Thermodynamic Computing: Fast, Cheap, and Under Control*

James P. Crutchfield  
Complexity Sciences Center  
Physics Department  
University of California, Davis  
http://csc.ucdavis.edu/~chaos/Talks/

Workshop on  
Agency at the Interface of Quantum and Complexity Science  
Nanyang Technological University  
Singapore  
13-16 January 2020

Joint work with Cina Aghamohammadi, Alec Boyd, Chris Ellison, Warren Fon, Christopher Jarzynski, Alexandra Jurgens, Sam Loomis, Dibyendu Mandal, Sarah Marzen, Matthew H. Matheny, Ayoti Patra, Paul Riechers, Michael Roukes, Olli-Pentti Saira, Susanne Still, Ariadna Venegas-Li, Greg Wimsatt & …

*With apologies to Errol Morris’ & Rodney Brooks’ Fast, Cheap, and Out of Control
Thermodynamic Computing: Fast, Cheap, and Under Control*

James P. Crutchfield
Complexity Sciences Center
Physics Department
University of California, Davis
http://csc.ucdavis.edu/~chaos/Talks/

Workshop on
Agency at the Interface of Quantum and Complexity Science
Nanyang Technological University
Singapore
13-16 January 2020

Joint work with Cina Aghamohammadi, Alec Boyd, Chris Ellison,
Warren Fon, Christopher Jarzynski, Alexandra Jurgens, Sam Loomis, Dibyendu Mandal, Sarah
Marzen, Matthew H. Matheny, Ayoti Patra, Paul Riechers, Michael Roukes, Olli-Pentti Saira,
Susanne Still, Ariadna Venegas-Li, Greg Wimsatt & …

*With apologies to Errol Morris’ & Rodney Brooks’ Fast, Cheap, and Out of Control
Abstract

The paradigm of thermodynamic computing has arrived, driven by recent theoretical and experimental progress, pitched to circumvent the end of Moore’s multi-decadal exponential progress in computing speed and density, and offered as a complement to quantum computing. I will review a recent planning effort (Computing Community Consortium, January 2019, Honolulu) aimed to accelerate reducing our recent progress to practice. As part of this I will also give a rather synoptic and optimistic survey of that progress, somewhat biased to the outputs from our multiyear Information Engines workshop series. Looking forward, I will address several open challenges. The first is to understand how the recent progress was built out of a calculus of limitations from deterministic unpredictability, quantum uncertainty, undecidability, and uncomputability to our current inability to track the flow of information, identify causal mechanisms, define structural complexity, and converge on unique explanatory models. What new limitations and innovations can we anticipate? The second challenge is to unpack what Landauer meant by “information-bearing degrees of freedom”. Explicitly addressing this is key to an objective theory of physical computing. The final challenge is what I call Landauer’s Stack: What are the actual thermodynamic costs of information processing? If we add up currently-identified thermodynamic costs–Landauer erasure, Information Processing Second Law, synchronization and error correction, implementation modularity, high reliability, and the like–can we accurately predict the energetics of contemporary and future computing? If not, how far are we from doing so and what might we be missing?
Physics of Information?

- Information Age!
- How can information be harnessed?
- What does it mean for a physical system to compute?
- Fundamental physical limits of information processing?
Physics of Computation Meeting (MIT, 1981)

Wheeler

Physics of Computation Conference

MIT May 6-8, 1981

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernad Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Roberth
24 Robert Lewis
25 Robert Susya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
29 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-Bi
34 Danny Hills
35 Arthur Burks
36 John Cooke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levin
45 Peter Gacs
46 Dan Greenberger
Physics of Computation Meeting (MIT, 1981)

Me

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Büttiker
23 Otto Röberth
24 Robert Lewis
25 Robert Susya
26 Stan Kugel
27 Bill Gosper
28 Lutz Fiese
29 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Martin Pour-El
34 Danny Hills
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerhard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger
Physics of Computation Conference Endicott House MIT May 6-8, 1981

