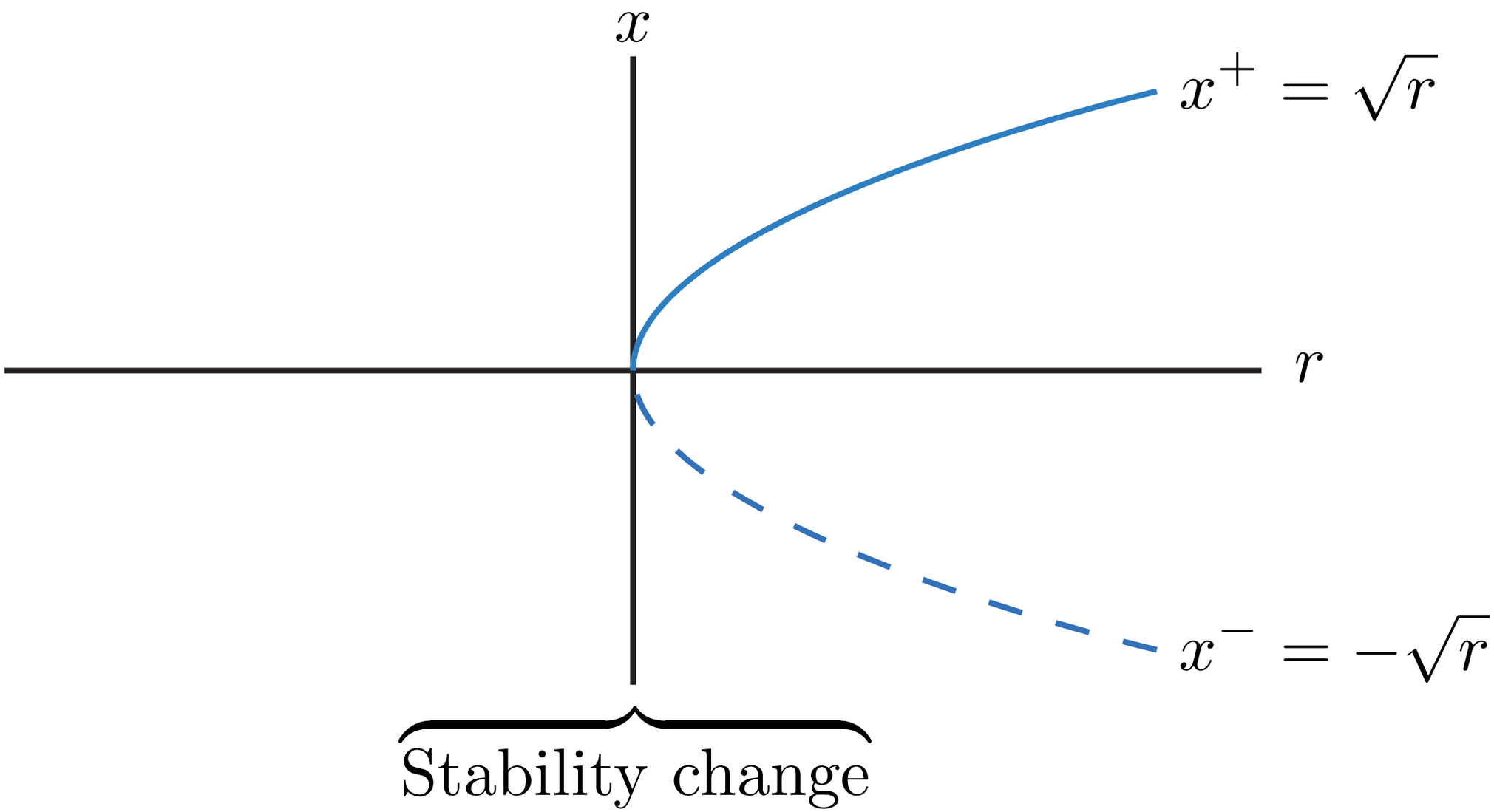
The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of 1D Flows ...

Bifurcation diagram: Attractor(s) versus parameter

Saddle node bifurcation diagram:

Fixed point(s) versus parameter

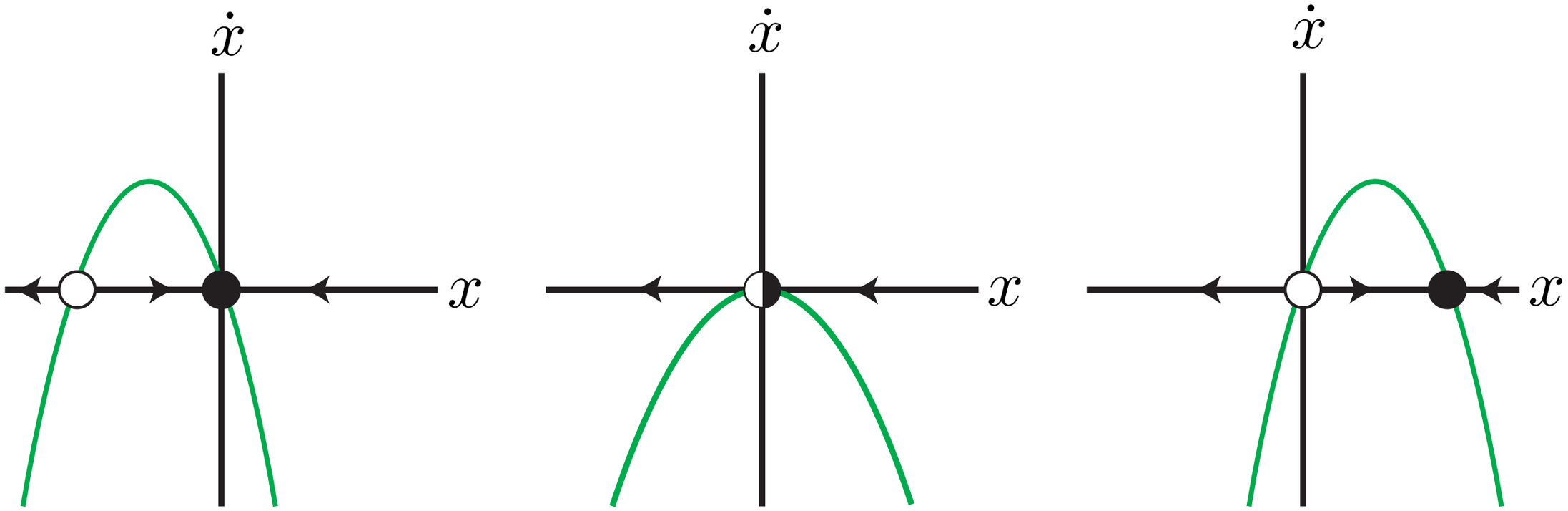


The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of ID Flows ...

Transcritical bifurcation: exchange of stability

Normal form: $\dot{x} = rx - x^2$

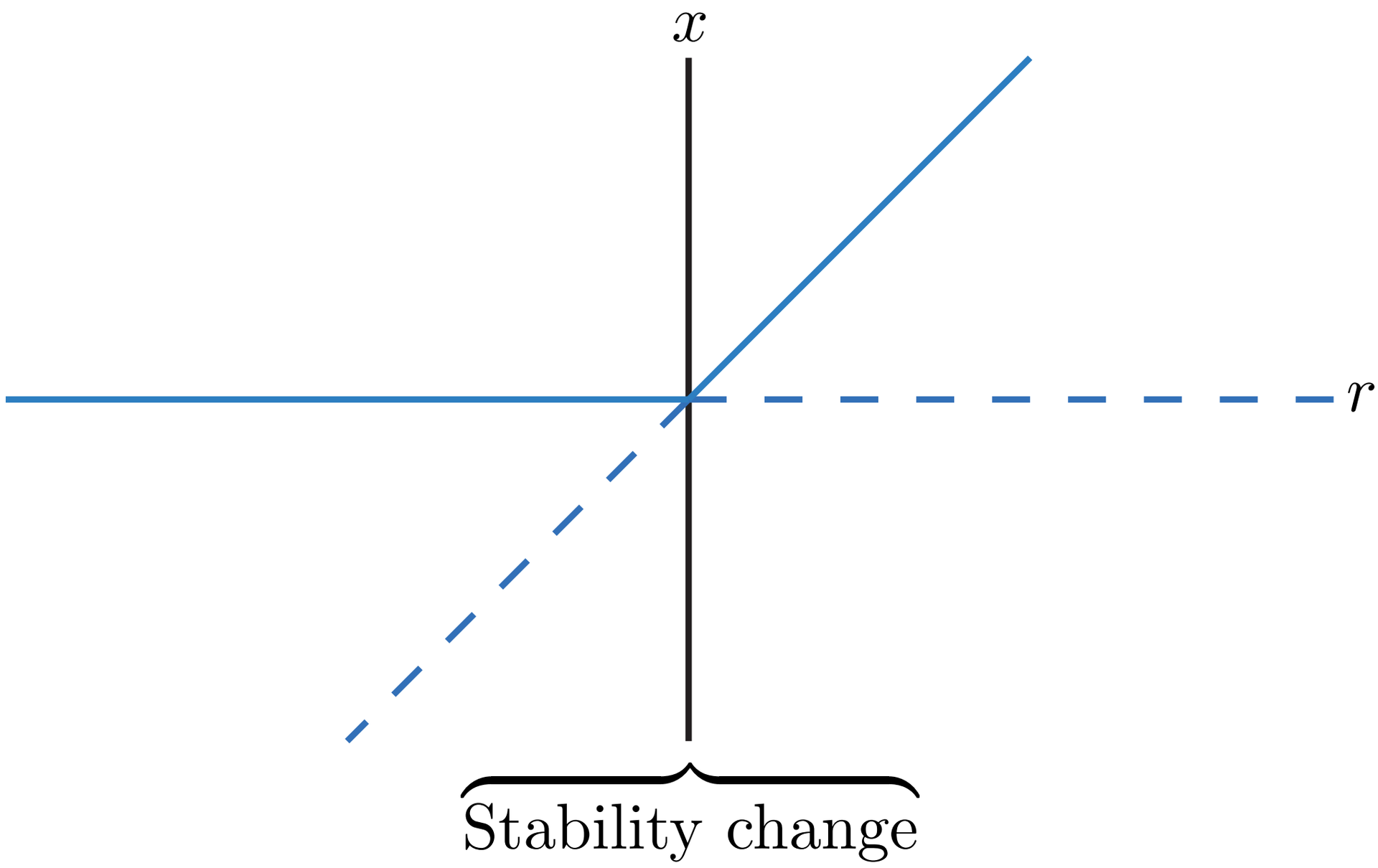


The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of ID Flows ...

