Thermodynamic Computing: Fast, Cheap, and Under Control*

James P. Crutchfield
Complexity Sciences Center
Physics Department
University of California, Davis
http://csc.ucdavis.edu/~chaos/Talks/

Workshop on
Agency at the Interface of Quantum and Complexity Science
Nanyang Technological University
Singapore
13-16 January 2020

Joint work with Cina Aghamohammadi, Alec Boyd, Chris Ellison, Warren Fon, Christopher Jarzynski, Alexandra Jurgens, Sam Loomis, Dibyendu Mandal, Sarah Marzen, Matthew H. Matheny, Ayoti Patra, Paul Riechers, Michael Roukes, Olli-Pentti Saira, Susanne Still, Ariadna Venegas-Li, Greg Wimsatt & …

*With apologies to Errol Morris' & Rodney Brooks' *Fast, Cheap, and Out of Control*

SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
1951

Thermodynamic Computing: Fast, Cheap, and Under Control*

James P. Crutchfield
Complexity Sciences Center
Physics Department
University of California, Davis
http://csc.ucdavis.edu/~chaos/Talks/

Workshop on
Agency at the Interface of Quantum and Complexity Science
Nanyang Technological University
Singapore
13-16 January 2020

Joint work with Cina Aghamohammadi, **Alec Boyd**, Chris Ellison, Warren Fon, Christopher Jarzynski, **Alexandra Jurgens**, **Sam Loomis**, Dibyendu Mandal, **Sarah Marzen**, Matthew H. Matheny, Ayoti Patra, **Paul Riechers**, Michael Roukes, Olli-Pentti Saira, Susanne Still, **Ariadna Venegas-Li**, Greg Wimsatt & …

*With apologies to Errol Morris' & Rodney Brooks' *Fast, Cheap, and Out of Control*

Abstract

The paradigm of thermodynamic computing has arrived, driven by recent theoretical and experimental progress, pitched to circumvent the end of Moore's multi-decadal exponential progress in computing speed and density, and offered as a complement to quantum computing. I will review a recent planning effort (Computing Community Consortium, January 2019, Honolulu) aimed to accelerate reducing our recent progress to practice. As part of this I will also give a rather synoptic and optimistic survey of that progress, somewhat biased to the outputs from our multiyear Information Engines workshop series. Looking forward, I will address several open challenges. The first is to understand how the recent progress was built out of a calculus of limitations from deterministic unpredictability, quantum uncertainty, undecidability, and uncomputability to our current inability to track the flow of information, identify causal mechanisms, define structural complexity, and converge on unique explanatory models. What new limitations and innovations can we anticipate? The second challenge is to unpack what Landauer meant by "information-bearing degrees of freedom". Explicitly addressing this is key to an objective theory of physical computing. The final challenge is what I call Landauer's Stack: What are the actual thermodynamic costs of information processing? If we add up currently-identified thermodynamic costs–Landauer erasure, Information Processing Second Law, synchronization and error correction, implementation modularity, high reliability, and the like–can we accurately predict the energetics of contemporary and future computing? If not, how far are we from doing so and what might we be missing?

Physics of Information?

- Information Age!
- How can information be harnessed?
- What does it mean for a physical system to compute?
- Fundamental physical limits of information processing?

Physics of Computation Meeting (MIT, 1981)

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis
25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Feynman

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler

13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis

25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke

37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Wheeler

Physics of Computation Conferenc MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler

13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis

25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke

37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis
25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis
25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis
25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler
13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis
25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke
37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

Physics of Computation Meeting (MIT, 1981)

Physics of Computation Conference Endicott House MIT May 6-8, 1981

1 Freeman Dyson
2 Gregory Chaitin
3 James Crutchfield
4 Norman Packard
5 Panos Ligomenides
6 Jerome Rothstein
7 Carl Hewitt
8 Norman Hardy
9 Edward Fredkin
10 Tom Toffoli
11 Rolf Landauer
12 John Wheeler

13 Frederick Kantor
14 David Leinweber
15 Konrad Zuse
16 Bernard Zeigler
17 Carl Adam Petri
18 Anatol Holt
19 Roland Vollmar
20 Hans Bremerman
21 Donald Greenspan
22 Markus Buettiker
23 Otto Floberth
24 Robert Lewis

25 Robert Suaya
26 Stan Kugell
27 Bill Gosper
28 Lutz Priese
39 Madhu Gupta
30 Paul Benioff
31 Hans Moravec
32 Ian Richards
33 Marian Pour-El
34 Danny Hillis
35 Arthur Burks
36 John Cocke

37 George Michaels
38 Richard Feynman
39 Laurie Lingham
40 Thiagarajan
41 ?
42 Gerard Vichniac
43 Leonid Levin
44 Lev Levitin
45 Peter Gacs
46 Dan Greenberger

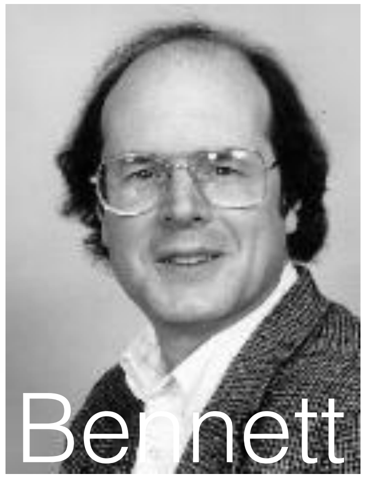
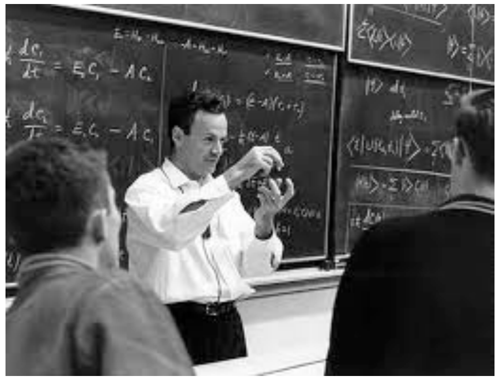
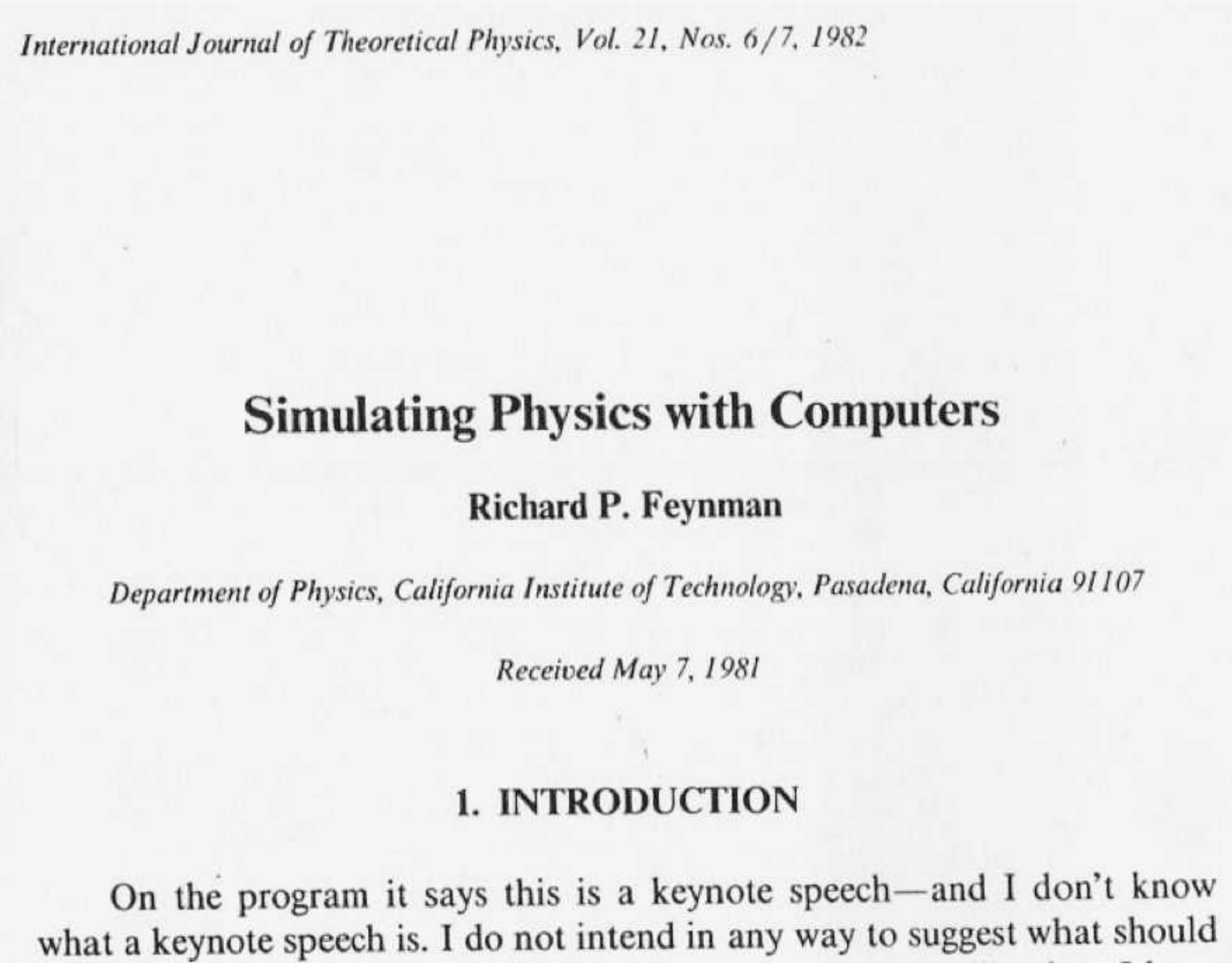


Photo: Charlie Bennett

Physics of Computation Meeting (MIT, 1981)

- Feynman introduces quantum computing





International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

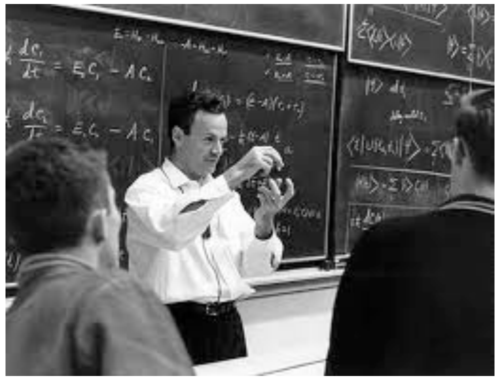
Received May 7, 1981

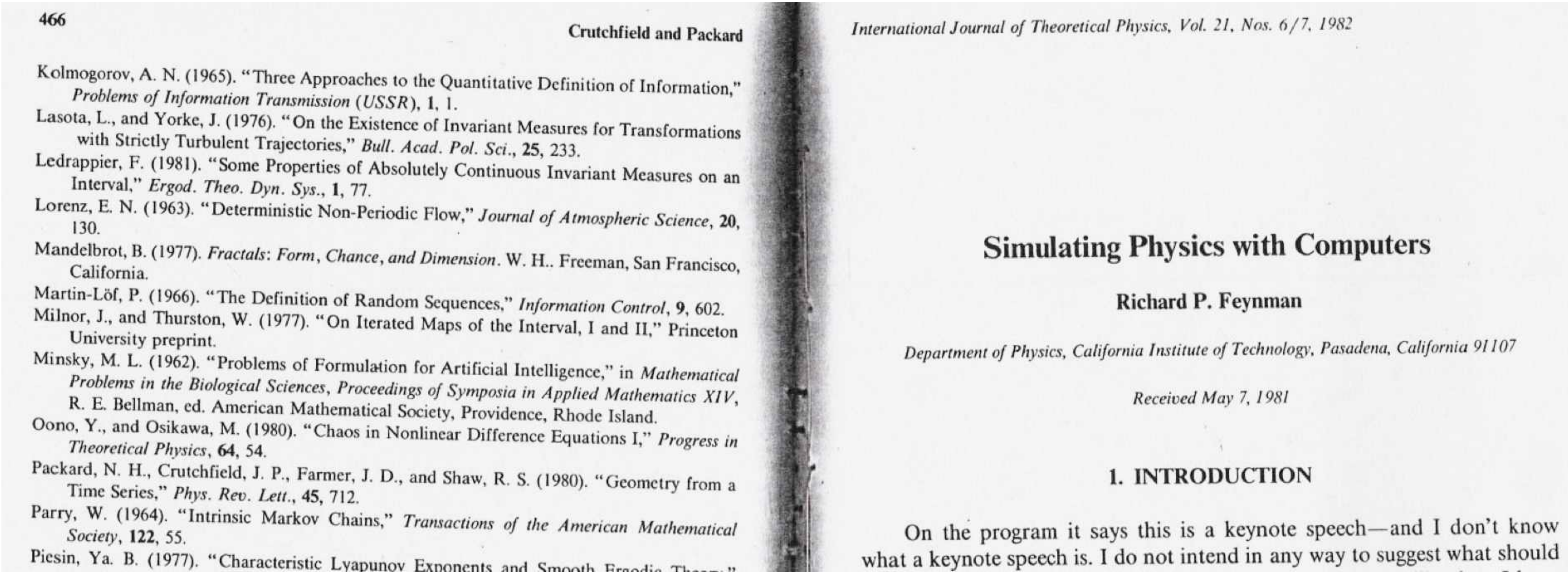
1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should

Physics of Computation Meeting (MIT, 1981)

- Feynman introduces quantum computing





Kolmogorov, A. N. (1965). "Three Approaches to the Quantitative Definition of Information," *Problems of Information Transmission (USSR)*, **1**, 1.

Lasota, L., and Yorke, J. (1976). "On the Existence of Invariant Measures for Transformations with Strictly Turbulent Trajectories," *Bull. Acad. Pol. Sci.*, **25**, 233.

Ledrappier, F. (1981). "Some Properties of Absolutely Continuous Invariant Measures on an Interval," *Ergod. Theo. Dyn. Sys.*, **1**, 77.

Lorenz, E. N. (1963). "Deterministic Non-Periodic Flow," *Journal of Atmospheric Science*, **20**, 130.

Mandelbrot, B. (1977). *Fractals: Form, Chance, and Dimension*. W. H.. Freeman, San Francisco, California.

Martin-Löf, P. (1966). "The Definition of Random Sequences," *Information Control*, **9**, 602.

Milnor, J., and Thurston, W. (1977). "On Iterated Maps of the Interval, I and II," Princeton University preprint.

Minsky, M. L. (1962). "Problems of Formulation for Artificial Intelligence," in *Mathematical Problems in the Biological Sciences, Proceedings of Symposia in Applied Mathematics XIV*, R. E. Bellman, ed. American Mathematical Society, Providence, Rhode Island.

Oono, Y., and Osikawa, M. (1980). "Chaos in Nonlinear Difference Equations I," *Progress in Theoretical Physics*, **64**, 54.

Packard, N. H., Crutchfield, J. P., Farmer, J. D., and Shaw, R. S. (1980). "Geometry from a Time Series," *Phys. Rev. Lett.*, **45**, 712.

Parry, W. (1964). "Intrinsic Markov Chains," *Transactions of the American Mathematical Society*, **122**, 55.