41. ?  42. Gerard Vichniac  43. Leonard Levin  44. Lev Levitin  45. Peter Gacs
46. Dan Greenberger

Photo: Charlie Bennett
Physics of Computation Meeting (MIT, 1981)

- Feynman introduces quantum computing

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don’t know what a keynote speech is. I do not intend in any way to suggest what should
Physics of Computation Meeting (MIT, 1981)

• Feynman introduces quantum computing

Intrinsic computation there, too!
Information Engines

(2012-202X)

Welcome

Synthetic nanoscale systems can behave as information engines, performing tasks that involve the manipulation of both information and energy. This remarkable fact, highlighted by recent theoretical and experimental breakthroughs, motivates our research. Our goal is to develop a unified framework for understanding, designing, and implementing information-processing engines.

Materials

NEWS!

$12.5M in grants for complexity, networks research (UCD article): CSC wins two Multidisciplinary University Research Initiatives, greatly expanding its complex systems research. Also, see Funding
Information Engines

PIs

+ **Gavin Crooks (Berkeley):**
   Thermodynamics of molecular machines, modeling and theoretical predictions, and search for basic thermodynamic and information-theoretic principles of nanoscale systems.

+ **Jim Crutchfield (Davis) (Lead):**
   Theoretical methods to analyze the intrinsic computational properties of nanoscale systems and devices; algorithms to analyze experimental data.

+ **Mike DeWeese (Berkeley):**
   Optimal control protocols with the goal of efficient control of nanoscale system behaviors. Devices and systems.

+ **Michael Roukes (Caltech):**
   Nanoscale devices, Mesoscale physics, BRAIN Initiative

+ **Chris Jarzynski (Maryland):**
   Adapt nonequilibrium thermodynamics to predict and control nanoscale system behavior and information processing.

+ **P. S. “Krishna” Krishnaprasad (Maryland):**
   Extending control theory to apply to nanoscale devices and systems.
# Thermodynamic Computing Workshop Report

DRAFT: Not for Distribution

## Table of Contents

Thermodynamic Computing Workshop Report

1. **Overview and Motivation**

   1.1. Introduction
   
   1.2. A Brief History of Thermodynamics and Computation
   
   1.3. Current State of Computing
   
   1.4. Recasting Computing in Thermodynamic Terms
   
   1.5. A Vision for Thermodynamic Computing
   
   1.5.1 TC Roadmap
   
   1.6. Summary of Scientific Challenges and Research Directions
   
   1.6.1. Scientific Challenges
   
   1.6.2. Research Directions
   
   1.7. Summary of Social and Technological Impacts
   
   1.7.1. Scientific Impacts
   
   1.7.2. Social and Technological Impacts

https://cra.org/ccc/events/thermodynamic-computing/

Thermodynamic Computing

- Theory
- Design
- Diagnosis
- Experiment
Thermodynamic Computing

• Theory
• Design
• Diagnosis
• Experiment
Intrinsic Computation
Information-Theoretic Analysis of Complex Systems ...

- Process \( \Pr(\vec{X}, \vec{X}) \) is a communication channel from the past \( \vec{X} \) to the future \( \vec{X} \):
Process $\Pr(\vec{X}, \vec{X})$ is a communication channel from the past $\vec{X}$ to the future $\vec{X}$:

- **Past** → **Present** → **Future**

Information Rate $h_\mu$

Channel Capacity $C$
Process \( \text{Pr}(\vec{X}, \vec{X}) \) is a communication channel from the past \( \vec{X} \) to the future \( \vec{X} \):

\[ E = I[\vec{X}; \vec{X}] \]

- Information-Theoretic Analysis of Complex Systems ...