Transcritical bifurcation diagram:

Fixed points versus parameter

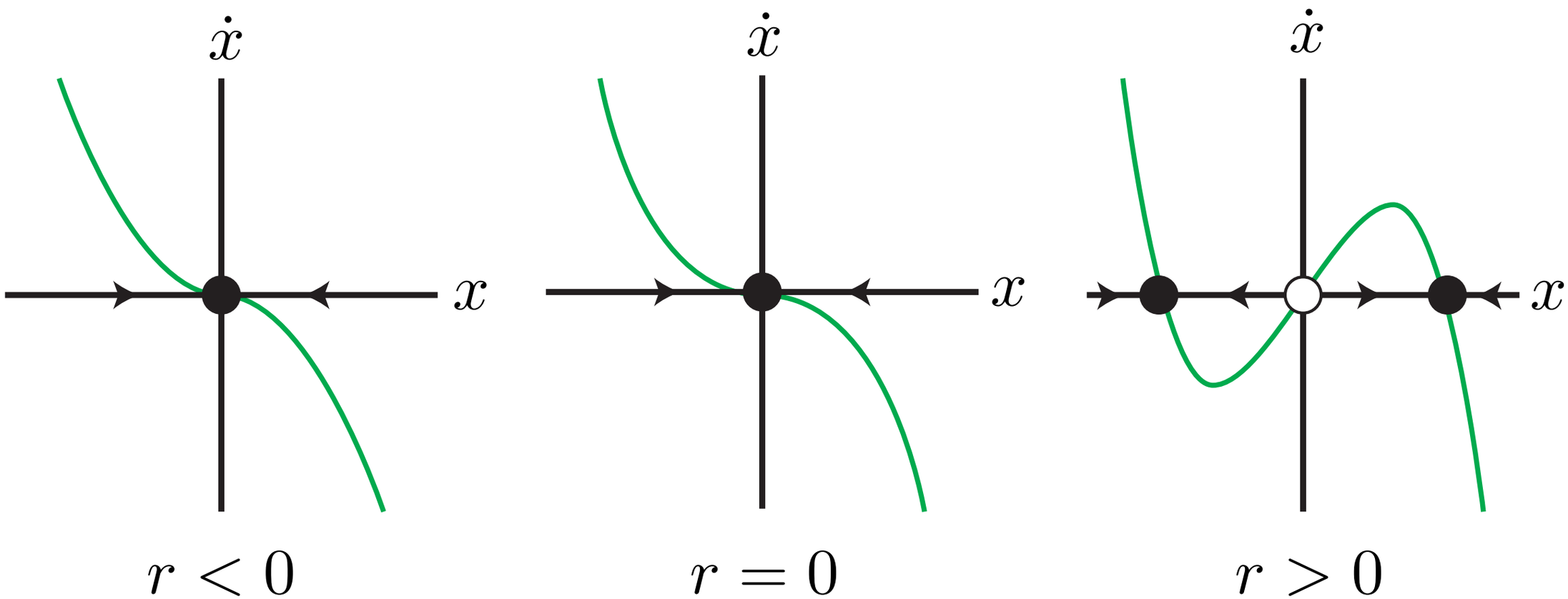


The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of 1D Flows ...

Pitchfork bifurcation:

Normal Form: $\dot{x} = rx - x^3$



$r < 0$ $\quad$ $r = 0$ $\quad$ $r > 0$

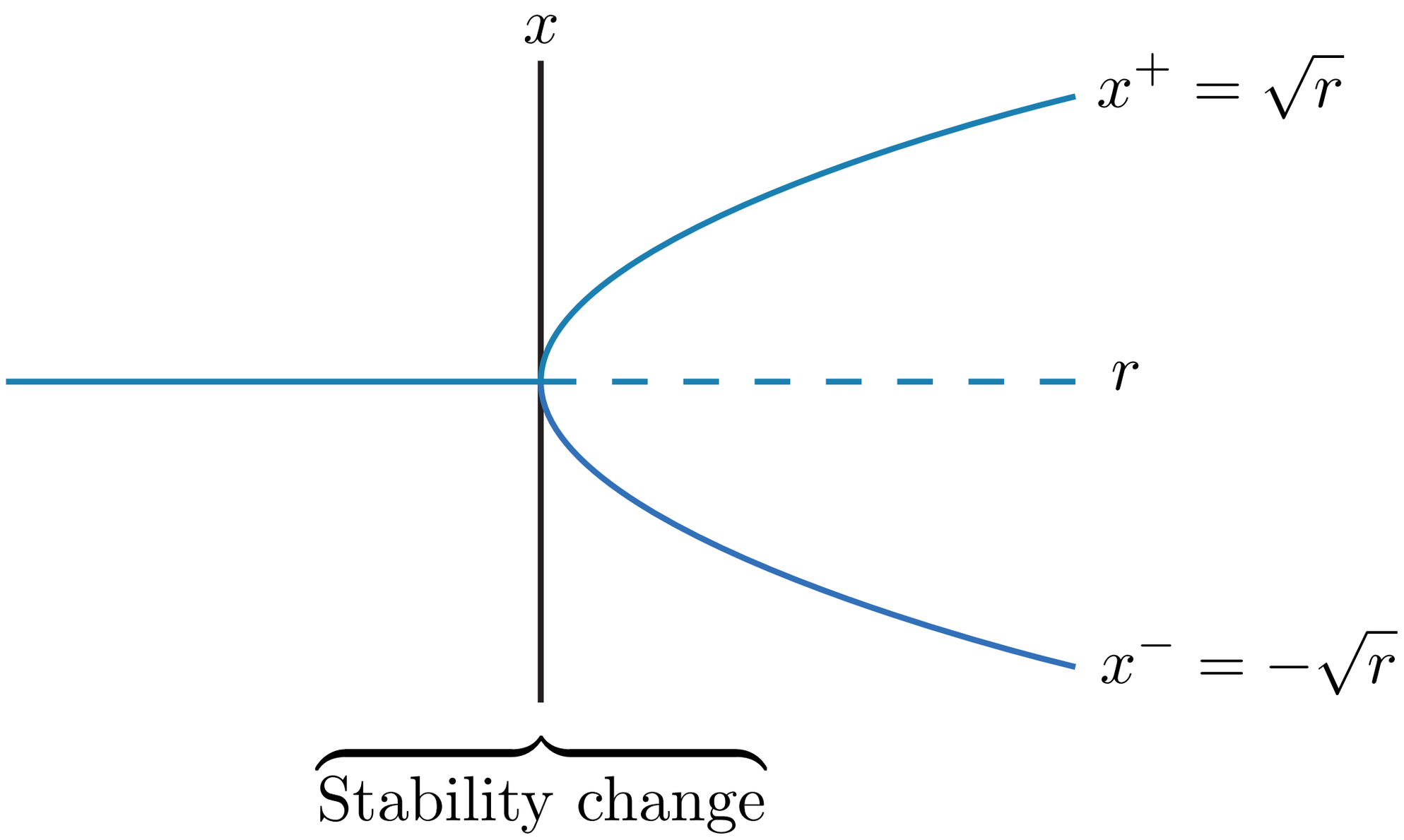
$\overbrace{\text{Stability change}}$

The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of 1D Flows:

Pitchfork bifurcation diagram:

Fixed points versus parameter



The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of 1D Flows ...

Effective Potential: $V(x)$

Given: $\dot{x} = f(x)$

Can we find $\dot{x} = -\dfrac{\partial V(x)}{\partial x}$?

$$V(x) = -\int dx\, f(x)$$

Pitchfork bifurcation:

$$\dot{x} = rx - x^3$$

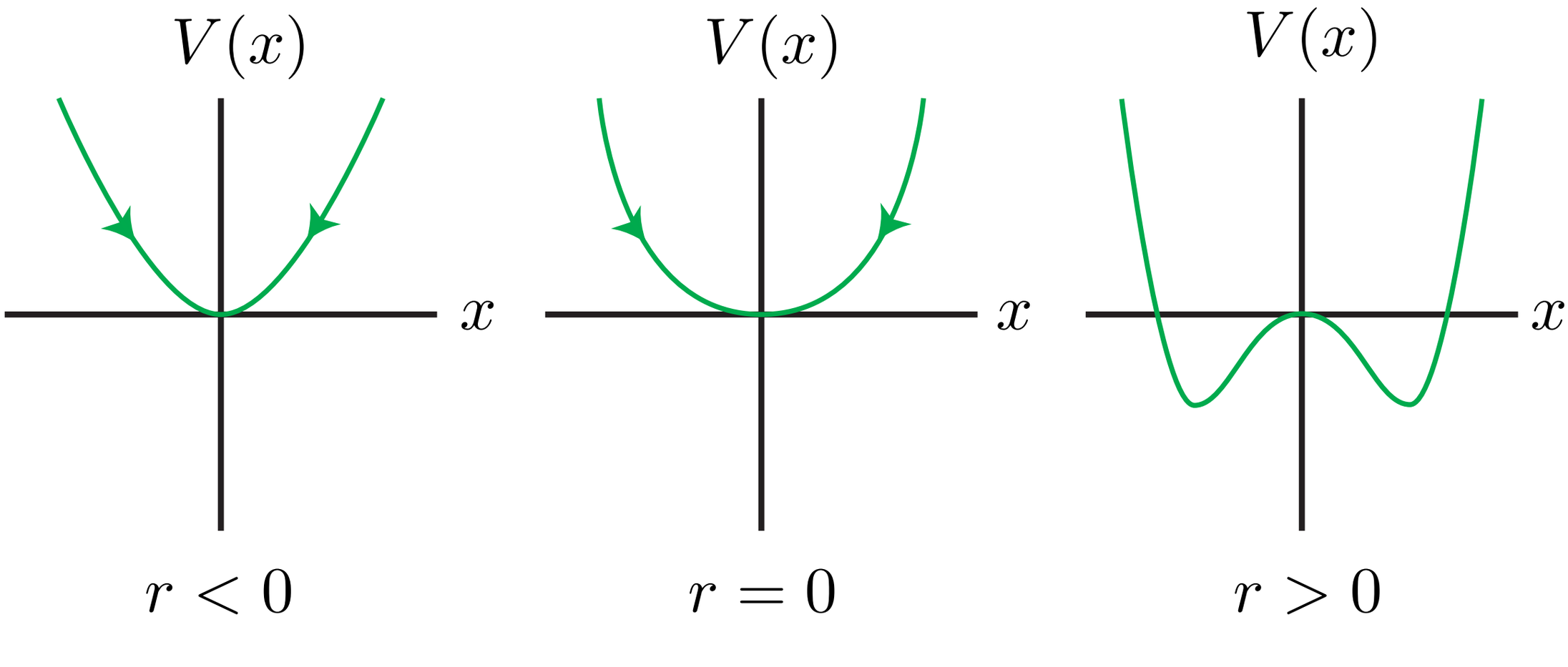
$$V(x) = -\frac{1}{2}rx^2 + \frac{1}{4}x^4$$

The Big, Big Picture (Bifurcations I) ...

Bifurcation Theory of 1D Flows ...

Pitchfork bifurcation:

Effective Potential: $V(x) = -\frac{1}{2}rx^2 + \frac{1}{4}x^4$



$\overbrace{\text{Stability change}}$

The Big, Big Picture (Bifurcations I) ...

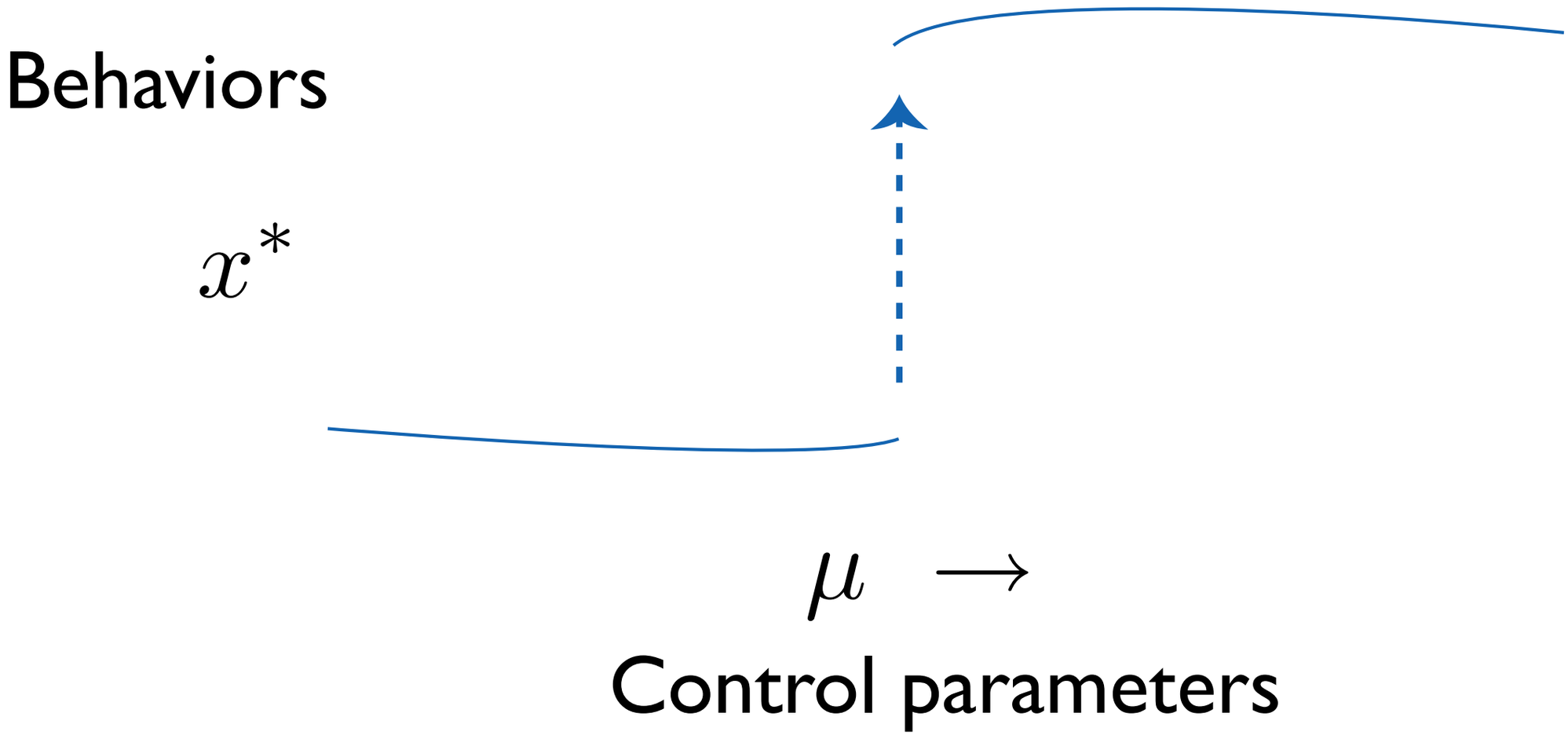
Catastrophe Theory:

Bifurcations from a fixed point to a fixed point

beyond "gradual" changes in equilibria

a model of sudden, large change in behavior

... and a new classification scheme



The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

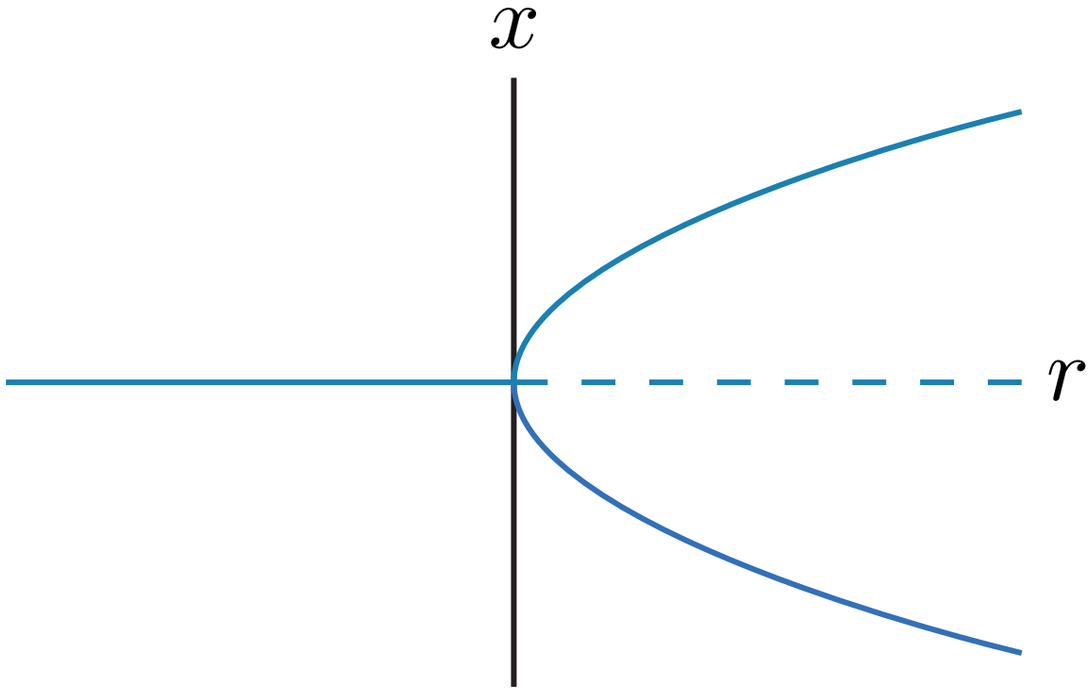
Cusp Catastrophe: two control parameters, one behavior