Piesin, Ya. B. (1977). "Characteristic Lyapunov Exponents and Smooth

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

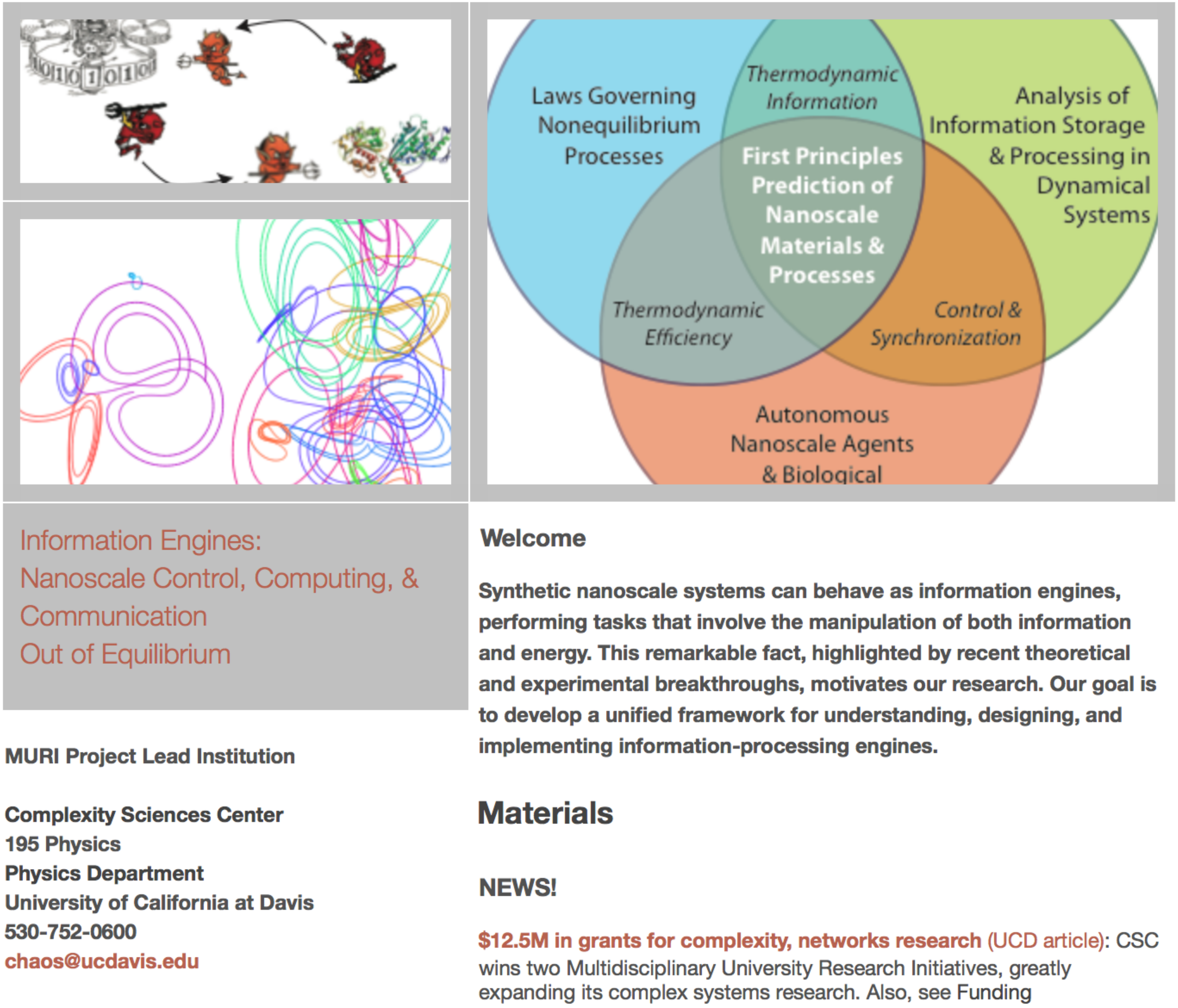
Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should

Intrinsic computation there, too!

Information Engines (2012-202X)



Information Engines:
Nanoscale Control, Computing, & Communication
Out of Equilibrium

Welcome

Synthetic nanoscale systems can behave as information engines, performing tasks that involve the manipulation of both information and energy. This remarkable fact, highlighted by recent theoretical and experimental breakthroughs, motivates our research. Our goal is to develop a unified framework for understanding, designing, and implementing information-processing engines.

MURI Project Lead Institution

Complexity Sciences Center
195 Physics
Physics Department
University of California at Davis
530-752-0600
chaos@ucdavis.edu

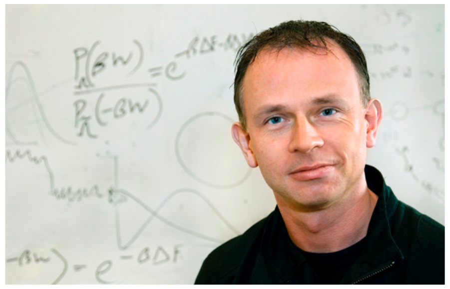
Materials

NEWS!

$12.5M in grants for complexity, networks research (UCD article): CSC wins two Multidisciplinary University Research Initiatives, greatly expanding its complex systems research. Also, see Funding

Information Engines

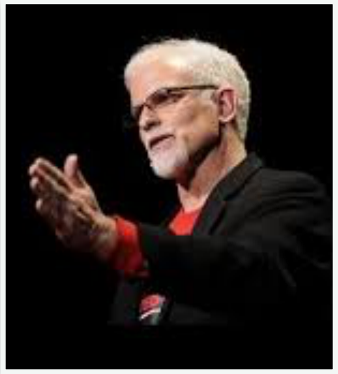
PIs



- **Gavin Crooks (Berkeley):** Thermodynamics of molecular machines, modeling and theoretical predictions, and search for basic thermodynamic and information-theoretic principles of nanoscale systems.
- **Jim Crutchfield (Davis) (Lead):** Theoretical methods to analyze the intrinsic computational properties of nanoscale systems and devices; algorithms to analyze experimental data.

Photo credit: S. Still

- **Mike DeWeese (Berkeley):** Optimal control protocols with the goal of efficient control of nanoscale system behaviors. devices and systems.
- **Michael Roukes (Caltech):** Nanoscale devices, Mesoscale physics, BRAIN Initiative

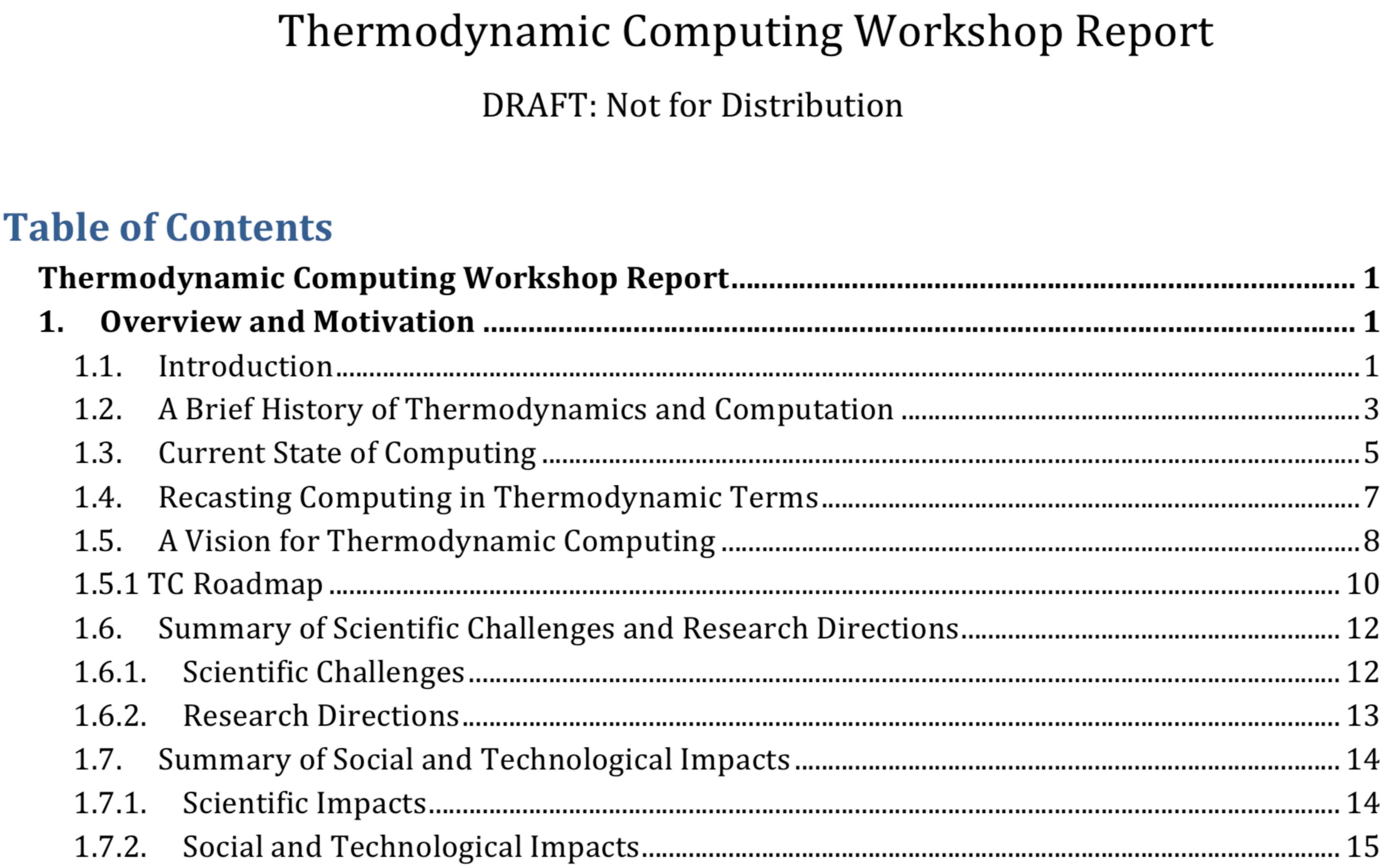


- **Chris Jarzynski (Maryland):** Adapt nonequilibrium thermodynamics to predict and control nanoscale system behavior and information processing.
- **P. S. "Krishna" Krishnaprasad (Maryland):** Extending control theory to apply to nanoscale devices and systems.

Visioning workshop on thermodynamic computing
3-5 January 2019
Honolulu, Hawaii

https://cra.org/ccc/events/thermodynamic-computing/
https://cra.org/ccc/wp-content/uploads/sites/2/2019/10/CCC-Thermodynamic-Computing-Reportv3.pdf

CCC
Computing Community Consortium
Catalyst



Thermodynamic Computing Workshop Report

DRAFT: Not for Distribution

Table of Contents

Thermodynamic Computing

- Theory
- Design
- Diagnosis
- Experiment

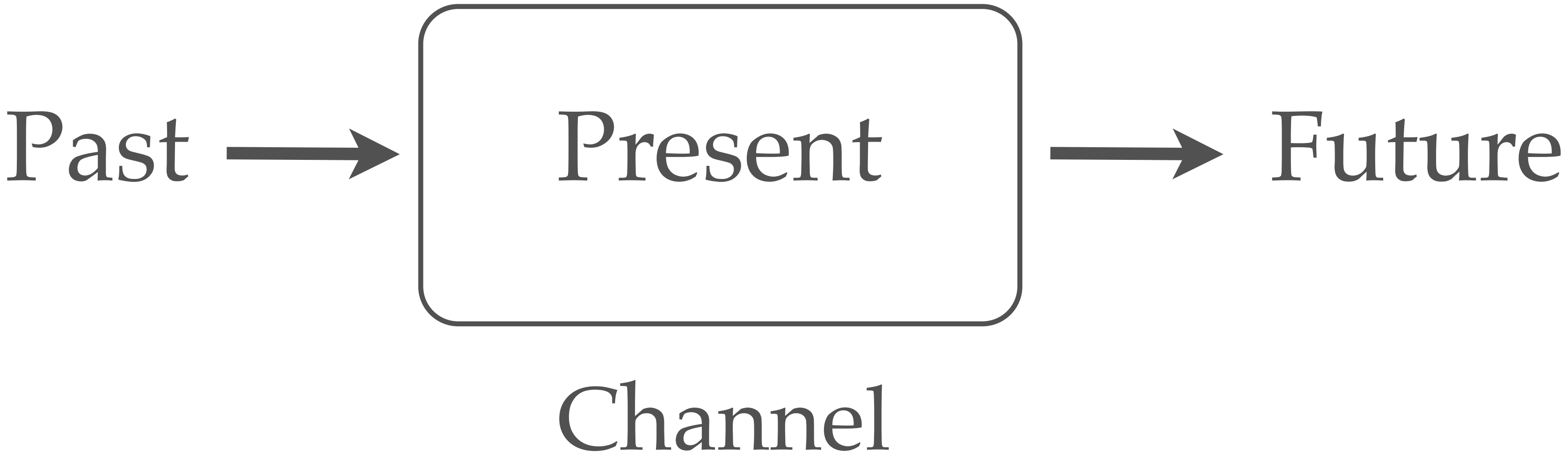
Thermodynamic Computing

- **Theory**
- Design
- Diagnosis
- Experiment

Intrinsic Computation

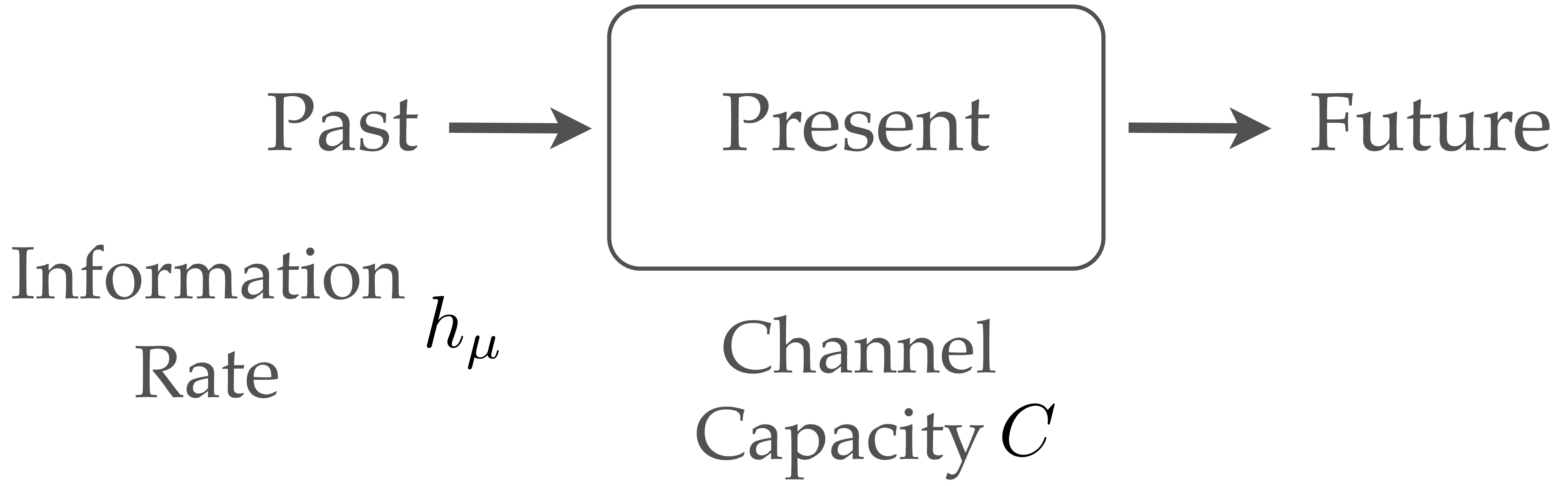
INFORMATION-THEORETIC ANALYSIS OF COMPLEX SYSTEMS ...