- **Channel Utilization:** Excess Entropy
**Foundations: Computational Mechanics**

**ε-Machine: Unique, minimal, & optimal predictor**

**Causal Equivalence:**

\[ \overline{x} \sim \overline{x'} \iff \Pr(\overline{X} | \overline{x}) = \Pr(\overline{X} | \overline{x'}) \]

**Stored versus Generated Information**

\[
C_\mu = - \sum_{\sigma \in \mathcal{S}} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \text{VERSUS} \quad h_\mu = - \sum_{\sigma \in \mathcal{S}} \Pr(\sigma) \sum_{\sigma' \in \mathcal{S}} \Pr(\sigma' | \sigma) \log_2 \Pr(\sigma' | \sigma)
\]
Causal Equivalence:
\[ \vec{x} \sim \vec{x}' \iff \Pr(\hat{X}|\vec{x}) = \Pr(\hat{X}|\vec{x}') \]

\varepsilon\text{-Machine: Unique, minimal, & optimal predictor}

Stored versus Generated Information
\[
C_{\mu} = - \sum_{\sigma \in \mathcal{S}} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \text{VERSUS} \quad h_{\mu} = - \sum_{\sigma \in \mathcal{S}} \Pr(\sigma) \sum_{\sigma' \in \mathcal{S}} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma)
\]

Structure versus Randomness

\begin{array}{c}
\text{Structural Complexity} C \\
0 \quad \text{Randomness} \quad (T, S, H, K, \ldots) \quad 1
\end{array}
Foundations: Computational Mechanics

\[ x \sim x' \iff \Pr(\vec{X} | \vec{x}) = \Pr(\vec{X} | \vec{x'}) \]

\( \varepsilon \)-Machine: Unique, minimal, & optimal predictor

**Intrinsic Computation:**

1. How much historical information does a process store?
2. In what architecture is it stored?
3. How is it used to produce future behavior?


Computational Mechanics

- **$\varepsilon$-Machine:**
  \[ M = \{ S, \{ T(x) : x \in A \} \} \]

- **Dynamic:**
  \[ T_{\sigma, \sigma'}^{(x)} = \Pr(\sigma'|\sigma, x) \]
  \[ \sigma, \sigma' \in S \]
Varieties of ε-Machine

- Denumerable Causal States
- Fractal
- Continuous

Varieties of \( \varepsilon \)-Machine

Continuous-Time Discrete-Event Processes

\( \varepsilon \)-Machine = Unifilar Hidden Semi-Markov Processes

Intrinsic Computation: Consequences

A system is **unpredictable**
if it has positive entropy rate: \( h_\mu > 0 \)

A system is **complex**
if it has positive structural complexity measures: \( C_\mu > 0 \)

A system is **emergent**
if its structural complexity increases over time:
\[
C_\mu(t') > C_\mu(t), \text{ if } t' > t
\]

A system is **hidden**
if its crypticity is positive: \( \chi = C_\mu - E > 0 \)

Information Ratchets
Beyond Maxwell+Szilard: Net Work Extraction!


Information Processing

Second Law of thermodynamics

• Asymptotic IPSL:

\[ \langle W \rangle \leq k_B T \ln 2 \left( h_{\mu}' - h_{\mu} \right) \]

• Information is fuel:

(Ordered inputs—correlations—are a thermodynamic resource.)

• Generalizes Landauer Principle; cf.:

\[ Q_{\text{erase}} \geq k_B T \ln 2 \quad \left( h_{\mu}' = 0; \quad h_{\mu} = 1 \right) \]

Information Processing

Second Law of thermodynamics

• IPSL constrains information processing done by any thermodynamic system.

• Upper bound on the maximum average work \(\langle W \rangle\) extracted per cycle.