Normal form: $\dot{x} = h + rx - x^3$

Effective potential: $V(x) = \frac{1}{4}x^4 - \frac{1}{2}rx^2 - hx$

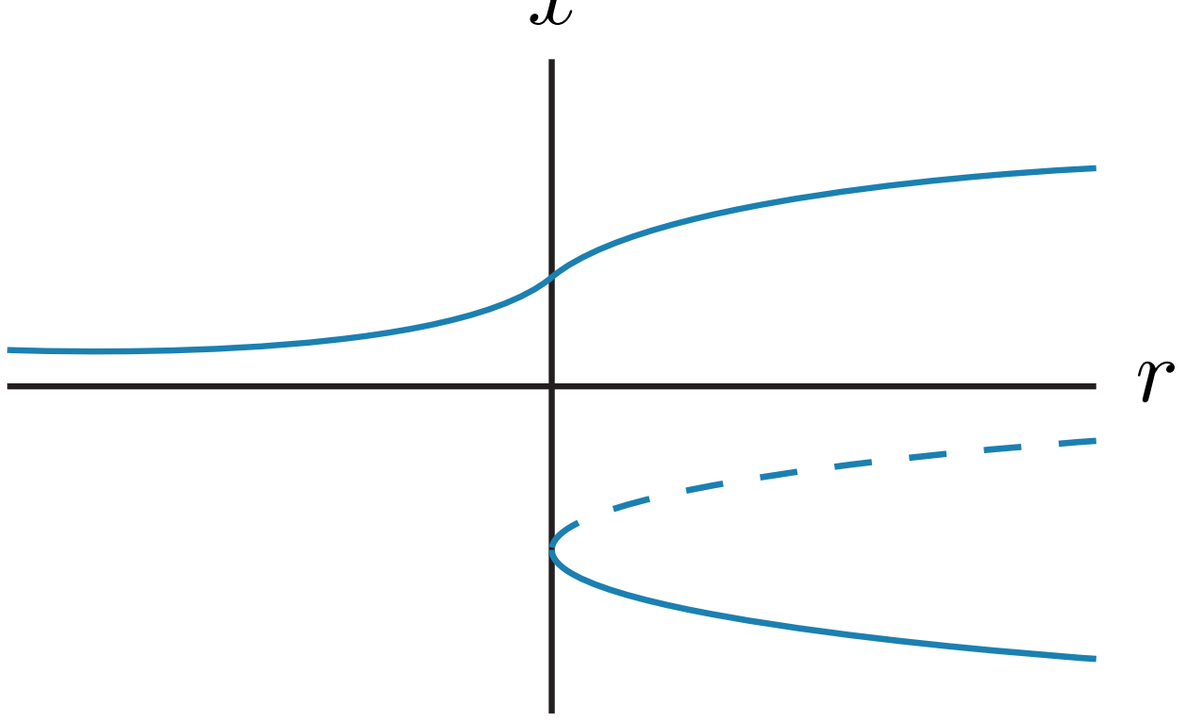
Pitchfork bifurcation:

$h = 0$



Broken symmetry:

$h \neq 0$



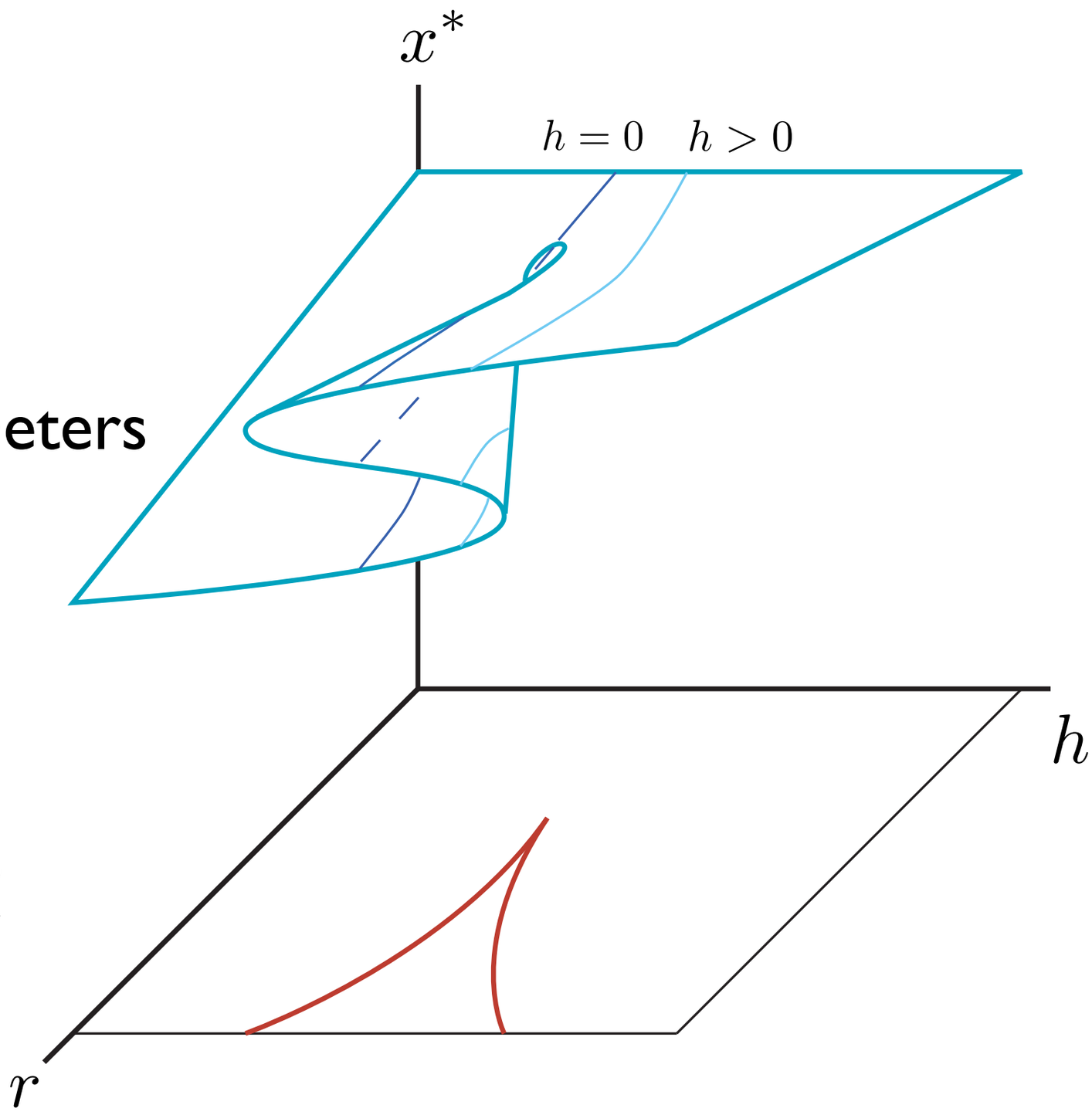
The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Cusp Catastrophe ...

Behavior Surface:

fixed points $\{x^*\}$ v. parameters



Control Space: $(r, h) \in \mathcal{M}$

The Big, Big Picture (Bifurcations I) ...

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Behavior Surface:

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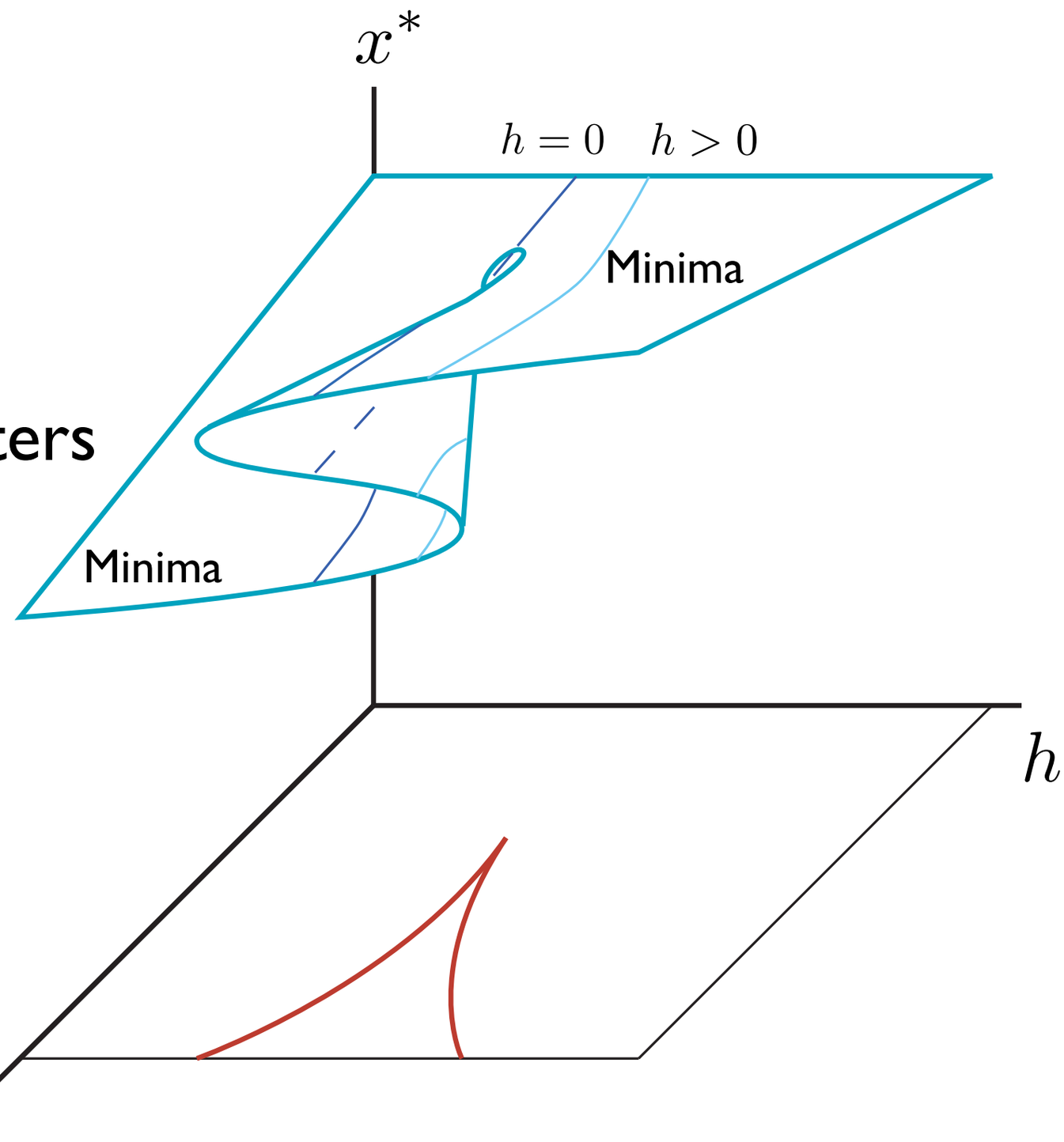