- Process $\Pr(\overleftarrow{X}, \overrightarrow{X})$ is a communication channel from the past $\overleftarrow{X}$ to the future $\overrightarrow{X}$:



INFORMATION-THEORETIC ANALYSIS OF COMPLEX SYSTEMS ...

- Process $\Pr(\overleftarrow{X}, \overrightarrow{X})$ is a communication channel from the past $\overleftarrow{X}$ to the future $\overrightarrow{X}$:



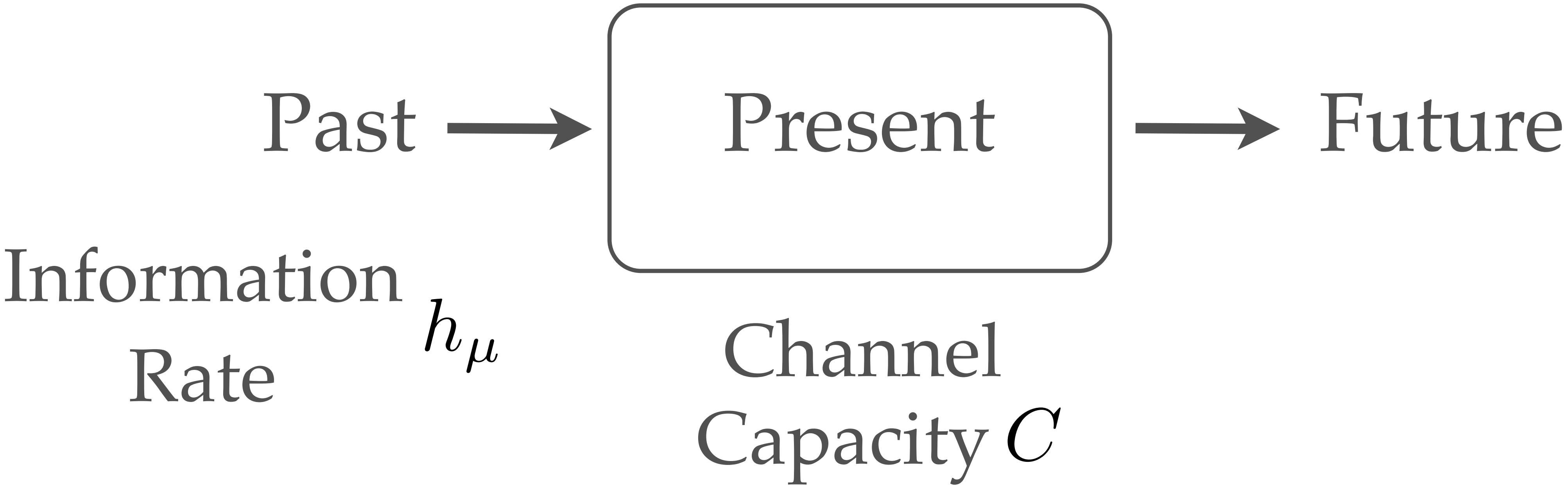
Past → Present → Future

Information Rate h_μ

Channel Capacity C

INFORMATION-THEORETIC ANALYSIS OF COMPLEX SYSTEMS ...

- Process $\Pr(\overleftarrow{X}, \overrightarrow{X})$ is a communication channel from the past $\overleftarrow{X}$ to the future $\overrightarrow{X}$:


Past

Present

Future

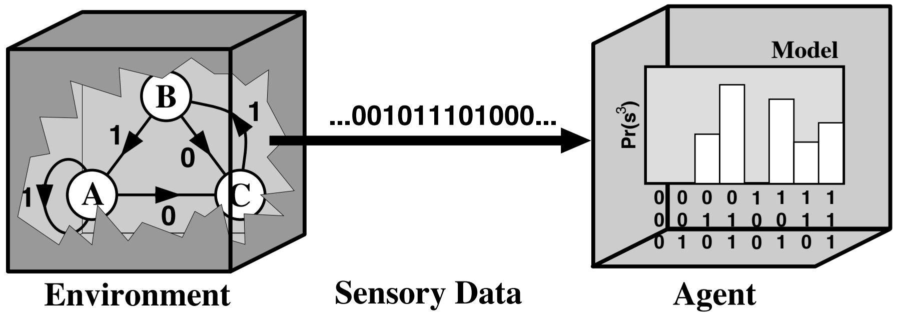
Information Rate h_μ

Channel Capacity C

- Channel Utilization: **Excess Entropy**

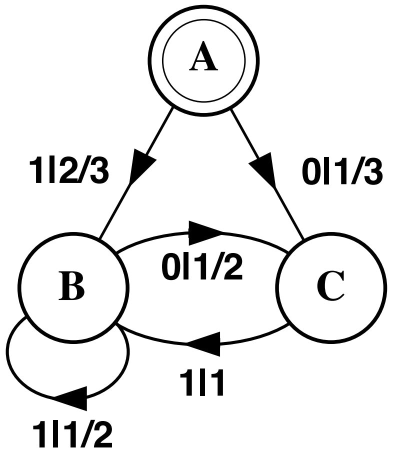
$$\mathbf{E} = I[\overleftarrow{X} ; \overrightarrow{X}]$$

Foundations: Computational Mechanics



Causal Equivalence:

$$\overleftarrow{x} \sim \overleftarrow{x}' \Leftrightarrow \Pr(\overrightarrow{X}|\overleftarrow{x}) = \Pr(\overrightarrow{X}|\overleftarrow{x}')$$

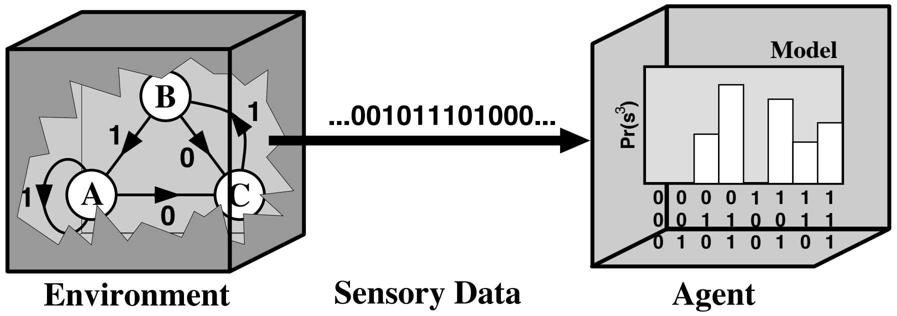


ε-Machine: Unique, minimal, & optimal predictor

Stored versus Generated Information

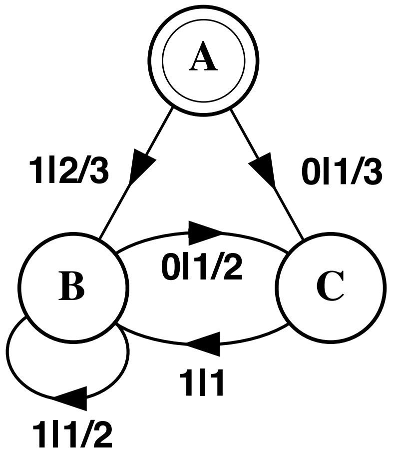
$$C_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \textbf{versus} \quad h_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \sum_{\sigma' \in \boldsymbol{\mathcal{S}}} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma)$$

Foundations: Computational Mechanics



Causal Equivalence:

$$\overleftarrow{x} \sim \overleftarrow{x}' \Leftrightarrow \Pr(\overrightarrow{X} | \overleftarrow{x}) = \Pr(\overrightarrow{X} | \overleftarrow{x}')$$

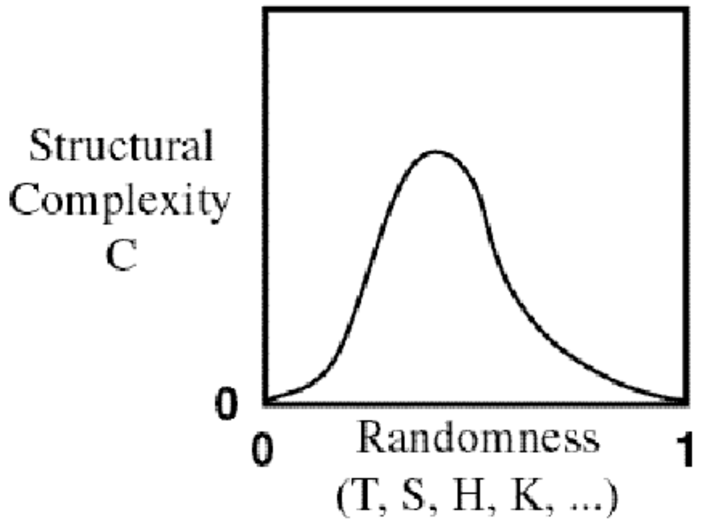


ε-Machine: Unique, minimal, & optimal predictor

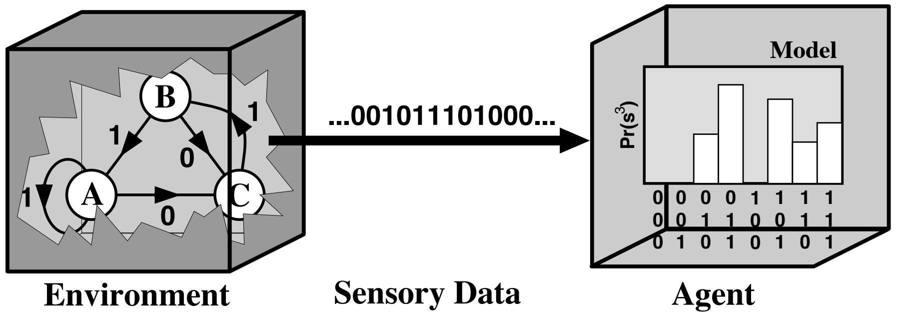
Stored versus Generated Information

$$C_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \textbf{versus} \quad h_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \sum_{\sigma' \in \boldsymbol{\mathcal{S}}} \Pr(\sigma' | \sigma) \log_2 \Pr(\sigma' | \sigma)$$

Structure versus Randomness

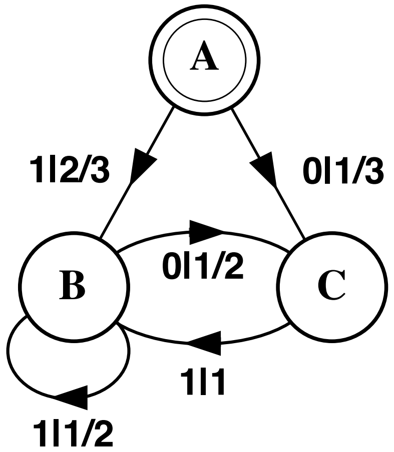


Foundations: Computational Mechanics



Causal Equivalence:

$$\overleftarrow{x} \sim \overleftarrow{x}' \Leftrightarrow \Pr(\overrightarrow{X}|\overleftarrow{x}) = \Pr(\overrightarrow{X}|\overleftarrow{x}')$$



ε-Machine: Unique, minimal, & optimal predictor

Stored versus Generated Information

$$C_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \text{versus} \quad h_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \sum_{\sigma' \in \boldsymbol{\mathcal{S}}} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma)$$

Intrinsic Computation:

1. How much historical information does a process store?
2. In what architecture is it stored?
3. How is it used to produce future behavior?

J.P. Crutchfield & K. Young, “Inferring Statistical Complexity”, Physical Review Letters 63 (1989) 105-108.

J.P. Crutchfield, “Between Order and Chaos”,
Nature Physics 8 (January 2012) 7-24.

COMPUTATIONAL MECHANICS

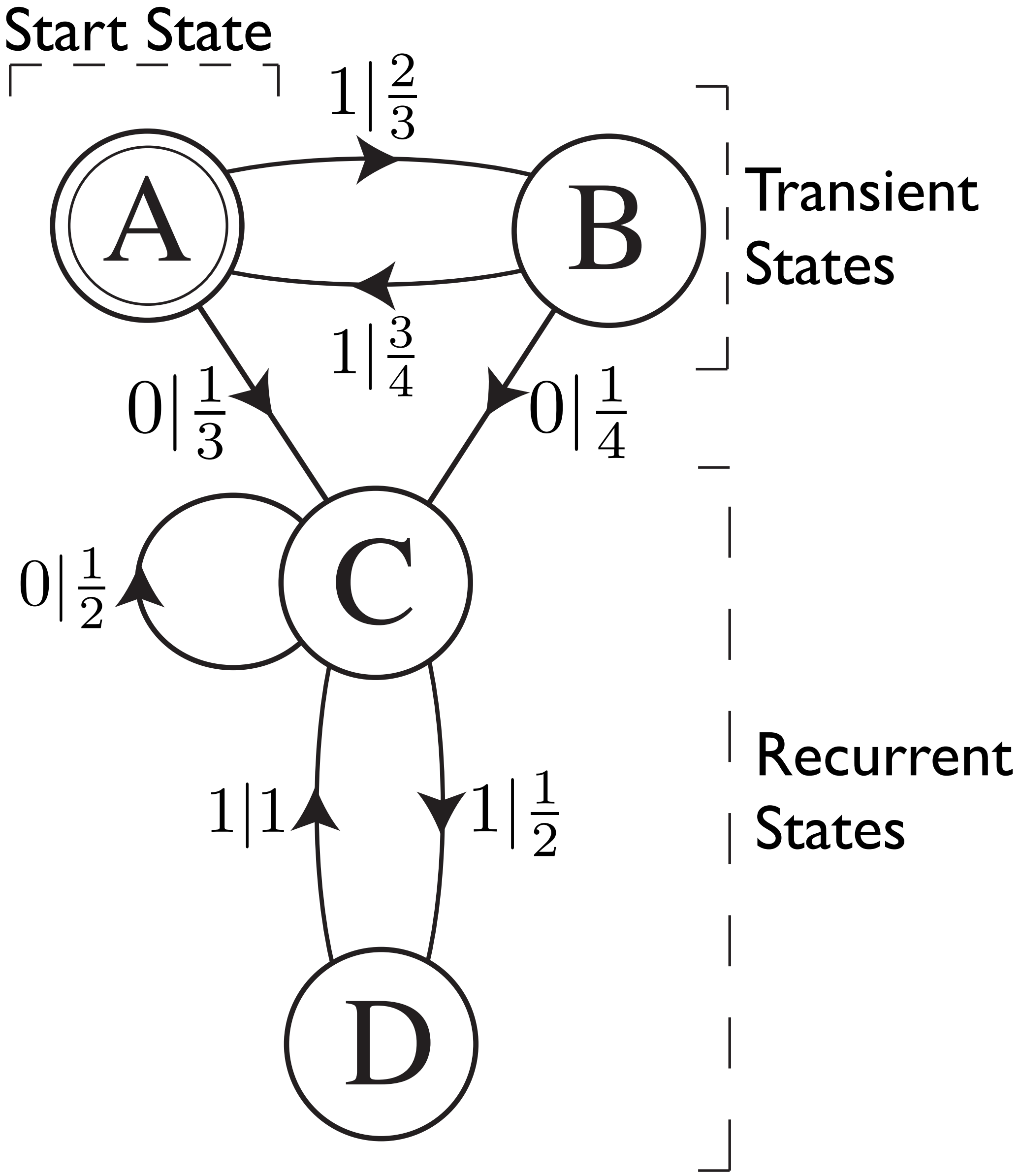
- ε-Machine:

$$M = \left\{ \boldsymbol{\mathcal{S}}, \{T^{(x)} : x \in \mathcal{A}\} \right\}$$