• Lower bounds the amount \(-\langle W \rangle\) of input work required for a physical system to support a given rate of intrinsic computation.
# Information Processing

## Second Law of Thermodynamics

IPSL determines Thermodynamic Functionality

<table>
<thead>
<tr>
<th>Function</th>
<th>Feature</th>
<th>Operation</th>
<th>Net Work</th>
<th>Net Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td></td>
<td>Extracts energy from the thermal reservoir, converts it into work by randomizing input information</td>
<td>$\langle W \rangle &gt; 0$</td>
<td>$h'<em>{\mu} - h</em>{\mu} &gt; 0$</td>
</tr>
<tr>
<td>Eraser</td>
<td></td>
<td>Uses external input of work to remove input information</td>
<td>$\langle W \rangle &lt; 0$</td>
<td>$h'<em>{\mu} - h</em>{\mu} &lt; 0$</td>
</tr>
<tr>
<td>Dud</td>
<td></td>
<td>Uses (wastes) stored work energy to randomize output</td>
<td>$\langle W \rangle &lt; 0$</td>
<td>$h'<em>{\mu} - h</em>{\mu} &gt; 0$</td>
</tr>
</tbody>
</table>

$$\langle W \rangle \leq k_B T \ln 2 \left( h'_{\mu} - h_{\mu} \right)$$
Fluctuations in Thermodynamic Function

\[ h_\mu(U_{\text{in}}) \quad h'_\mu(U_{\text{in}}) \quad \langle W \rangle(U_{\text{in}}) \]
Fluctuations in Thermodynamic Function

Informational Second Law

\[ \langle W \rangle \leq k_B T \ln 2 \left( h_{\mu}' - h_{\mu} \right) \]

<table>
<thead>
<tr>
<th>Feature</th>
<th>Operation</th>
<th>Net Work</th>
<th>Net Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>Extracts energy from the thermal reservoir, converts it into work by randomizing input information</td>
<td>( \langle W \rangle &gt; 0 )</td>
<td>( k_B - h_{\mu} &gt; 0 )</td>
</tr>
<tr>
<td>Eraser</td>
<td>Uses external input of work to remove input information</td>
<td>( \langle W \rangle &lt; 0 )</td>
<td>( k_B - h_{\mu} &lt; 0 )</td>
</tr>
<tr>
<td>Dud</td>
<td>Uses (wastes) stored work energy to randomize output</td>
<td>( \langle W \rangle &lt; 0 )</td>
<td>( k_B - h_{\mu} &gt; 0 )</td>
</tr>
</tbody>
</table>

⇒ Thermodynamic Function
Fluctuations in Thermodynamic Function

Information Ratchet
+ Fluctuation Spectroscopy
+ Informational Second Law (IPSL)

Informational Second Law \implies Thermodynamic Function

\[ \langle W \rangle \leq k_B T \ln 2 (h'_\mu - h_\mu) \]
**Requisite Complexity**

<table>
<thead>
<tr>
<th>Input Process</th>
<th>Ratchet Transducer</th>
<th>Output Process</th>
<th>Thermal Relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>memoryless input memoryless ratchet</td>
<td>$D \rightarrow 0:b 1:1-b$</td>
<td>$A \rightarrow 0:0:1-q 1:0:q 0:1:p 1:1:1-p$</td>
<td>$D \rightarrow 0:b(1-q)+(1-b)p 1:q+(1-b)(1-p)$</td>
</tr>
<tr>
<td>memoryless input memoryful ratchet</td>
<td>$D \rightarrow 0:b 1:1-b$</td>
<td>$A \rightarrow 0:0:1-p 1:0:p 1:1:1$</td>
<td>$D \rightarrow 0:b(1-p) 1:q+(1-b)$</td>
</tr>
<tr>
<td>memoryful input memoryless ratchet</td>
<td>$1 : 0.5$</td>
<td>$0 : 0.5$</td>
<td>$1 : 1.0$</td>
</tr>
<tr>
<td>memoryful input memoryful ratchet</td>
<td>$1 : 0.5$</td>
<td>$0 : 0.5$</td>
<td>$1 : 1.0$</td>
</tr>
</tbody>
</table>

• Lessons: Information processing thermodynamic systems should match the complexity of their inputs/environment:
  • Memoryless ratchets optimal for uncorrelated environments.
  • Memoryful ratchets optimal for correlated environments.