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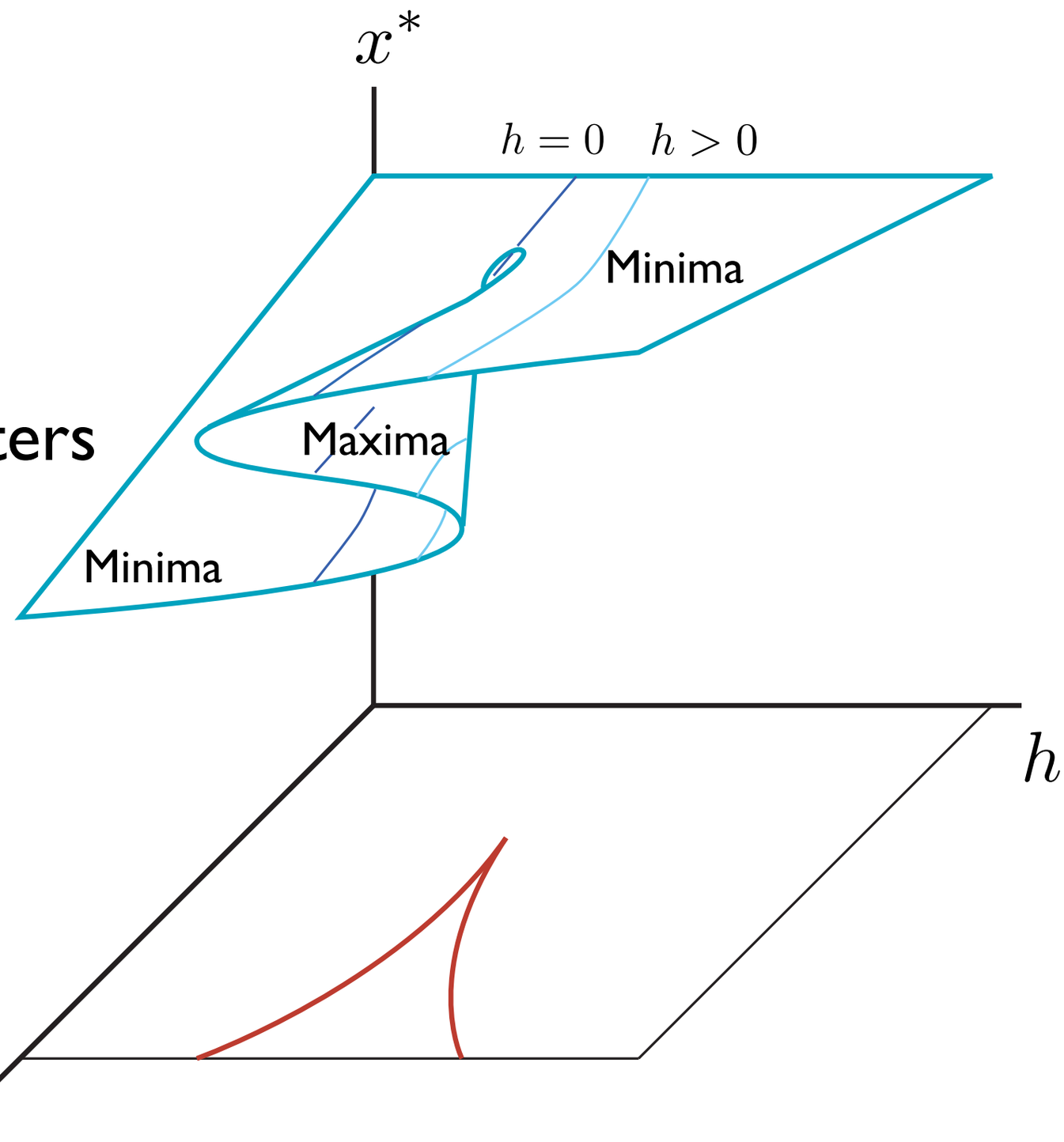

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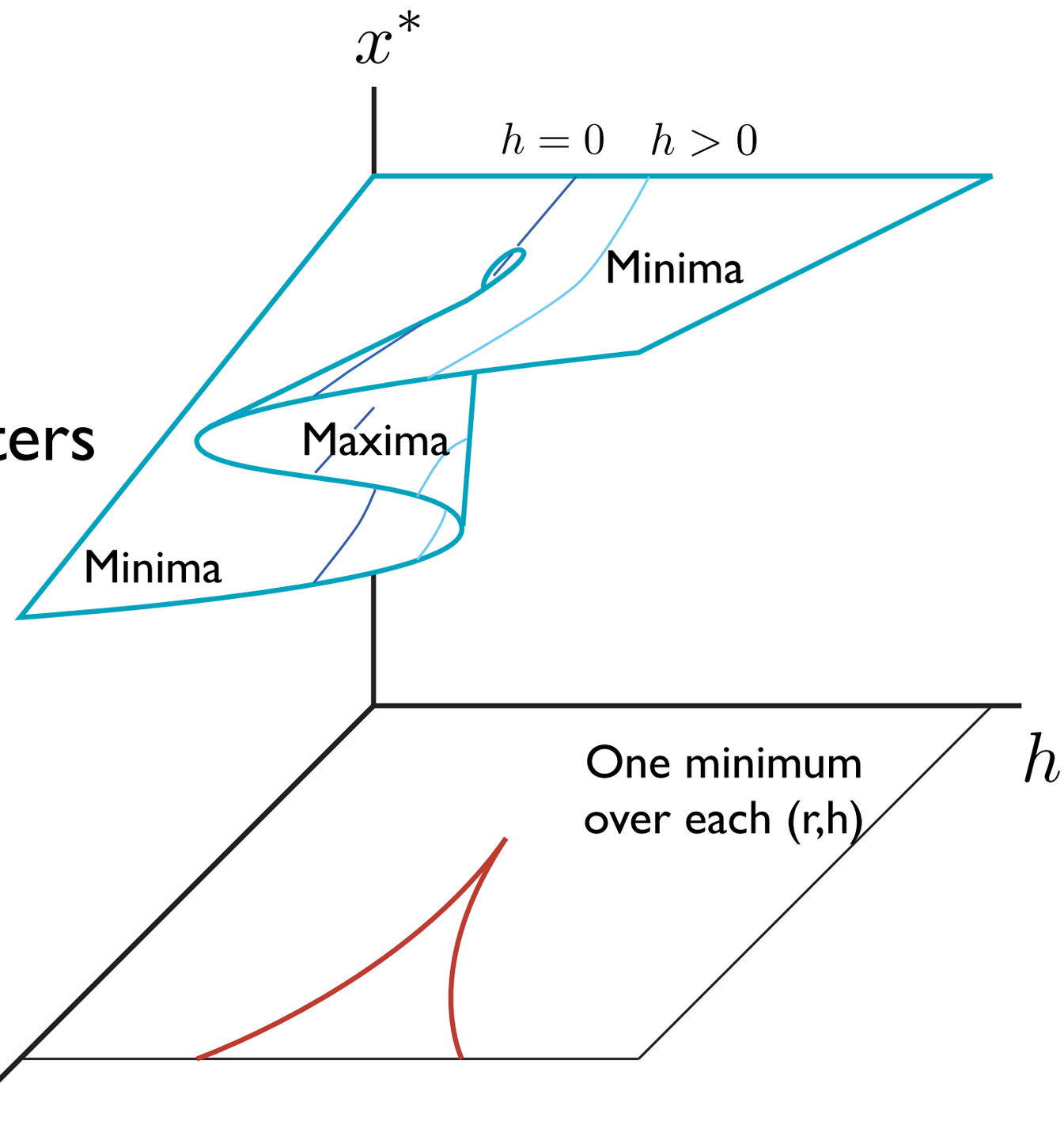


Control Space: $(r, h) \in \mathcal{M}$

The Big, Big Picture (Bifurcations I) ...

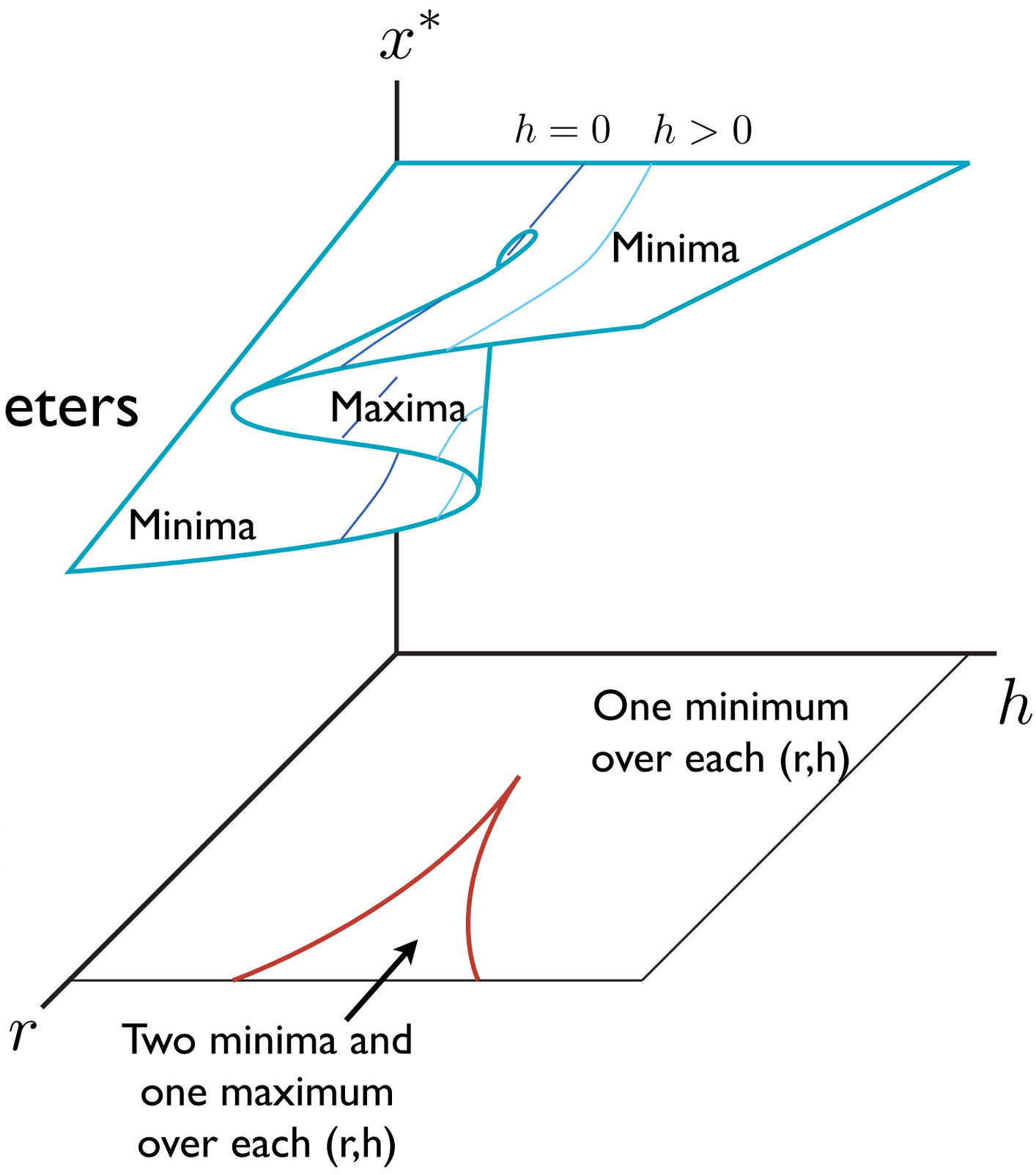
Catastrophe Theory ...