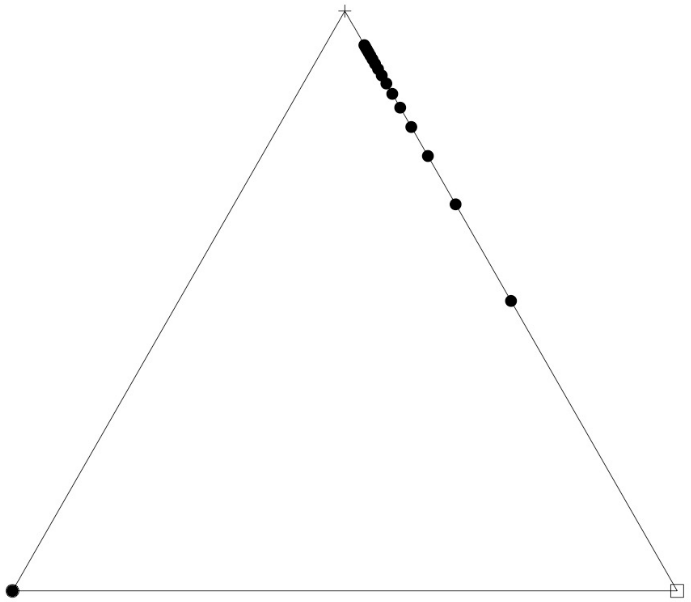
- Dynamic:

$$T^{(x)}_{\sigma,\sigma'} = \Pr(\sigma' | \sigma, x)$$

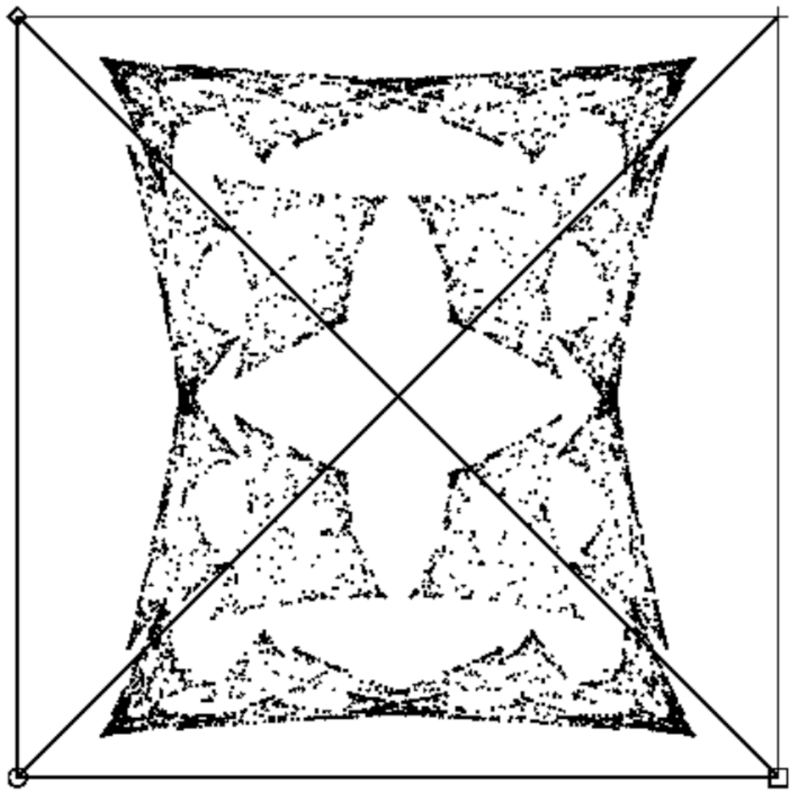
$$\sigma, \sigma' \in \boldsymbol{\mathcal{S}}$$



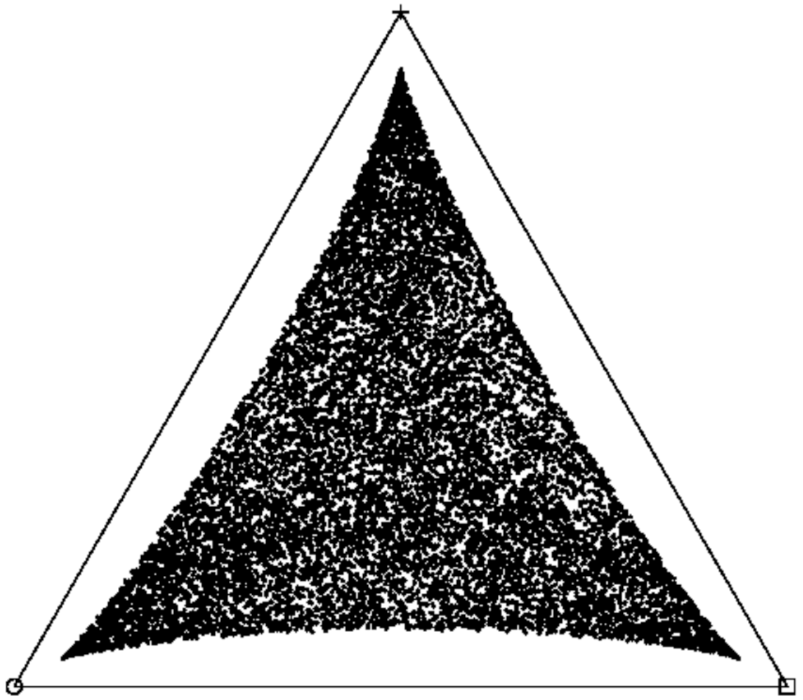
Varieties of ε-Machine



Denumerable
Causal States



Fractal



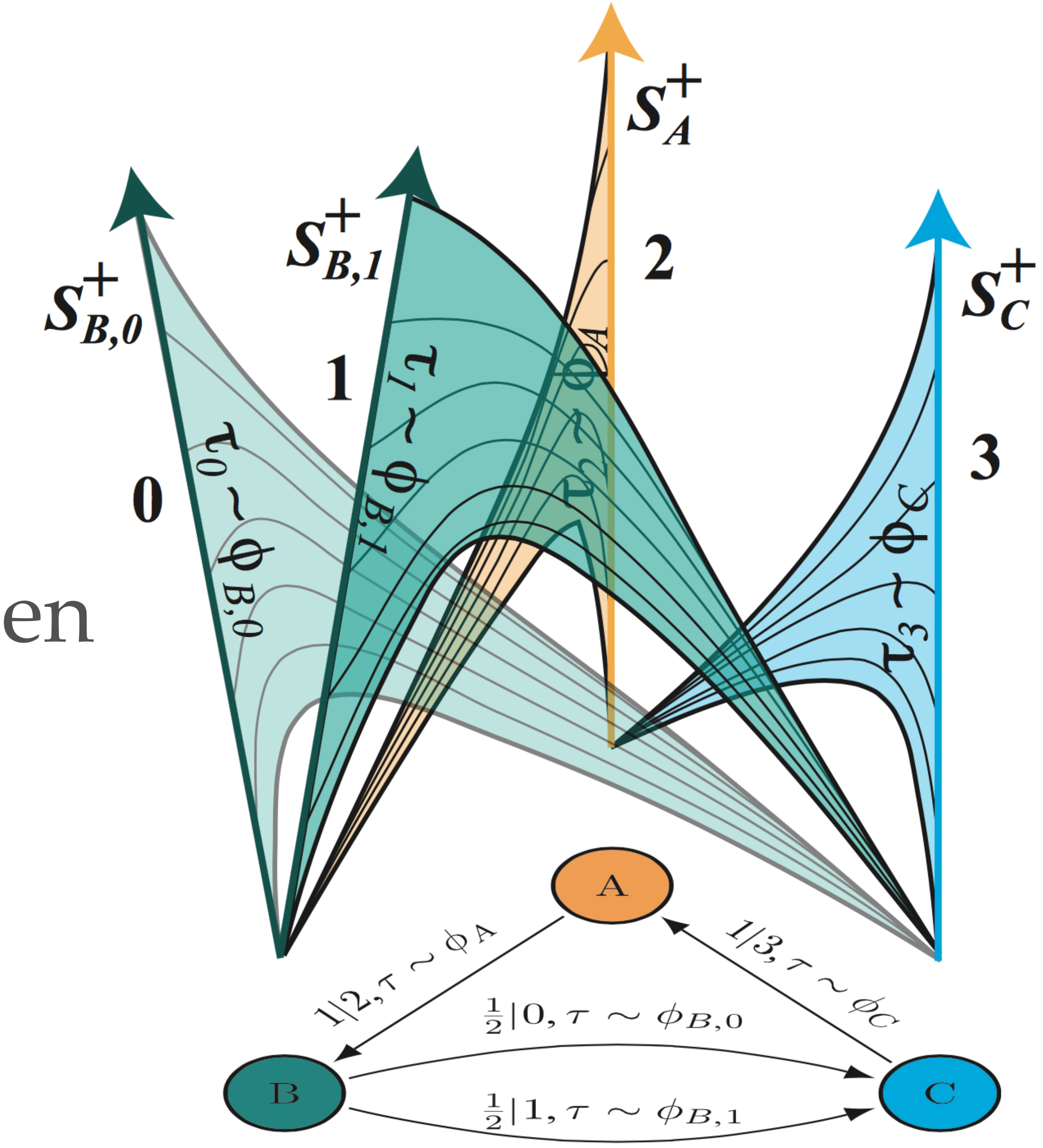
Continuous

J. P. Crutchfield, *Calculi of Emergence: Computation, Dynamics, and Induction*, **Physica D 75** (1994) 11-54.

Varieties of ε-Machine

Continuous-Time
Discrete-Event Processes

ε-Machine = Unifilar Hidden
Semi-Markov Processes



S. E. Marzen and J. P. Crutchfield, *Structure and Randomness of Continuous-Time Discrete-Event Processes*, **Journal of Statistical Physics** 169:2 (2017) 303-315.

Intrinsic Computation: Consequences

A system is unpredictable
if it has positive entropy rate: $h_\mu > 0$

A system is complex
if it has positive structural complexity measures: $C_\mu > 0$

A system is emergent
if its structural complexity increases over time:

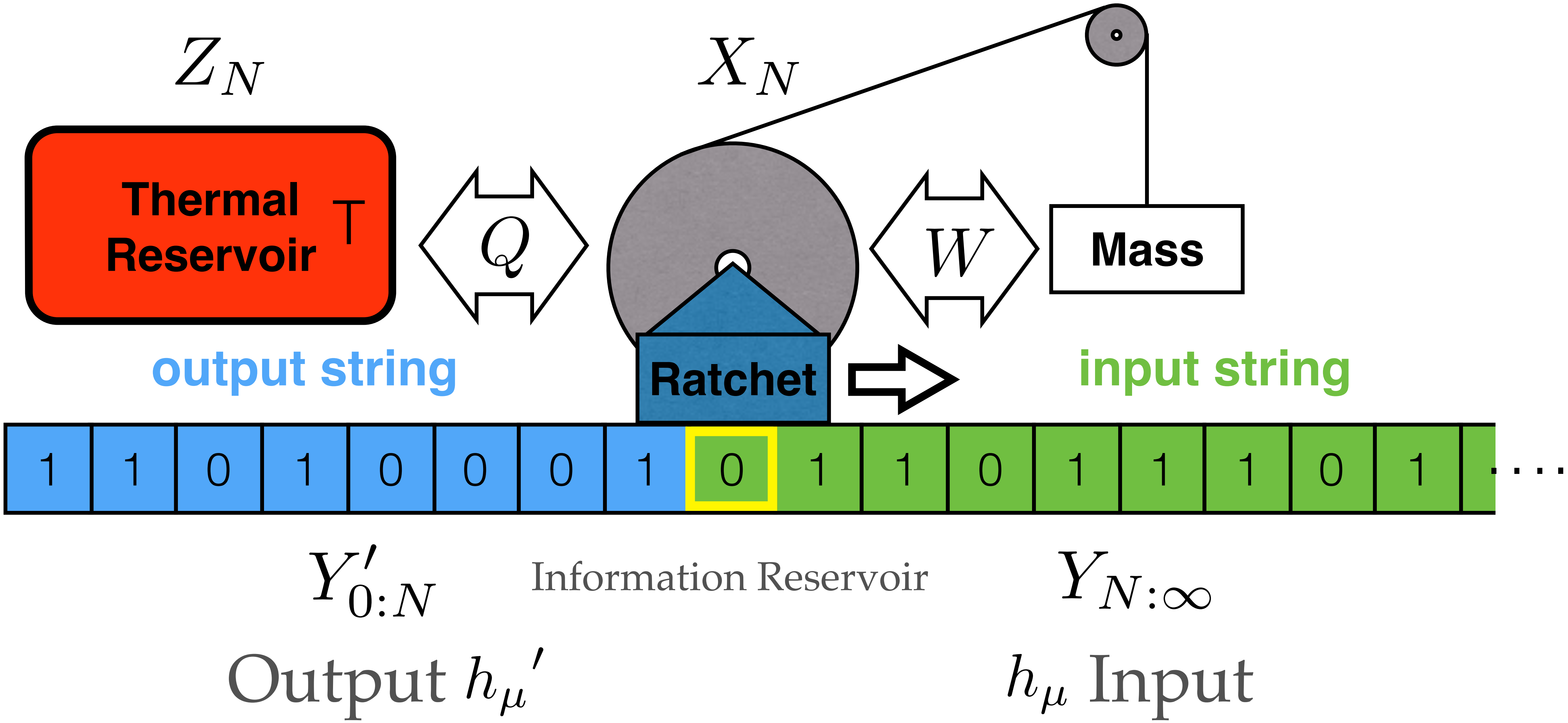
$$C_\mu(t') > C_\mu(t), \text{ if } t' > t$$

A system is hidden
if its crypticity is positive: $\chi = C_\mu - \mathbf{E} > 0$

Thermodynamics of Organization: Information Processing Second Law of Thermodynamics (IPSL)

INFORMATION RATCHETS

Beyond Maxwell+Szilard: Net Work Extraction!