Thermodynamic Computing

- Theory
- **Design**
- Diagnosis
- Experiment
Erasing a Bit
Erasing a Bit

Potential

Erasure Protocol

Position

Control Parameters
Shortcuts via Counterdiabatic Control
Shortcuts via Counterdiabatic Control

(A) $t = 0$

(B) $t = 0^+$

(C) $0^+ < t < \tau^-$

(D) $t = \tau^-$

(E) $t = \tau$

\begin{align*}
\langle W_{t:0\rightarrow 0^+} \rangle &= F_{t=0^+}^{\text{eq}} - F_{t=0}^{\text{neq}} \\
\langle W_{t:0^+ \rightarrow \tau^-} \rangle &= \Delta F_{t:0^+ \rightarrow \tau^-}^{\text{eq}} + \langle W_{CD} \rangle \\
\langle W_{t:0 \rightarrow \tau^-} \rangle &= F_{t=\tau^-}^{\text{neq}} - F_{t=\tau^-}^{\text{eq}}
\end{align*}

Pr($X_t = x$): ---

$V(x,t)$: ---

$\frac{k_B T}{k_B T}$: ---
Shortcuts via Counterdiabatic Control

Dissipated work v. initial $b_i$ & final bit bias $b_f$

\[ b_i = 0.0 \quad b_i = 0.25 \quad b_i = 0.5 \]

\[
\begin{align*}
\pi^2 : \quad &L^2 F_1[p(\cdot)] = F_2[b(\cdot)]:
\end{align*}
\]
Shortcuts via Counterdiabatic Control

Dissipated work v. initial $b_i$ & final bit bias $b_f$

$b_i = 0.0$

$b_i = 0.25$

$b_i = 0.5$

$\pi^2$:

$\frac{\tau \langle W^{CD} \rangle}{L^2 F_1[p(\cdot)]} = F_2[b(\cdot)]$:

Perfect erasure in finite time at finite cost!
Shortcuts via Counterdiabatic Control

• Performance scaling:

\[ \langle W \rangle = k_B T \ln 2(H[Y_0] - H[Y_\tau]) + \frac{L^2}{\tau} F_1[p(\cdot)] F_2[b(\cdot)] \]

General Landauer

Counterdiabatic Work

Barrier Indepedent of \( \tau \) & \( L \)

Bit bias

• Trade-offs: work, speed, size of the information-bearing degrees of freedom, fidelity, and storage robustness
Thermodynamic Computing

- Theory
- Design
- **Diagnosis**
- Experiment
Fluctuation Theorems

\[
\frac{\Pr_F(+\beta W)}{\Pr_R(-\beta W)} = e^{\Delta F} e^{+\beta W}
\]

Fluctuation Theorems

- Work distribution during erasure: Complex!
Trajectory-Class Fluctuation Theorems

- Work distributions as a diagnostic
- Track microscopic information processing with only mesoscopic thermodynamics
- TCFTs for trajectory class C:

\[
\frac{\Pr_R(C^R)}{\Pr(C)} = \langle e^{-\beta W} \rangle_C
\]

- TCFTs interpolate btw integral and detailed FTs.