Cusp Catastrophe ...

Behavior Surface:

fixed points $\{x^*\}$ v. parameters

Control Space: $(r, h) \in \mathcal{M}$



The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Cusp Catastrophe ...

Behavior Surface:

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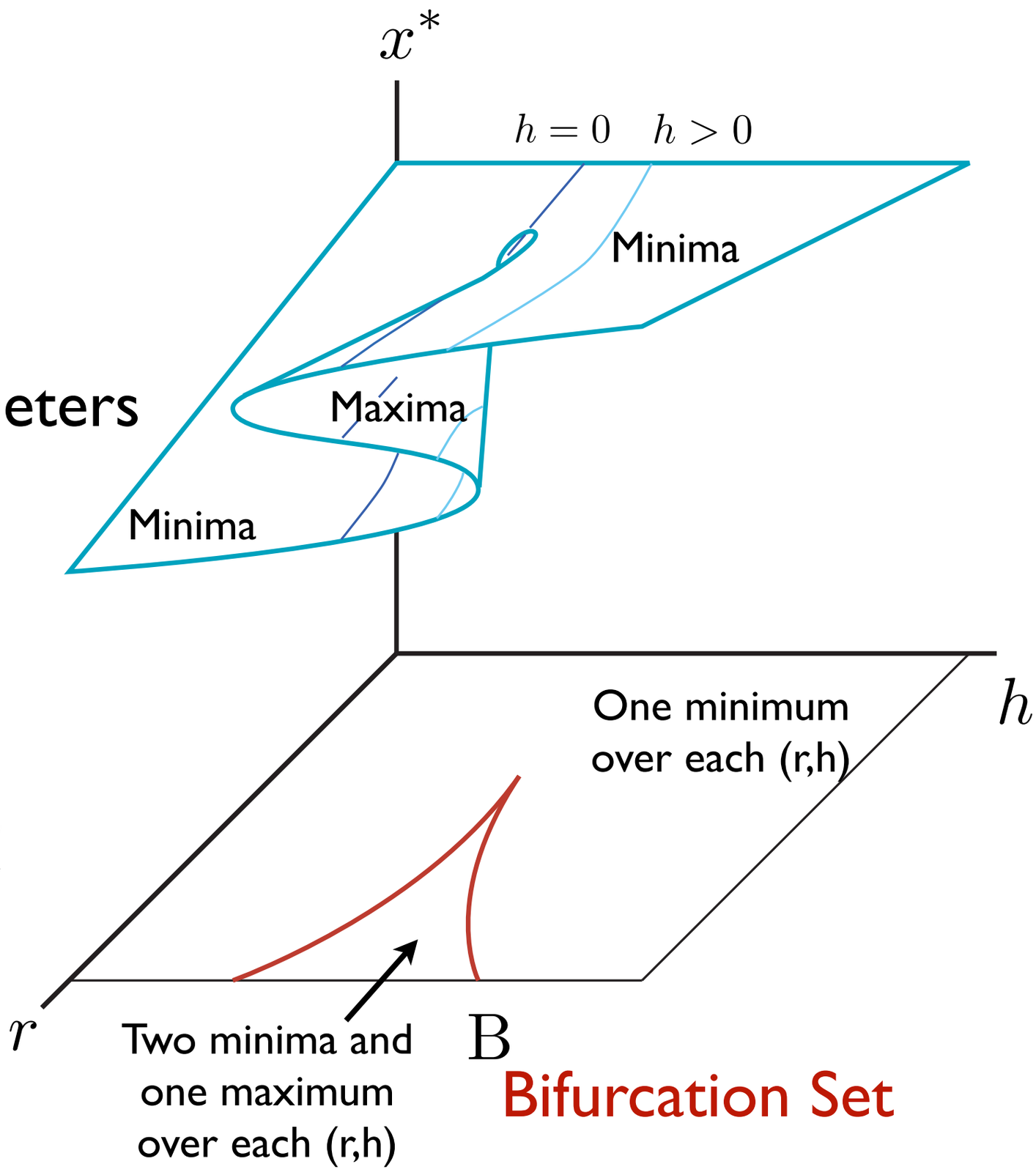
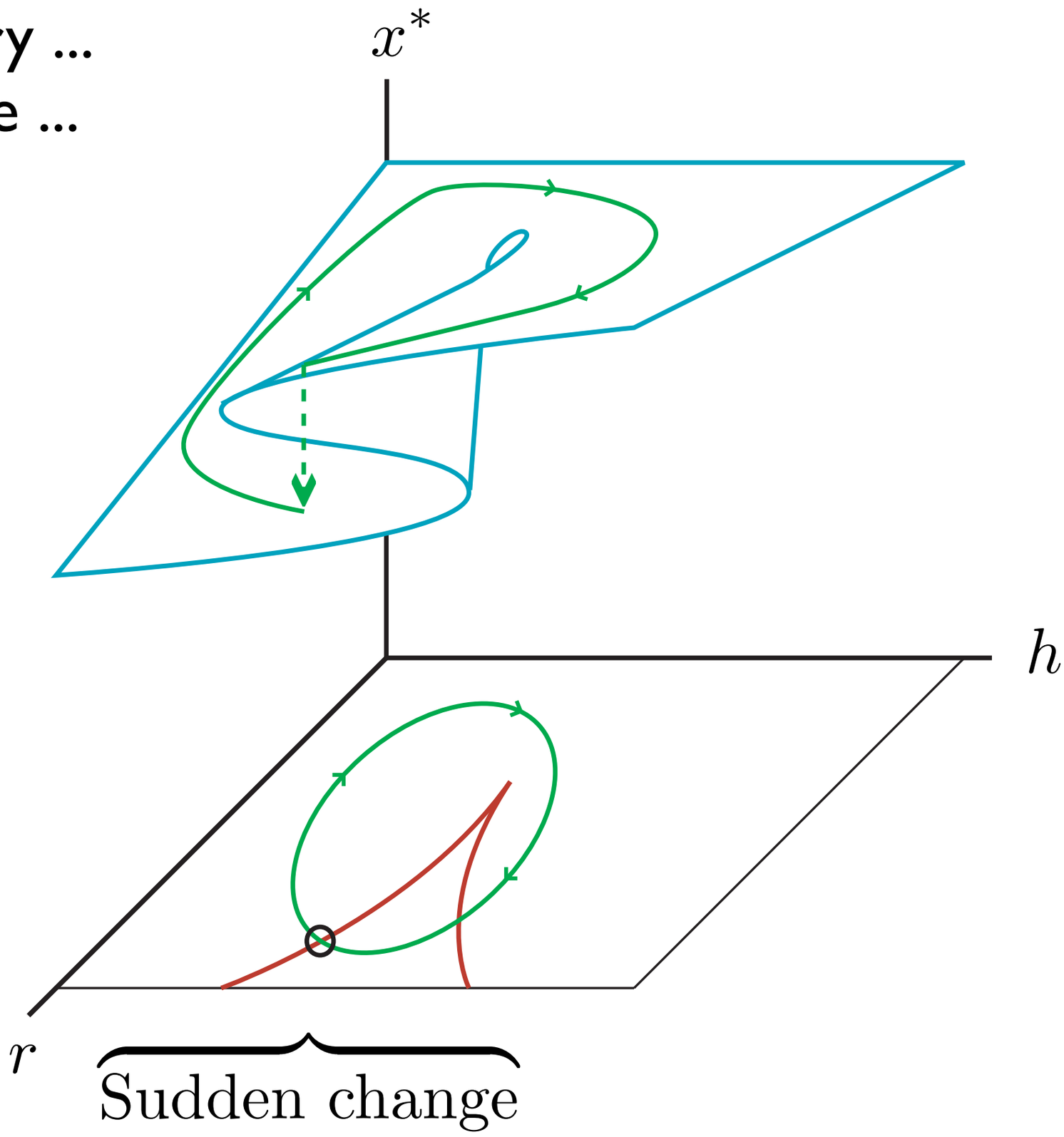