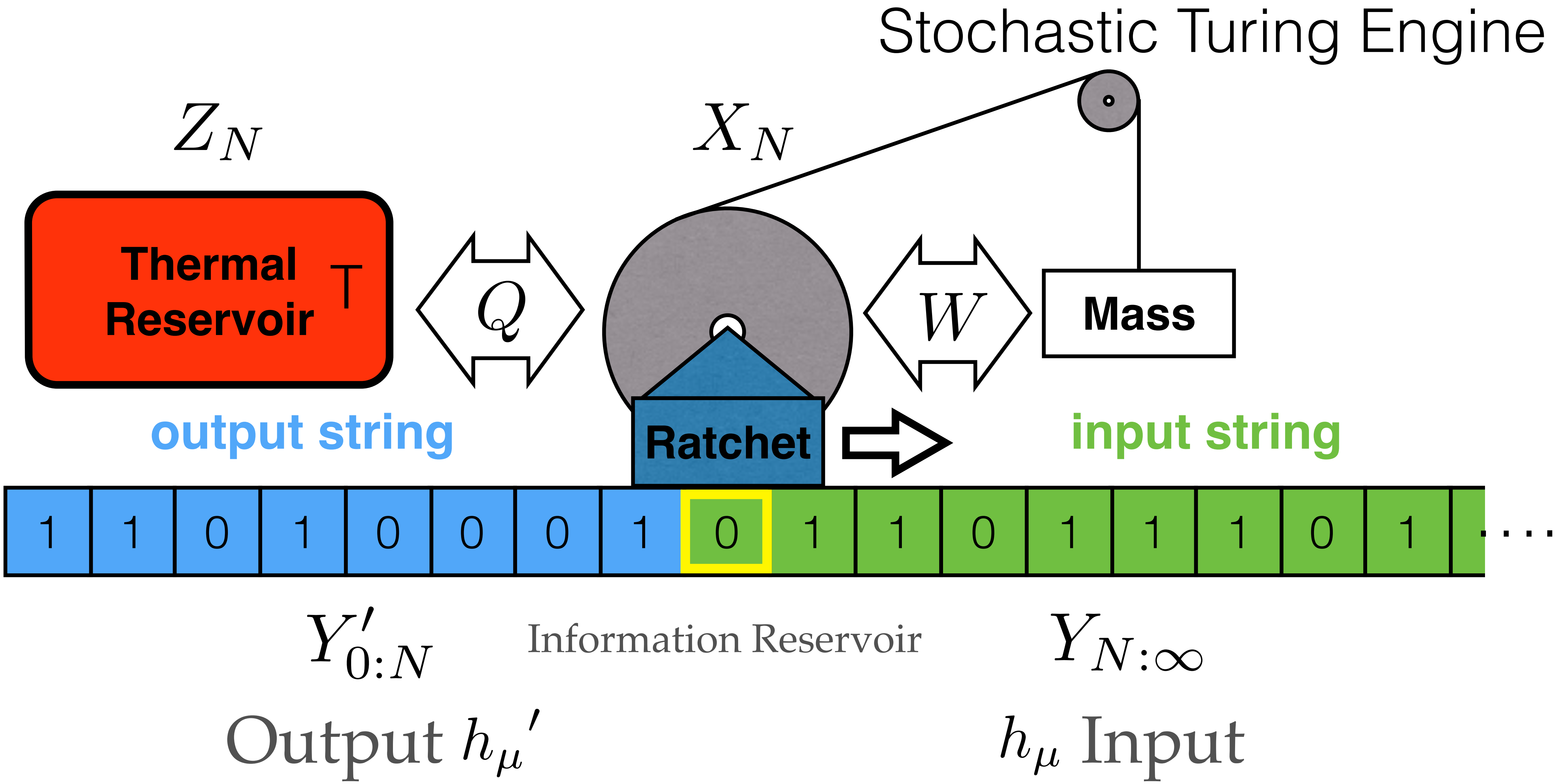


D. Mandal and C. Jarzynski. “Work and information processing in a solvable model of Maxwell’s demon”. Proc. Natl. Acad. Sci. USA, **109**(29):11641–11645, 2012.

A. Boyd, D. Mandal, and JPC, “Identifying Functional Thermodynamics in Autonomous Maxwellian Ratchets”. New Journal of Physics **18** (2016) 023149.

INFORMATION RATCHETS

Beyond Maxwell+Szilard: Net Work Extraction!



D. Mandal and C. Jarzynski. “Work and information processing in a solvable model of Maxwell’s demon”. Proc. Natl. Acad. Sci. USA, **109**(29):11641–11645, 2012.

A. Boyd, D. Mandal, and JPC, “Identifying Functional Thermodynamics in Autonomous Maxwellian Ratchets”. New Journal of Physics **18** (2016) 023149.

INFORMATION PROCESSING SECOND LAW OF THERMODYNAMICS

- Asymptotic IPSL:

$$\langle W \rangle \;\leq\; k_{\mathrm{B}} T \ln 2 \;\left({h_\mu}' - h_\mu \right)$$

- Information is fuel:

(Ordered inputs—correlations—are a thermodynamic resource.)

- Generalizes Landauer Principle; cf.:

$$Q_{\mathrm{erase}} \geq k_B T \ln 2 \qquad\qquad \big(\underbrace{{h_\mu}' = 0}_{\text{output}};\; \underbrace{h_\mu = 1}_{\text{input}}\big)$$

A. B. Boyd, D. Mandal, and JPC, *Identifying Functional Thermodynamics in Autonomous Maxwellian Ratchets*, **New Journal of Physics** 18 (2016) 023149.

INFORMATION PROCESSING SECOND LAW OF THERMODYNAMICS

- IPSL constrains information processing done by any thermodynamic system.
- Upper bound on the maximum average work $\langle W$ extracted per cycle.
- Lower bounds the amount $-\langle W \rangle$ of input work required for a physical system to support a given rate of intrinsic computation.

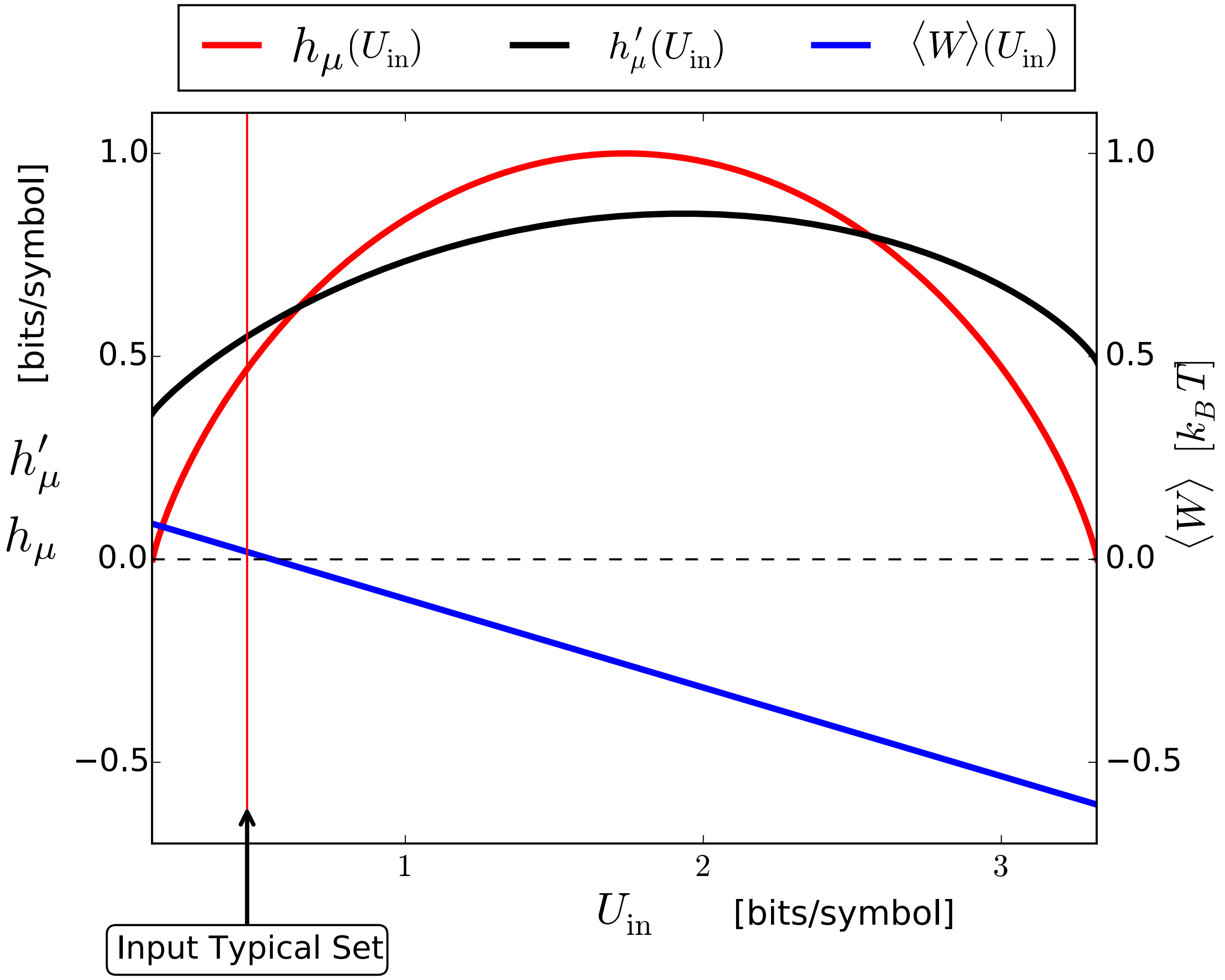
Information Processing Second Law of thermodynamics

IPSL determines Thermodynamic Functionality

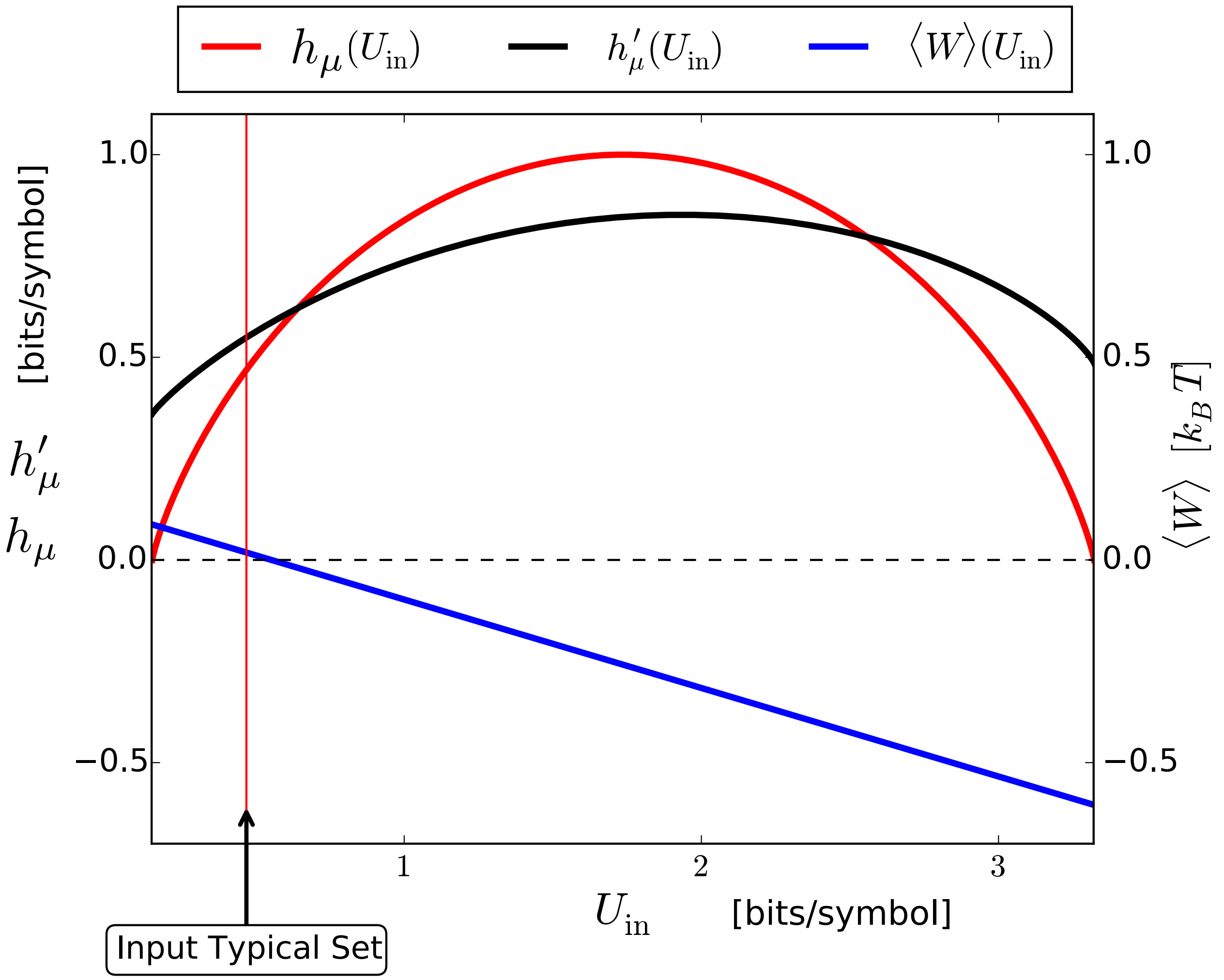
Feature Function	Operation	Net Work	Net Computation
Engine	Extracts energy from the thermal reservoir, converts it into work by randomizing input information	$\langle W \rangle > 0$	$h'_\mu - h_\mu > 0$
Eraser	Uses external input of work to remove input information	$\langle W \rangle < 0$	$h'_\mu - h_\mu < 0$
Dud	Uses (wastes) stored work energy to randomize output	$\langle W \rangle < 0$	$h'_\mu - h_\mu > 0$

$$\langle W \rangle \;\leq\; k_{\mathrm{B}} T \ln 2 \;\left({h_\mu}' - h_\mu \right)$$