G. Wimsatt, O-P. Saira, A. B. Boyd, M. H. Matheny, S. Han, M. L. Roukes, and J. P. Crutchfield,
Blue distribution is Pre-Untilt Success trajectories
Orange is that of Pre-Untilt Fail trajectories
Green remaining Untilt Active trajectories.
Black curve is their sum, reconstructs the total work distribution.
Thermodynamic Computing

- Theory
- Design
- Diagnosis
- **Experiment** (Sorry, not today: Roukes@Caltech)
Thermodynamic Computing

• Theory
• Design
• Diagnosis
• Experiment

• Summary
Nanoenergy Stack

- Power dissipation per logic operation (J)

- Year range: 1995 to 2015

- CMOS nodes:
  - 250nm
  - 130nm
  - 90nm
  - 65nm
  - 45nm
  - 32nm
  - 22nm
Nanoenergy Stack

power dissipation per logic operation (J)

10^{-14} - 10^{-22}

year

1995 - 2015

CMOS nodes

250nm - 22nm

room temperature 100k_B T reliability limit

Michael Roukes, California Institute of Technology © Caltech 2015
Nanoenergy Stack

Power dissipation per logic operation (J)

- 250nm
- 130nm
- 90nm
- 65nm
- 45nm
- 32nm
- 22nm

CMOS nodes

- Room temperature thermal energy, $k_B T$
- Room temperature 100$k_B T$ reliability limit

Year:
- 1995
- 2000
- 2005
- 2010
- 2015

Michael Roukes, California Institute of Technology  © Caltech 2015
Nanoenergy Stack

Power dissipation per logic operation (J)

- 250nm
- 130nm
- 90nm
- 65nm
- 45nm
- 32nm
- 22nm

- Room temperature Landauer entropy, $k_B T \ln 2$
- Room temperature 100$ k_B T$ reliability limit
- Room temperature thermal energy, $k_B T$

Year


Michael Roukes, California Institute of Technology © Caltech 2015
Nanoenergy Stack

power dissipation per logic operation (J)

room temperature Landauer entropy, $k_B T \ln 2$

room temperature thermal energy, $k_B T$

room temperature 100$k_B T$ reliability limit

overhead

CMOS nodes

250nm
130nm
90nm
65nm
45nm
32nm
22nm


year

MICHAEL ROUKES, CALIFORNIA INSTITUTE OF TECHNOLOGY

© CALTECH 2015
## Principles of Thermodynamic Computing

<table>
<thead>
<tr>
<th>Principle</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information Destruction</td>
<td>Logically irreversible operations dissipate energy (Landauer, 1961)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>Logically nonreciprocal operations dissipate energy</td>
</tr>
<tr>
<td>Information Creation</td>
<td>Creating information dissipates heat (Aghamohammdi &amp; Crutchfield, 2017)</td>
</tr>
<tr>
<td>Information Process Second Law</td>
<td>Work to drive (or energy dissipated) during computation (Boyd, Mandal, &amp; Crutchfield, 2016a)</td>
</tr>
<tr>
<td>Requisite Complexity</td>
<td>Advantage maximized when controller matches environment (Boyd, Mandal, &amp; Crutchfield, 2016b)</td>
</tr>
<tr>
<td>Synchronization &amp; Error Correction</td>
<td>Work to correct errors or synchronize to environment (Boyd, Mandal, &amp; Crutchfield, 2017)</td>
</tr>
<tr>
<td>Modularity</td>
<td>Controller modularity is thermodynamically expensive (Boyd, Mandal, &amp; Crutchfield, 2018)</td>
</tr>
<tr>
<td>Information Dynamics</td>
<td>Maxwellian demons are chaotic dynamical systems (Boyd &amp; Crutchfield, 2016)</td>
</tr>
<tr>
<td>Steady-State Transitions</td>
<td>Work to drive transitions between information storage states (Riechers &amp; Crutchfield, 2017)</td>
</tr>
<tr>
<td>Functional Fluctuations</td>
<td>Engine functionality fluctuates in small systems, short times (Crutchfield &amp; Aghamohammdi, 2016)</td>
</tr>
<tr>
<td>Control tradeoffs</td>
<td>Counterdiabatic control dissipation design (Campbell &amp; De, 2017) (Boyd, Patra, Jarzynski, &amp; Crutchfield, 2018)</td>
</tr>
<tr>
<td>Reliability</td>
<td>Dissipation costs of high-reliability information processing</td>
</tr>
<tr>
<td>Trajectory-class fluctuations</td>
<td>Success and failure have thermodynamic signatures (Wimsatt, et al., 2019)</td>
</tr>
</tbody>
</table>

