Structure or Noise?

Dynamics Days 2008
Knoxville, Tennessee

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5 January 2008
Why We Must Model I

- Nature spontaneously organizes
- Emergent structures
Emergent structures
Why We Must Model 2

- Engineered systems also spontaneously organize
  - Internet route flapping
  - Power-law Internet organization
  - Financial markets crash
  - Power grids fail spectacularly
  - Social pattern formation on the web
  - ...

Consequence

- Each needs its own explanatory (function) basis

- Problem:
  Emergent structures not given directly by the system coordinates or the governing equations of motion
Why we must Model 3

- Fundamental Mathematics: Intrinsic Randomness
  - Nonlinear dynamical systems [Kolmogorov 1958]:
    - Chaotic systems: Shannon entropy $h_\mu > 0$
  - Kolmogorov-Chaitin [1963] complexity of Data:
    - Size of shortest Turing Machine Program to predict Data
  - KC complexity = Shannon entropy [Brudno 1978]:

$$|\text{Program}| \propto e^{h_\mu |\text{Data}|}$$
Exponential Increase in Prediction Resources

Accuracy $\propto e^{-T}$

$|\text{Measurements}| \propto e^{T}$

$|\text{Compute time}| \propto e^{T}$

Prediction Horizon $T$
Consequence

- No short cuts!
  - No closed-form solutions
  - No computational speed-ups
  - Must compute full trajectory

- Right representation is critical for reducing the prediction error as far as possible (but no farther!)
Past:

Fundamental in Nonlinear Dynamics!

- Each nonlinear system requires its own representation
- Selecting balance between ascribing structure or noise to a measurement depends on representation
- Fundamental issue: Theory building
- Subsidiary issue: Statistical fluctuations due to finite data sample
Future: Fundamental in Designing Multiagent Systems
The Feedback Loop

- **World**
- **Observations**
- **Model**
- **Agent**
- **Policy**
- **Actions**
Knowledge + Action

- A central challenge:
  Actions change the world
  and so
  its statistics,
  and
  what is knowable.
Approaches

- Modeling:
  - Statistical inference
- Strategizing:
  - Game theory
- Adapting:
  - Reinforcement learning
- Group behavior:
  - Population dynamics (evolution & ecology)
- ...
Approaches: Sticking points

- **Modeling:**
  - Statistical inference: static, batch mode

- **Strategizing:**
  - Game theory: equilibria, no transients

- **Adapting:**
  - Reinforcement learning: a priori design, brittle

- **Group behavior:**
  - Population dynamics (evolution & ecology): individuals have no structure (don’t learn)

- Where are the basic principles?
Interactive Learning
(Susanne Still, Chris Ellison, & JPC)

- Problem: Experiment to Learn World Model
- The world behaves: $X = \overset{\leftarrow}{X} \overset{\rightarrow}{X}$
  - past
  - future
- Agent learns model of the world: States $\mathcal{R}$
- Agent takes actions $\mathcal{A}$
- Those actions affect the world
- Now the world is different!
- How to close the feedback loop?

Passively Learning a Model

- **Pattern discovery:**
  - Learn the world’s hidden states $\Pr(\mathcal{R} | \vec{X})$

- **Causal shielding:**
  $$\Pr(\vec{X} \vec{X}) = \Pr(\vec{X} | \mathcal{R})\Pr(\vec{X} | \mathcal{R})$$

- **Search in the space of models:** $\mathcal{R} \in \mathcal{M}$

- **Objective function**
  $$\min_{\Pr(\mathcal{R} | \vec{X})} \left( I[\vec{X}; \mathcal{R}] + \beta I[\vec{X}; \vec{X} | \mathcal{R}] \right)$$

  $\beta \sim 1/T$

- Model: Map from histories to states
- Info states contain about histories
- Reduce info history has about future
Passively Learning a Model

- Optimal states $\Pr(R|\vec{X})$ are Gibbs distributions:

$$\Pr_{\text{opt}}(R|\vec{X}) = \frac{\Pr(R)}{Z(\vec{X},\beta)} e^{-\beta E(R,\vec{X})}$$

where

$$E(R,\vec{X}) = \mathcal{D}\left(\Pr(\vec{X} | \vec{X})\|\Pr(\vec{X} | R)\right)$$

$$\Pr(\vec{X} | R) = \frac{1}{\Pr(R)} \sum_{\vec{X}} \Pr(\vec{X} | \vec{X}) \Pr(R | \vec{X}) \Pr(\vec{X})$$

$$\Pr(R) = \sum_{\vec{X}} \Pr(R | \vec{X}) \Pr(\vec{X})$$
Passively Learning a Model

- Solve these equations self-consistently

- Parametrized family of models $\Pr(R|\bar{X}): R_\beta$

- Structure or Noise?