FLUCTUATIONS IN THERMODYNAMIC FUNCTION



FLUCTUATIONS IN THERMODYNAMIC FUNCTION



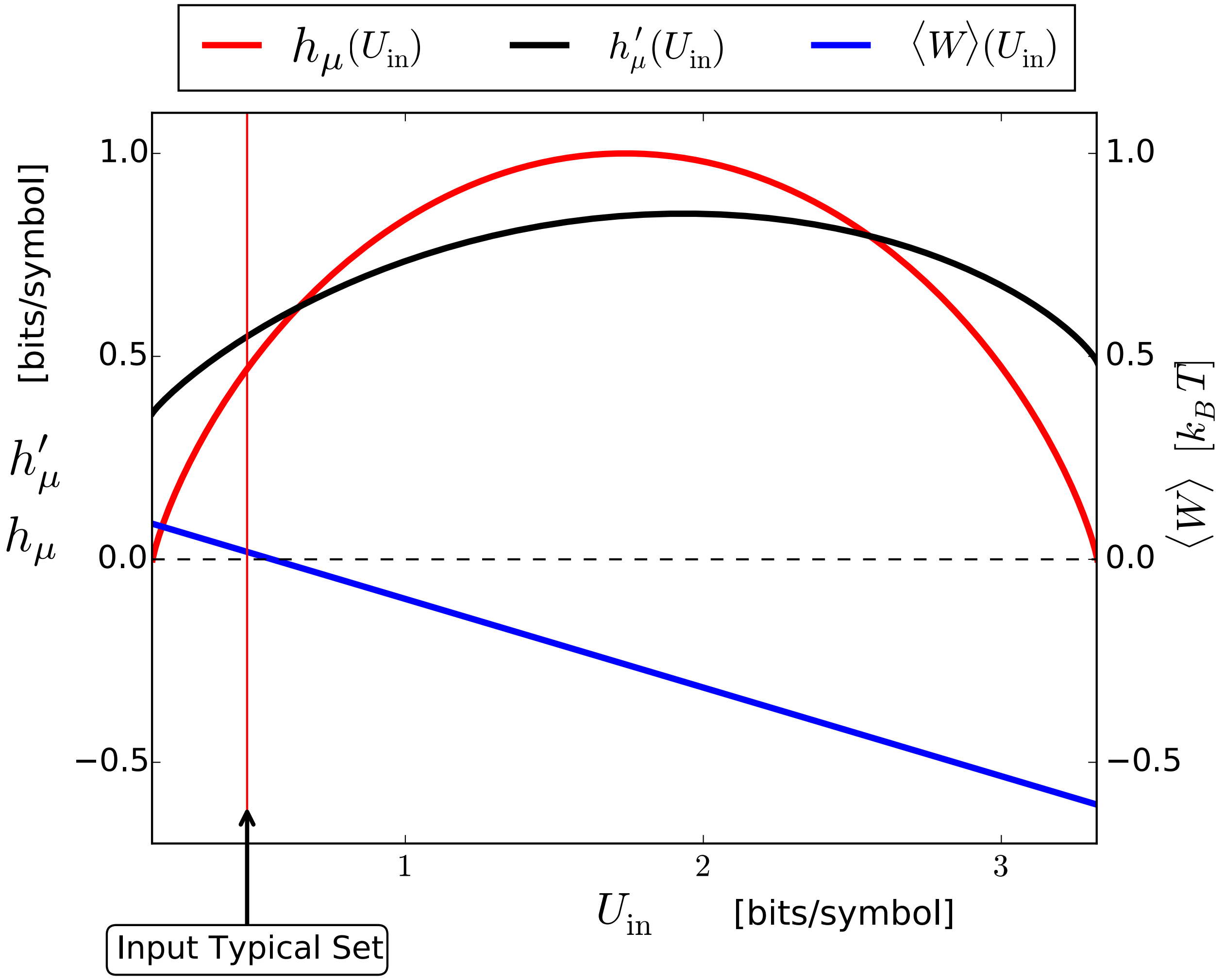
$$\langle W\rangle \;\leq\; k_{\mathrm{B}} T \ln 2 \;\left(h_\mu{}' - h_\mu\right)$$

Feature Function	Operation	Net Work	Net Computation
Engine	Extracts energy from the thermal reservoir, converts it into work by randomizing input information	$\langle W\rangle > 0$	$h'_\mu - h_\mu > 0$
Eraser	Uses external input of work to remove input information	$\langle W\rangle < 0$	$h'_\mu - h_\mu < 0$
Dud	Uses (wastes) stored work energy to randomize output	$\langle W\rangle < 0$	$h'_\mu - h_\mu > 0$

Informational Second Law

$\Rightarrow$ Thermodynamic Function

Fluctuations in Thermodynamic Function

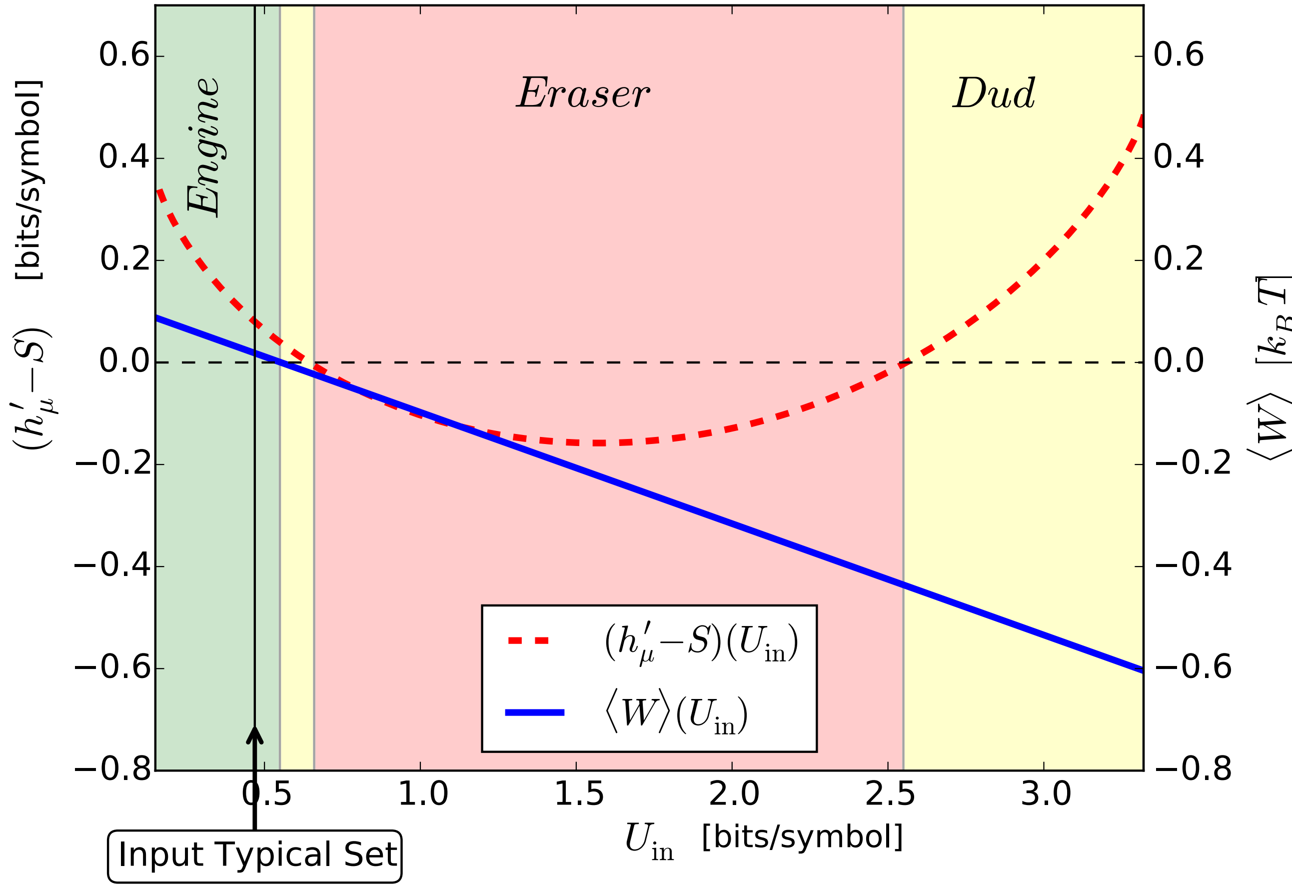


Information Ratchet
+ Fluctuation Spectroscopy
+ Informational Second Law (IPSL)

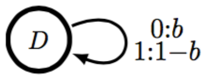
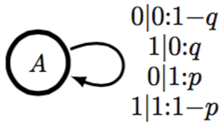
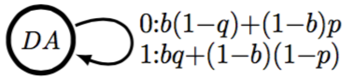
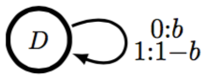
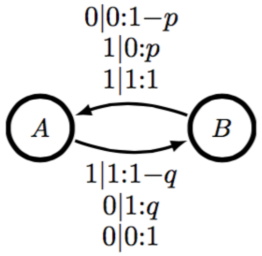
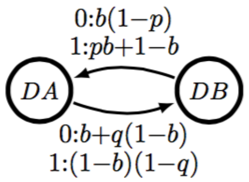
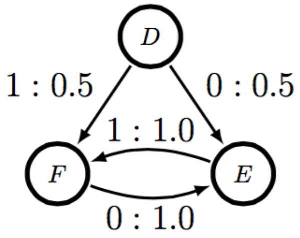
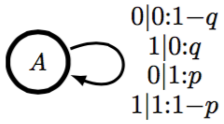
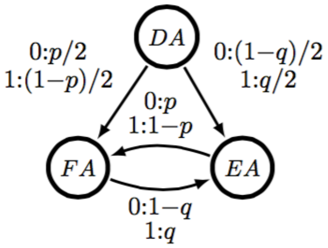
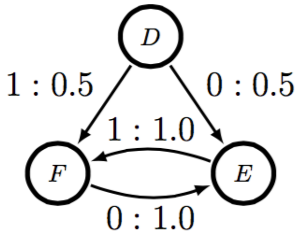
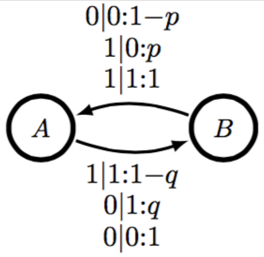
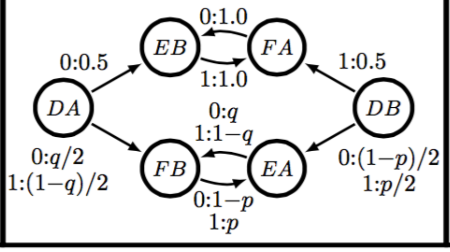
$$\langle W \rangle \leq k_{\text{B}} T \ln 2 \, (h_\mu{}' - h_\mu)$$

Feature / Function	Operation	Net Work	Net Computation
Engine	Extracts energy from the thermal reservoir, converts it into work by randomizing input information	$\langle W \rangle > 0$	$h'_\mu - h_\mu > 0$
Eraser	Uses external input of work to remove input information	$\langle W \rangle < 0$	$h'_\mu - h_\mu < 0$
Dud	Uses (wastes) stored work energy to randomize output	$\langle W \rangle < 0$	$h'_\mu - h_\mu > 0$

Informational Second Law
$\Rightarrow$ Thermodynamic Function



REQUISITE COMPLEXITY

	Input Process	Ratchet Transducer	Output Process	Thermal Relations
memoryless input memoryless ratchet				$0 = H_1 - h_\mu = H'_1 - h'_\mu$ $\langle W \rangle \le \Delta h_\mu = \Delta H_1$
memoryless input memoryful ratchet				$0 = H_1 - h_\mu \le H'_1 - h'_\mu$ $\langle W \rangle \le \Delta h_\mu \le \Delta H_1$
memoryful input memoryless ratchet				$0 \le H'_1 - h'_\mu \le H_1 - h_\mu$ $\langle W \rangle \le \Delta H_1 \le \Delta h_\mu$
memoryful input memoryful ratchet				$H'_1 - h'_\mu \overset{?}{=} H_1 - h_\mu$ $\langle W \rangle \le \Delta h_\mu$ ~~$\langle W \rangle \le \Delta H_1$~~

- Lessons: Information processing thermodynamic systems should match the complexity of their inputs/environment:
  - Memoryless ratchets optimal for uncorrelated environments.
  - Memoryful ratchets optimal for correlated environments.

A. B. Boyd, D. Mandal, and JPC, *Leveraging Environmental Correlations: The Thermodynamics of Requisite Variety*, **Journal of Statistical Physics** 167:6 (2017) 1555-1585. .