Table 2: Nonequilibrium thermodynamics of information processing in classical physical systems. For notation, refer to the cited works.
Power dissipation per logic operation ($J$)

- 45nm
- 32nm
- 22nm

Reliability limit

Energy, $k_B T$

Entropy, $k_B T \ln 2$

2010 | 2015

$10^{-16}$ | $10^{-17}$ | $10^{-18}$ | $10^{-19}$ | $10^{-20}$ | $10^{-21}$ | $10^{-22}$
power dissipation per logic operation (J)

Logical irreversibility

Reliability limit

Energy, $k_B T$

Entropy, $k_B T \ln 2$

45nm

32nm

22nm

overhead

2010 2015
power dissipation per logic operation (J)

reliability limit
energy, $k_B T$
entropy, $k_B T \ln 2$

overhead

Logical irreversibility

45nm  32nm  22nm
Landauer's Stack

Power dissipation per logic operation (J)

Reliability limit

Energy, $k_B T$

Entropy, $k_B T \ln 2$

Logical irreversibility

2010 2015
Landauer’s Stack

Power dissipation per logic operation ($J$)

Nonreciprocity
IPSL: $\langle W \rangle \leq k_B T \ln 2 \left( h_{\mu'} - h_\mu \right)$
Rate: $1/\tau$
Size: $L^2$
Reliability: $-\ln \epsilon$
Storage stability
Circuit modularity
NESS transitions
Trajectory-class FTs
Quantum Coherence

Logical irreversibility

45nm
32nm
22nm

Nanometer

Reliability limit

Energy, $k_B T$

Entropy, $k_B T \ln 2$

2010
2015
Landauer’s Stack

Nonreciprocity
IPSL: $\langle W \rangle \leq k_B T \ln 2 \ (h'_{\mu} - h_{\mu})$
Rate: $1/\tau$
Size: $L^2$
Reliability: $-\ln \epsilon$
Storage stability
Circuit modularity
NESS transitions
Trajectory-class FTs
Quantum Coherence

Experiment thermal energy resolution

Power dissipation per logic operation ($J$)
Landauer’s Stack

• Fundamental costs of information processing?

• Account for all currently-identified thermodynamic costs:
  Landauer irreversibility, nonreciprocity, IPSL, requisite complexity,
  randomness creation, prediction, functional fluctuations, sensing,
  synchronization, error correction, modularity, high reliability, …

• Predict total stack cost—energetics of extant computing

• If not, how far are we from doing so?

• What might we be missing?
Thermodynamic Computing Futures
Thermodynamic Computing

Roadmap To Date

• Design
  • Comparative optimality: Many ways (now) to design control protocols (geometric control, counterdiabatic, …)
  • Trade-offs: Speed v. energy v. accuracy v. …

• Diagnosis
  • Mesoscopics (e.g., work distributions) to diagnose success & failure

• Experiment
  • Flux qubits: an attractive platform
  • Thermodynamics of bit reset via a continuously-monitored flux cbit
  • Test current predictions
  • Work distributions display universal features of functional microscopic trajectories
Thermodynamic Computing Challenges

- Beyond overdamped: Fully underdamped dynamics
- Beyond detailed balance
- Beyond linear response
- Beyond Markovian: Hidden Markov dynamics, long-range correlations
- Beyond memoryless: Memoryful information processing, transducers
- Multivariate information theory: Beyond single bit, two-point mutual information, transfer entropy, …
- Information-bearing degrees of freedom?
- Strong system+environment & system+system coupling
- Nonlinear dynamics of information processing: Beyond NESS
- New analytics: Nonnormal, nondiagonalizable, …
- Information-engine circuits and lattices
Thanks!