  $\beta$ trades-off model size against prediction error
What Do Solutions Mean?

Causal Models

- Causal architecture given by \( \epsilon \)-Machine \( M \):
  - Optimal predictor:
    \[
    h_\mu(M) \leq h_\mu(\mathcal{R})
    \]
  - Minimal size (within optimal predictors \( \hat{\mathcal{R}} \)):
    \[
    C_\mu(M) \leq C_\mu(\hat{\mathcal{R}})
    \]
  - Unique (within min, opt predictors)

Passively Learning a Model

- Theorem: Low-temperature limit
  \[ \beta \to \infty \]
  Recover \( \epsilon \)-Machine:
  \[ R_\beta \to M \]

- Conclusion: Best causal approximates.
Passively Learning a Model

Optimal balance structure & error
At each level $\beta$ of approximation

Causal Rate Distortion Curve

In theory

$H[\text{Past}]$

$\epsilon M$ limit

$I[\text{Past};\text{Rivals}]$

$R(D)$

$I[\text{Past};\text{Future}]$

Slope = inverse $T = \beta$

IID limit

$E = I[\text{Past};\text{Future}]$

$E - I[\text{Future};\text{Rivals}]$

Distortion
Passively Learning a Model

Optimal balance structure & error
At each level $\beta$ of approximation

In practice: Learn an oo-state world (SNS: simple nondeterministic source)
Interactive Learning

- **Decision:** Using model, take actions
- **Policy:** $\Pr(A|\vec{X})$ (or from $\mathcal{R}$)
- **Experimentation objective function**

$$\max_{\Pr(\mathcal{R}|\vec{X}),\Pr(A|\vec{X})} \left( I[\{\mathcal{R},A\};\vec{X}] - \lambda I[\mathcal{R};\vec{X}] - \mu I[A;\vec{X}] \right)$$

- **Model:** Map from histories to states
- **Info states/actions contain about futures**
- **Policy:** Map from histories to actions
- **Info states contain about histories**
- **Info actions contain about histories**
Interactive Learning: Results

- Optimal model: Recover causal architecture
- Optimal policies
- Causally equivalent policies
- Curiosity: Take informative actions
- Control: Make world easier to model
- Balance of exploitation and control

arxiv.org: [0708.0654 [physics.gen-ph]] & [0708.1580 [cs.IT]]
Connections

- Note iLearning subsumes:
  - Causal modeling
  - Game theory
  - Equilibrium economics
  - Reinforcement learning
Knowledge + Action

- Deviation from knowledge:
  - Model evaluation (e.g., prediction error)

- Valuation: Commitment of resources to action
  - Policy evaluation (e.g., average reward)

- How? Augment objective function
  - Change relative weighting of Lagrange multipliers: $\lambda \& \mu$
  - Add new terms: e.g., ...

- Examples:
  - “Science”: Need accurate knowledge, at expense of producing it
  - “Politics”: Need world to behave, independent of knowledge or cost
  - These are positions on causal rate-distortion curve
The Feedback Loop

World

Observations

Actions

Model

Policy

Agent

Prediction Error

Deviation from knowledge

Valuation

Commitment of resources to actions
Main message

- Closing the loop:
  How interaction changes the world & how one adapts to those changes
- Theoretical foundations (& algorithms) for closing the feedback loop are now available.
Conclusion

- Basic principles follow from
  - Information theory (rate distortion)
  - Statistical physics
- Balance exploitation & exploration
- Balance structure & error
- Balance exploitation & control
- Challenge: Fold in risk
Prospects

- Collective Cognition:
  - Pattern discovery
  - Interactive learning
  - Adaptation dynamics
  - Emergent policy design
  - Multiagent dynamical systems

![Diagram with Agents and Environment](attachment:diagram.png)
Thanks!