W. Ross Asbhy, **An Introduction to Cybernetics,** John Wiley and Sons, New York, second edition, 1960.

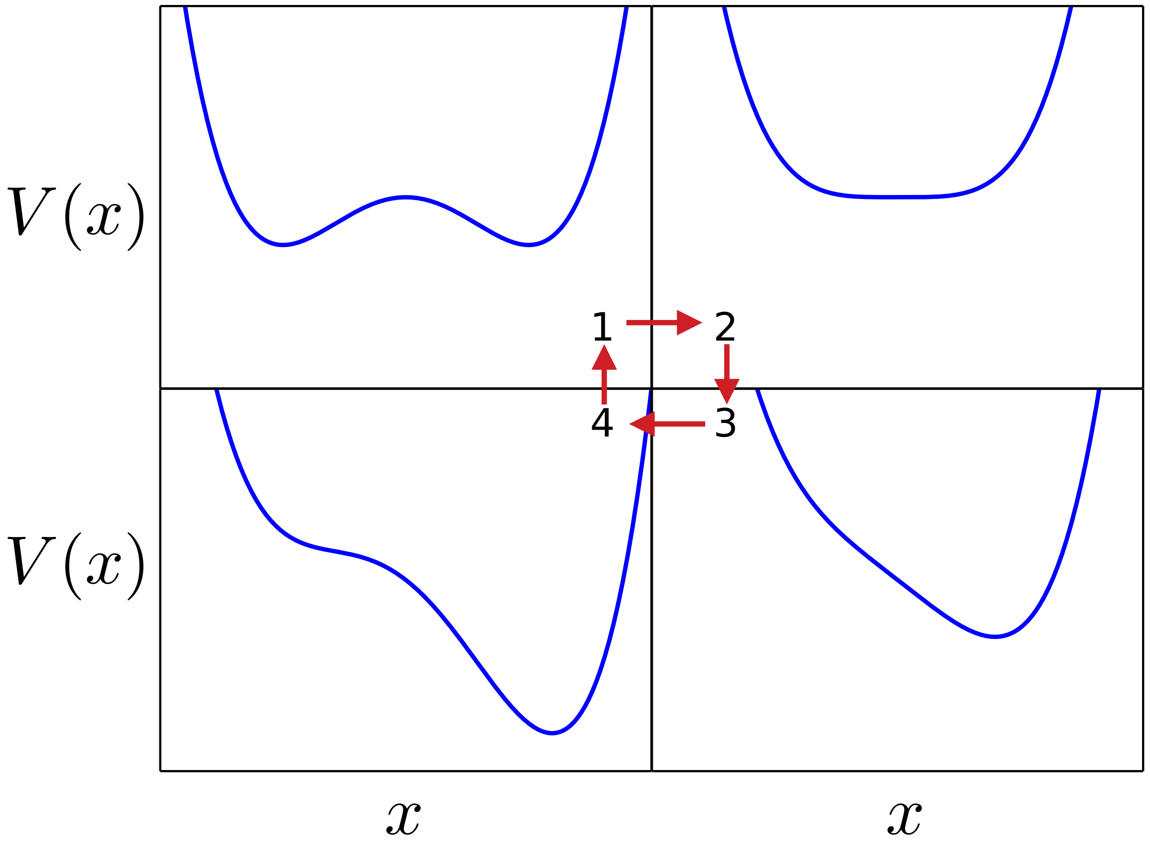
Thermodynamic Computing

- Theory
- **Design**
- Diagnosis
- Experiment

Erasing a Bit

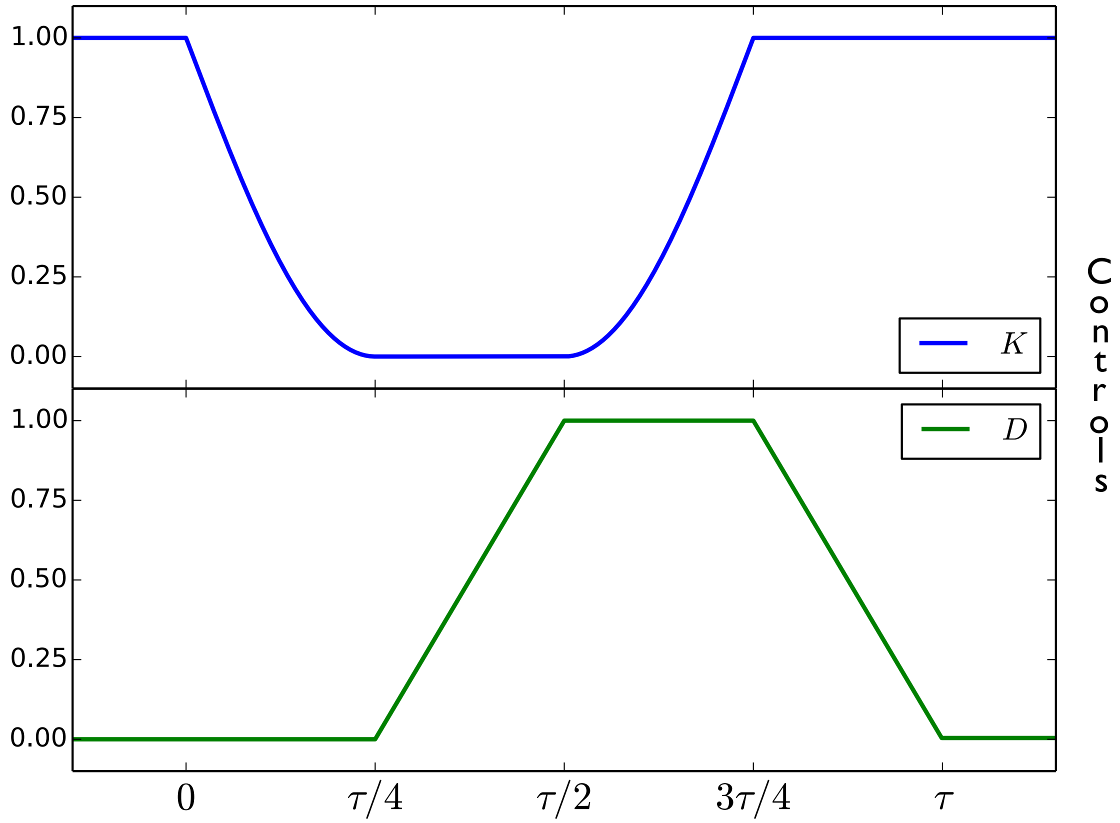
Erasing a Bit

Potential

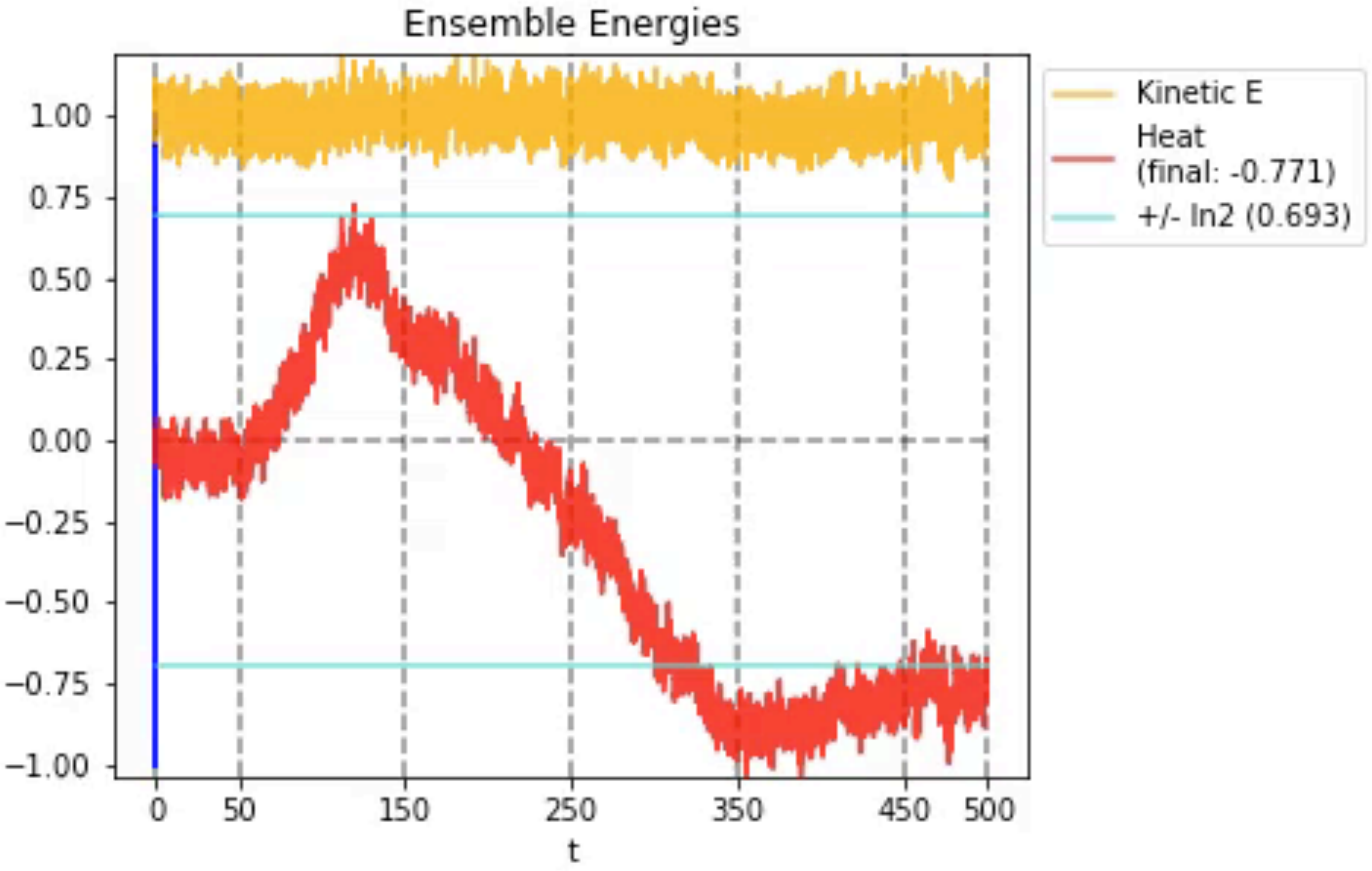


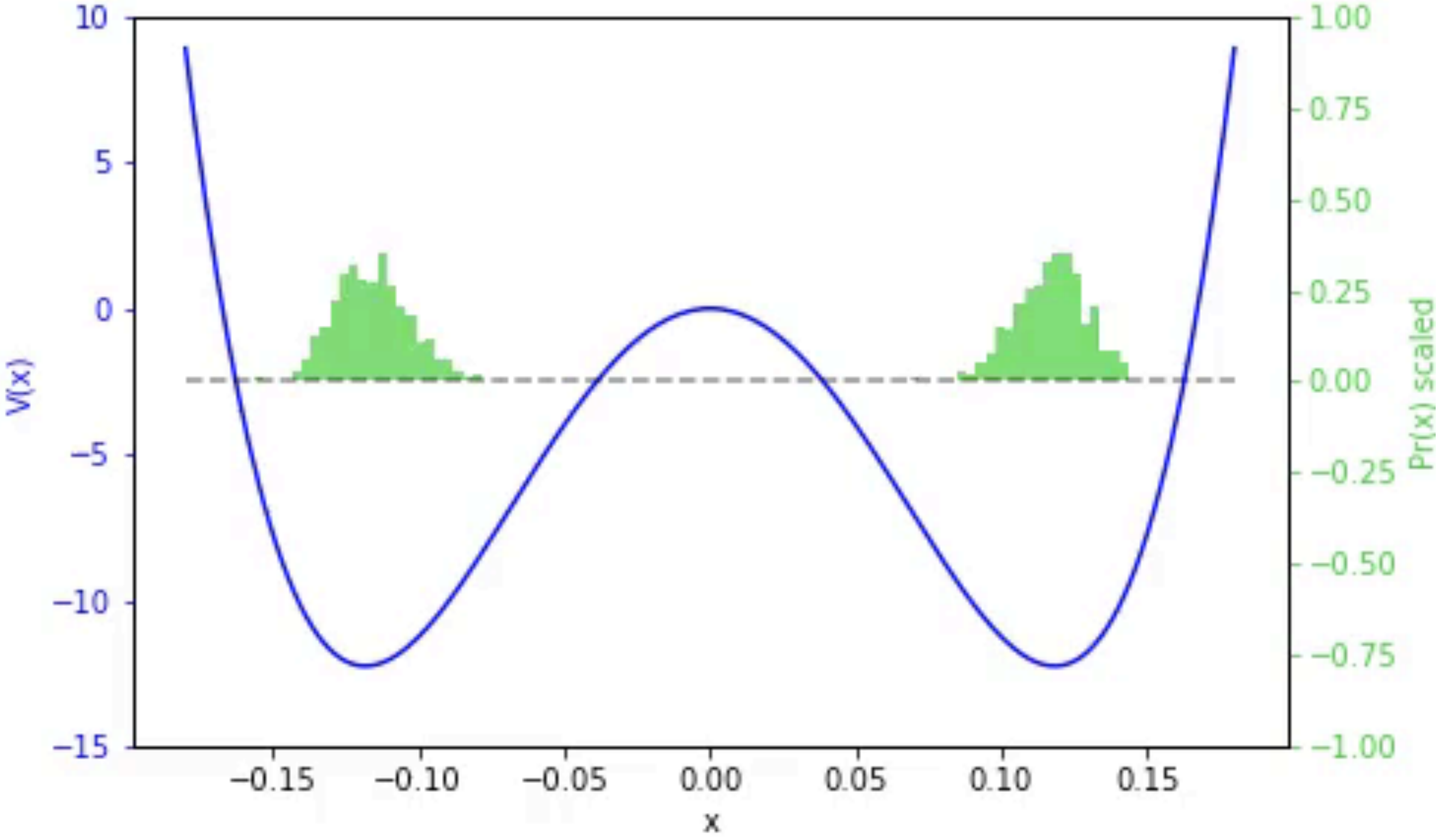
Position

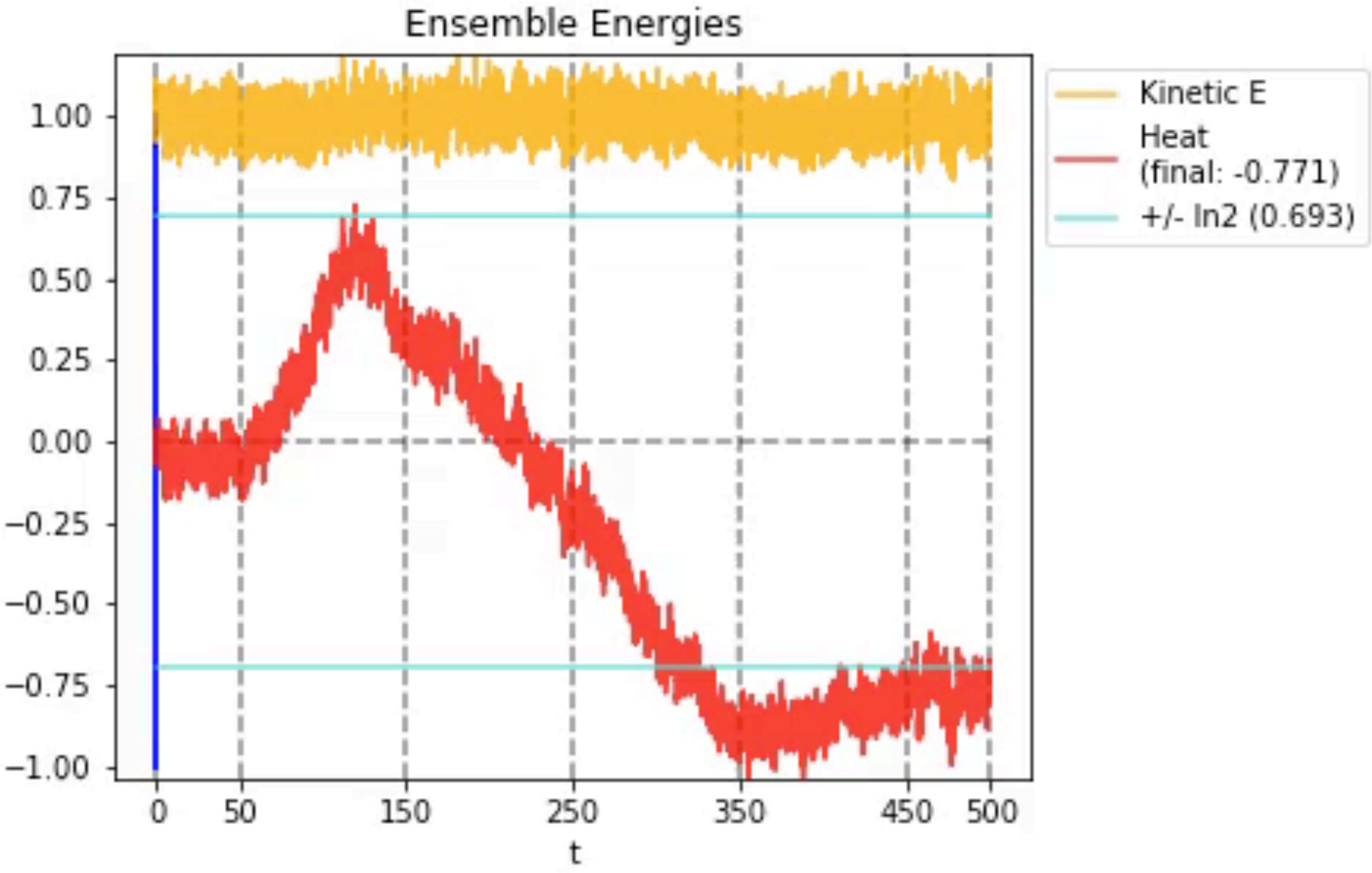
Erasure Protocol

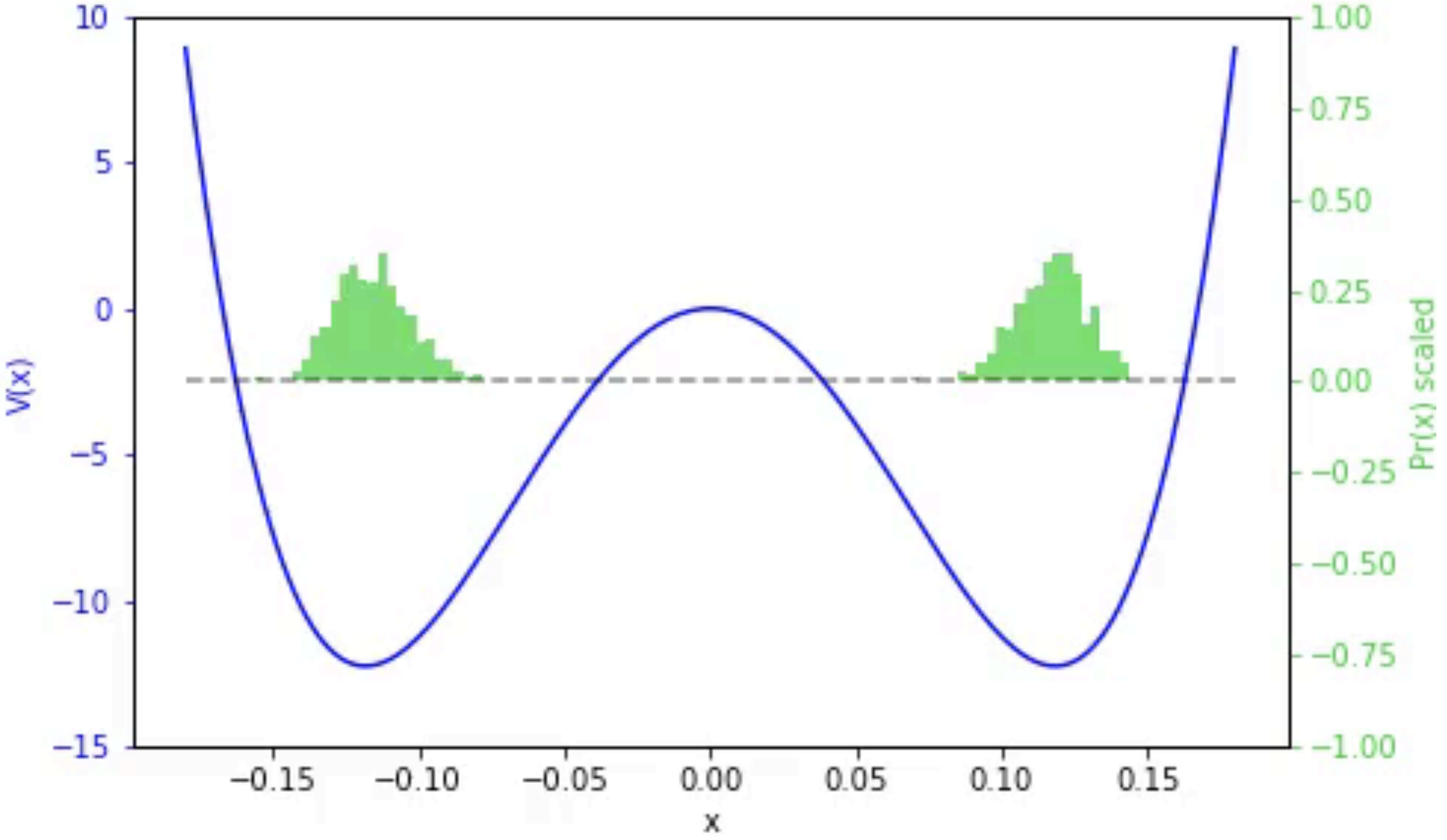


Control Parameters


Ensemble Energies

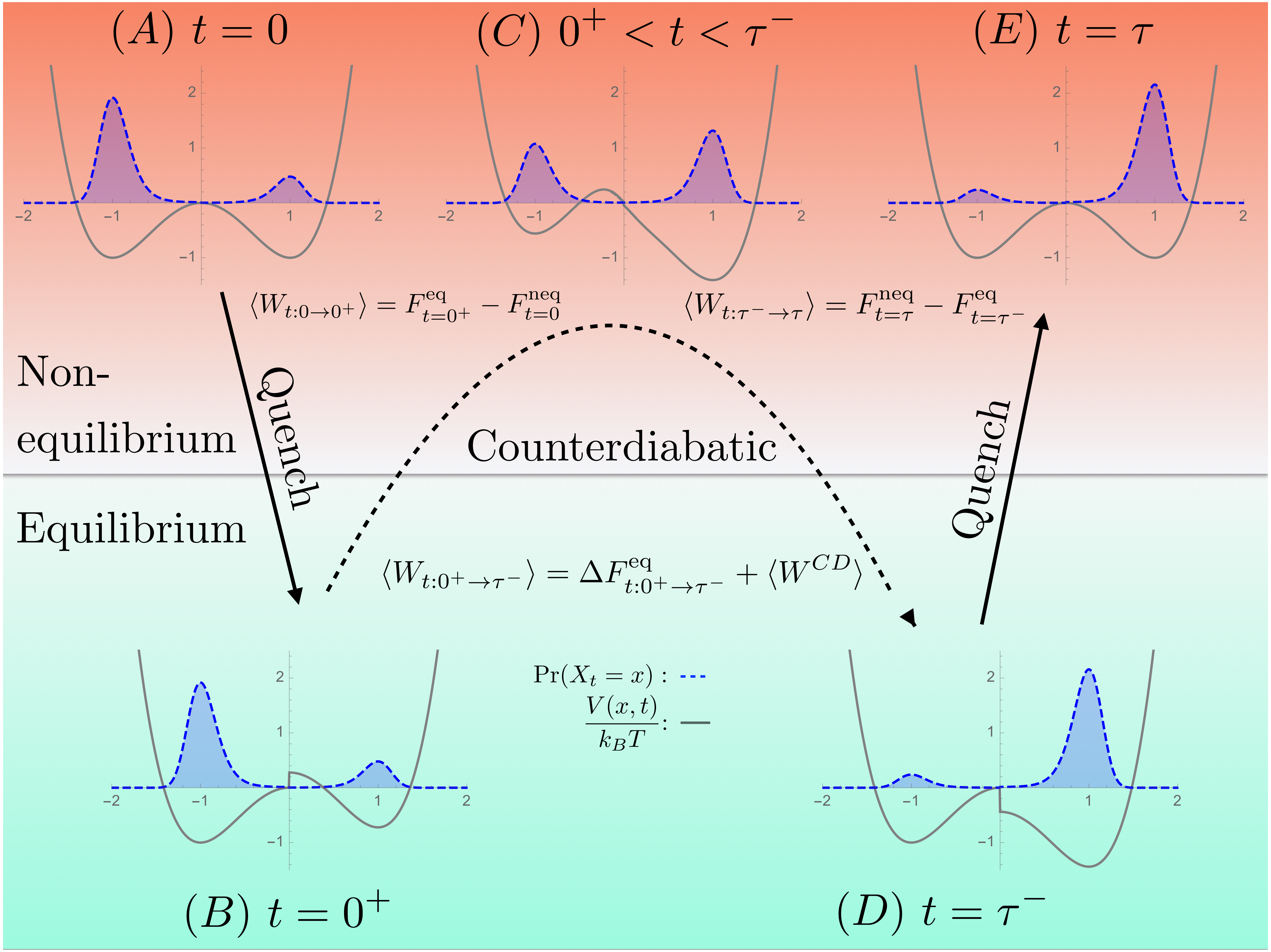



Ensemble Energies
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
50
150
250
350
450
500
t
Kinetic E
Heat
(final: -0.771)
+/- ln2 (0.693)


10
5
0
−5
−10
−15
V(x)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Pr(x) scaled
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
x

Shortcuts via Counterdiabatic Control

Shortcuts via Counterdiabatic Control



$(A)\ t = 0$

$(C)\ 0^+ < t < \tau^-$

$(E)\ t = \tau$