Control Space: $(r, h) \in \mathcal{M}$

The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

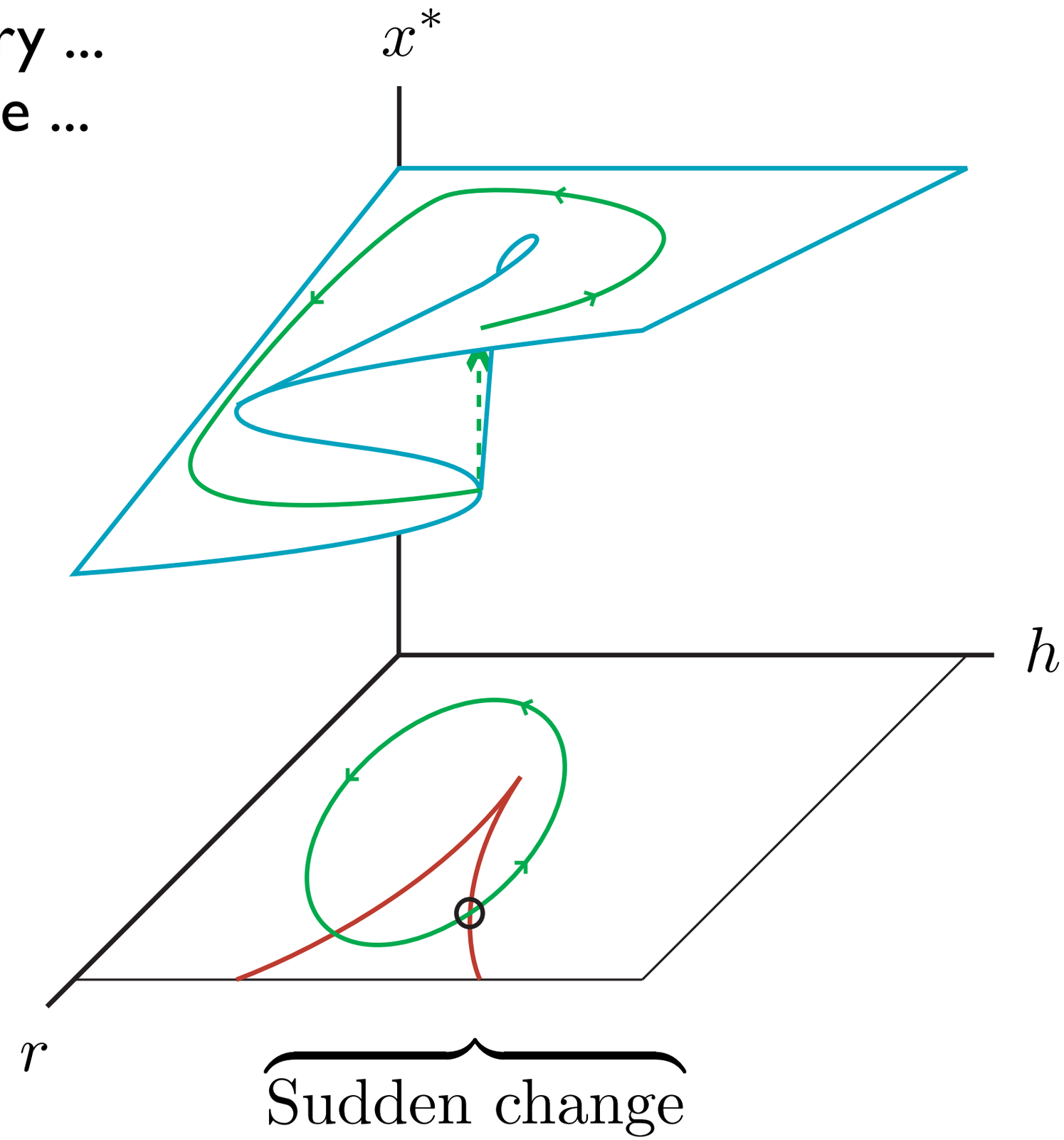
Cusp Catastrophe ...



The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

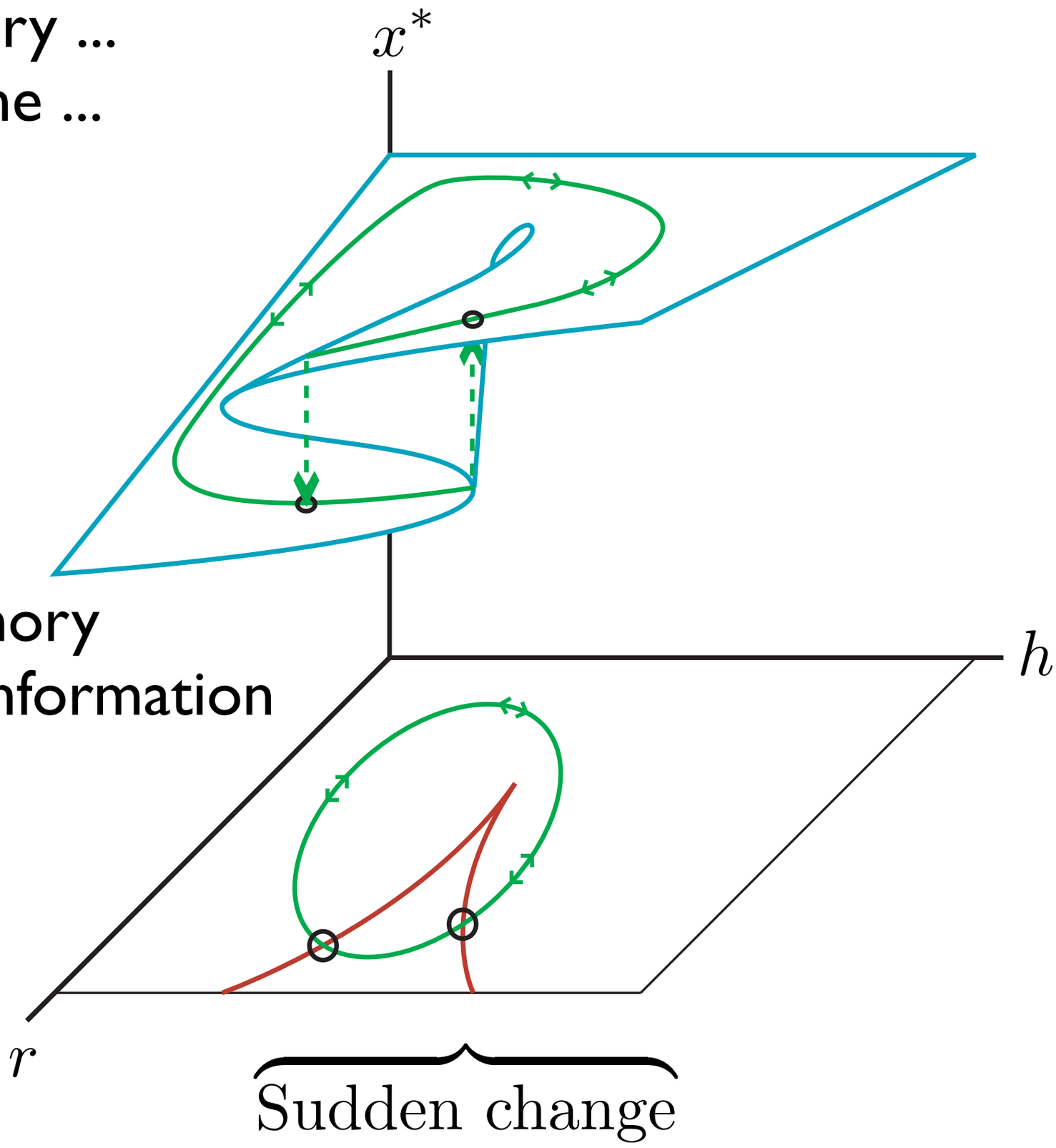
Cusp Catastrophe ...



The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Cusp Catastrophe ...



Hysteresis:

system has memory

a way to store information

The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Classification:

number of controls

number of behavior variables

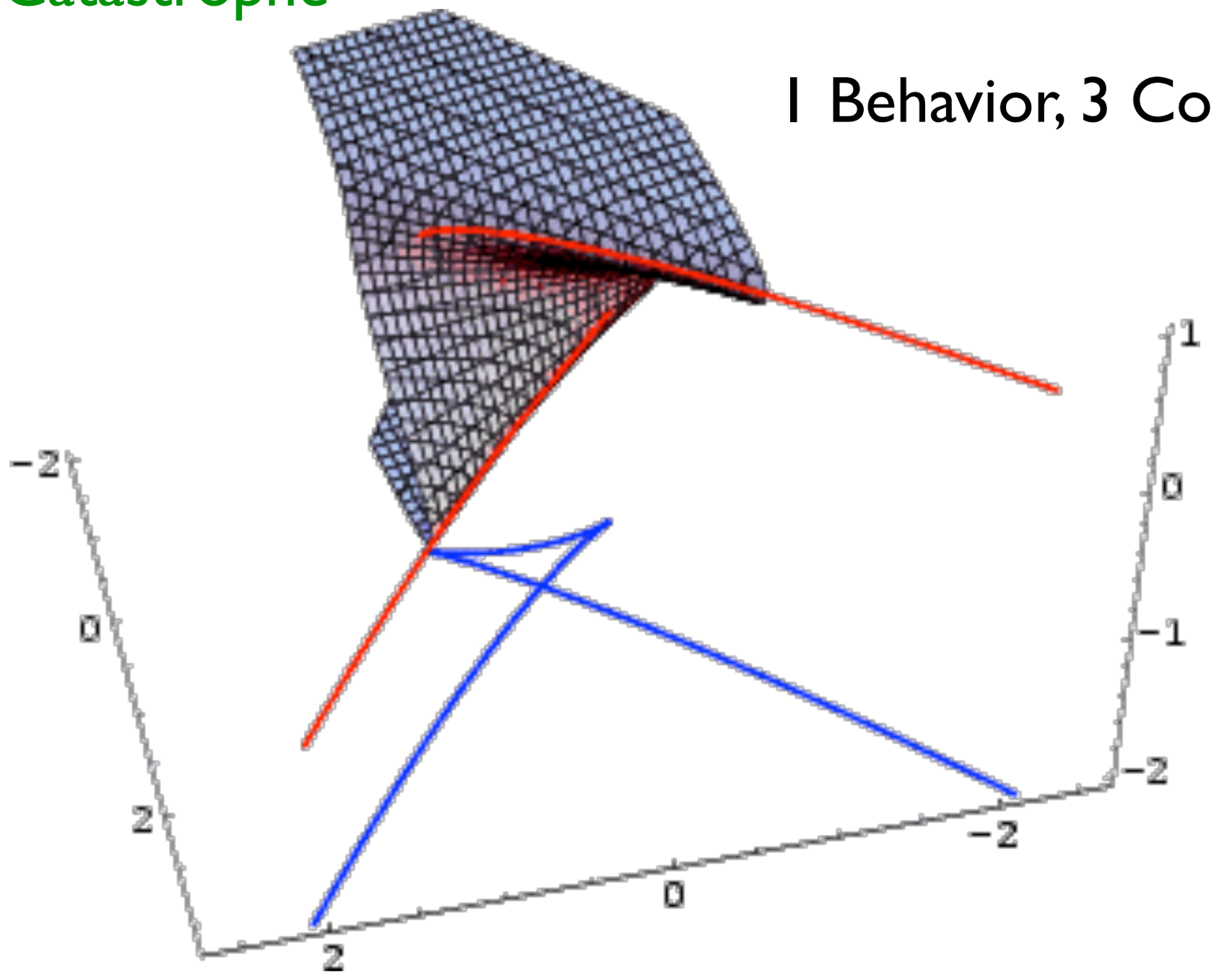
Controls	1 Behavior	2 Behaviors
1	Fold (Saddle Node)	
2	Cusp	
3	Swallowtail	Hyperbolic & Elliptic Umbilic
4	Butterfly	Parabolic Umbilic
5	?	?

The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Swallowtail Catastrophe

1 Behavior, 3 Controls

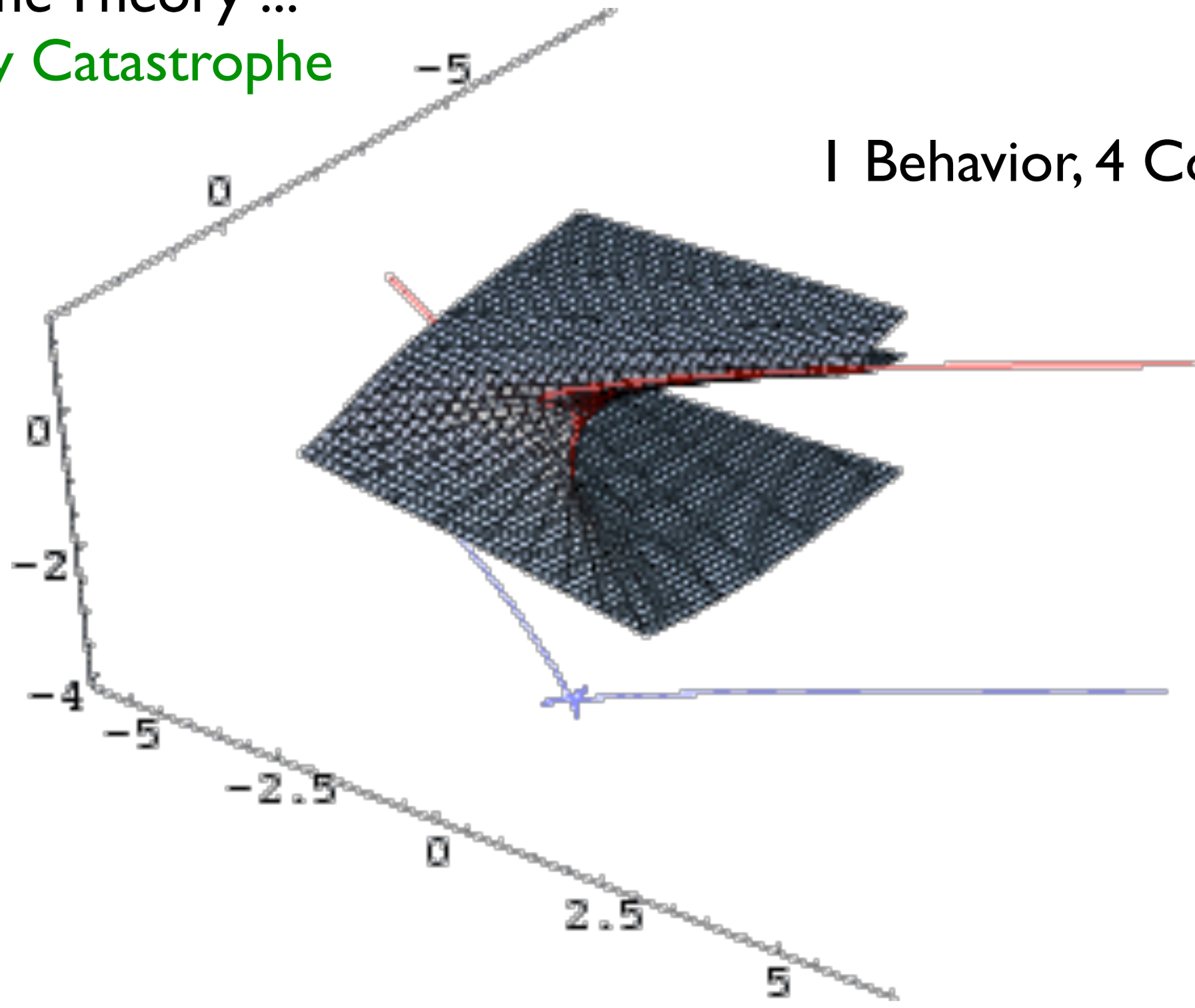


The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Butterfly Catastrophe

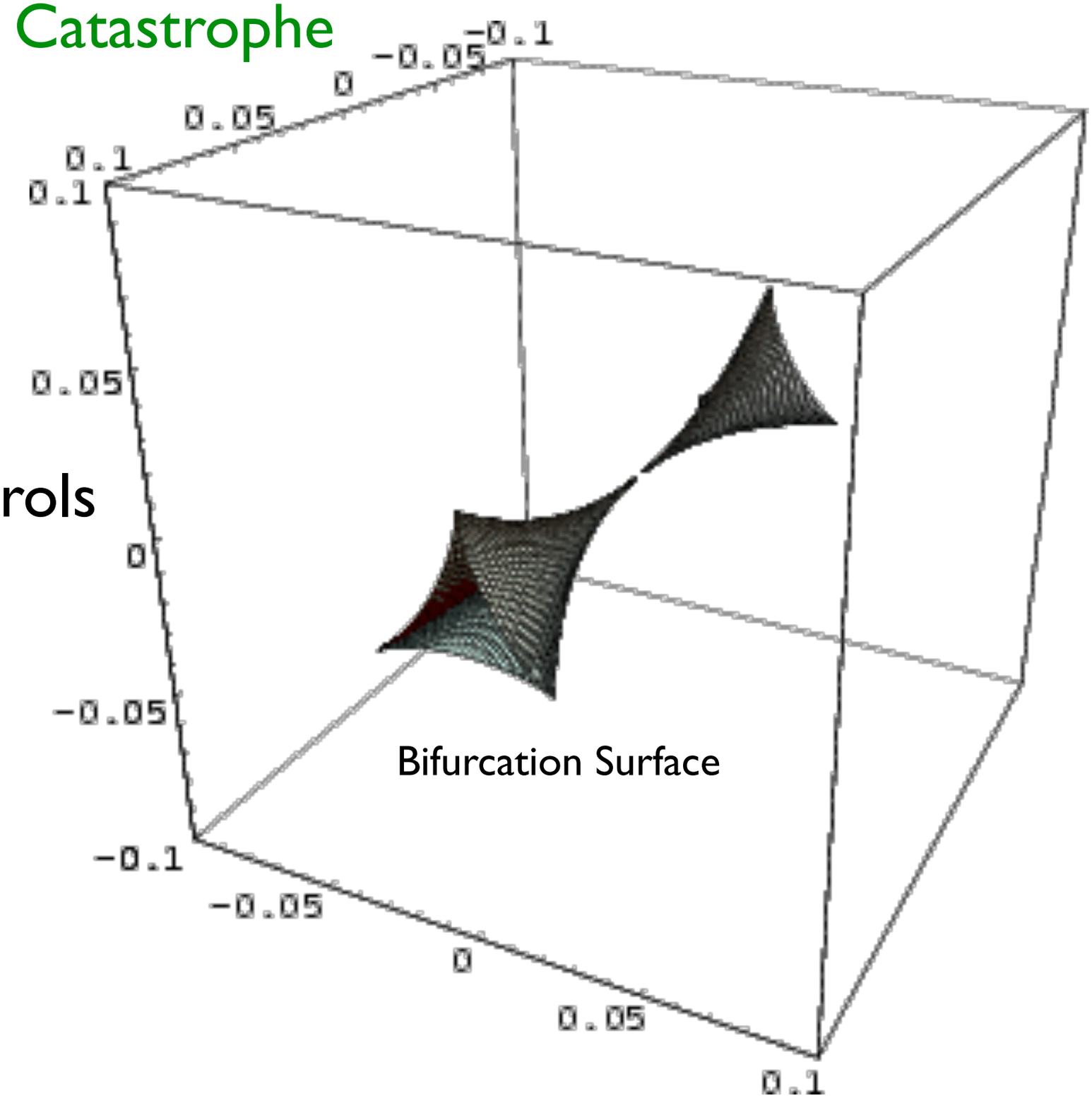
1 Behavior, 4 Controls



The Big, Big Picture (Bifurcations I) ...

Catastrophe Theory ...

Elliptic Umbilic Catastrophe



2 Behavior, 3 Controls

The Big, Big Picture (Bifurcations I) ...

Reading for next lecture:

NDAC, Chapter 8 and Sec. 10.0-10.4.