$\langle W_{t:0\to 0^+}\rangle = F^{\mathrm{eq}}_{t=0^+} - F^{\mathrm{neq}}_{t=0}$

$\langle W_{t:\tau^-\to \tau}\rangle = F^{\mathrm{neq}}_{t=\tau} - F^{\mathrm{eq}}_{t=\tau^-}$

Non-equilibrium

Quench

Counterdiabatic

Quench

Equilibrium

$\langle W_{t:0^+\to \tau^-}\rangle = \Delta F^{\mathrm{eq}}_{t:0^+\to \tau^-} + \langle W^{CD}\rangle$

$\Pr(X_t = x)$: - - -

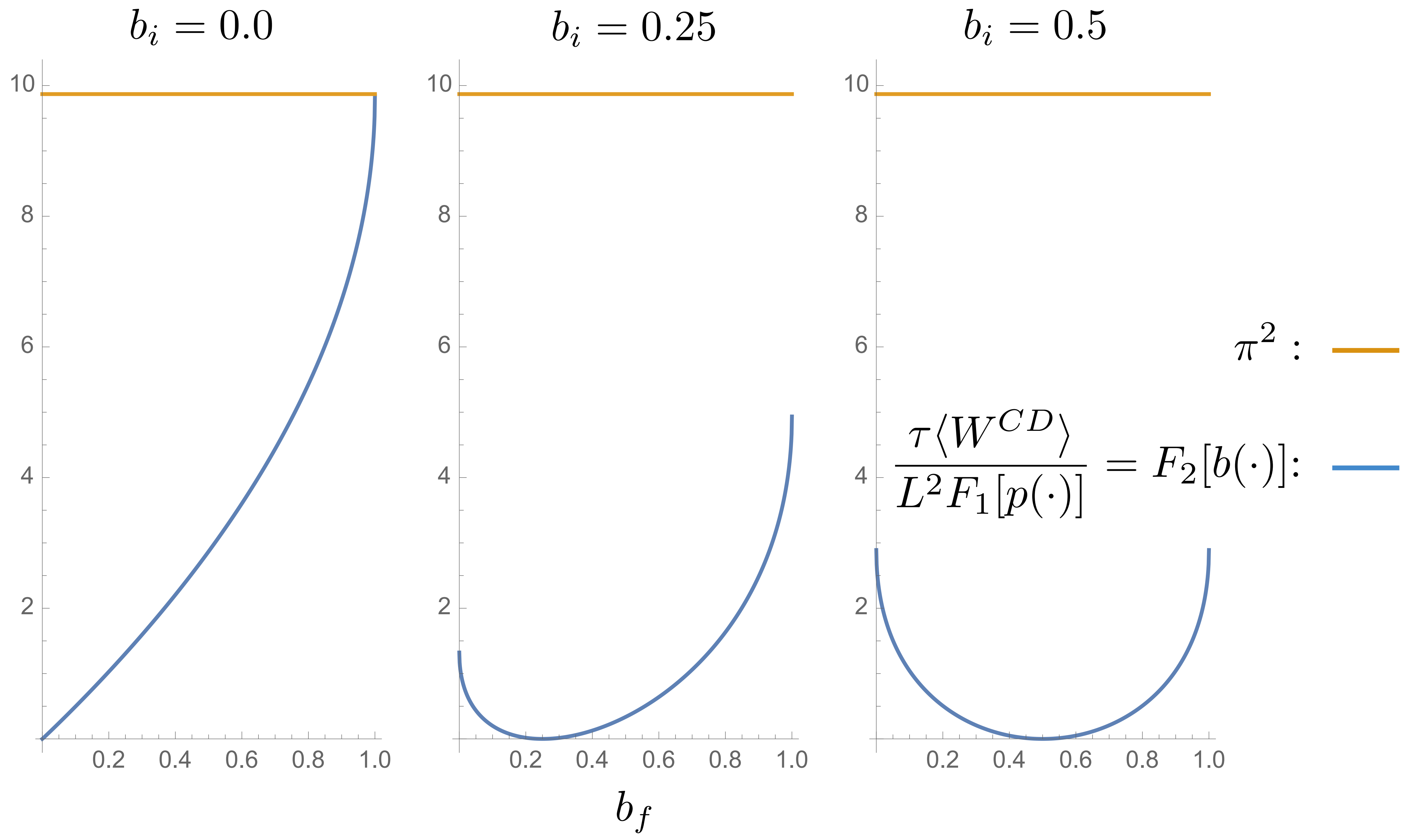
$\dfrac{V(x,t)}{k_B T}$: —

$(B)\ t = 0^+$

$(D)\ t = \tau^-$

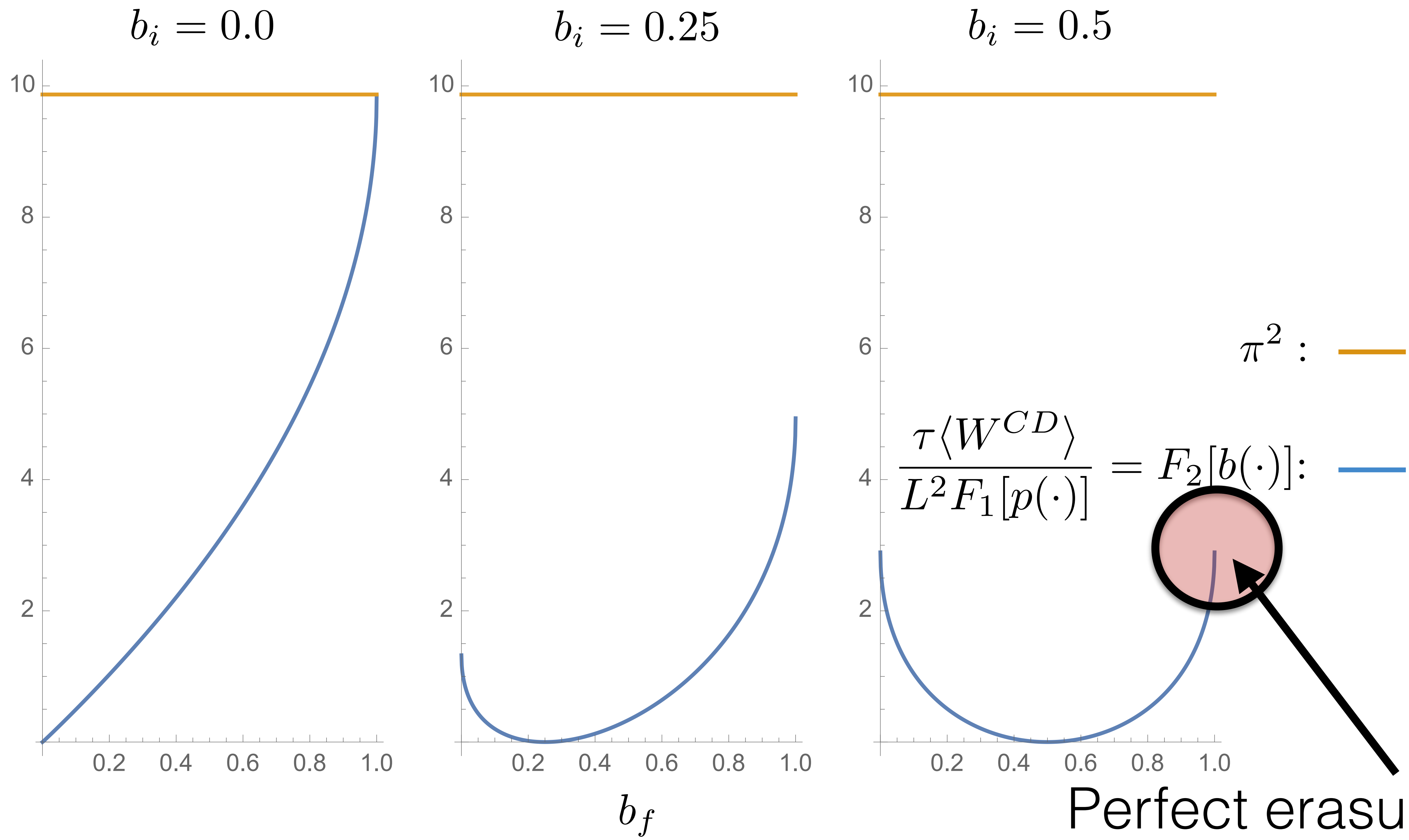
Shortcuts via Counterdiabatic Control

Dissipated work v. initial b_i & final bit bias b_f



Shortcuts via Counterdiabatic Control

Dissipated work v. initial b_i & final bit bias b_f



Perfect erasure
in finite time
at finite cost!

Shortcuts via Counterdiabatic Control

- Performance scaling:

General Landauer

Counterdiabatic Work

$$\langle W \rangle = k_{\mathrm{B}} T \ln 2 (H[Y_0] - H[Y_\tau]) + \frac{L^2}{\tau} F_1[p(\cdot)] F_2[b(\cdot)]$$

Barrier Bit bias

Independent of τ & L

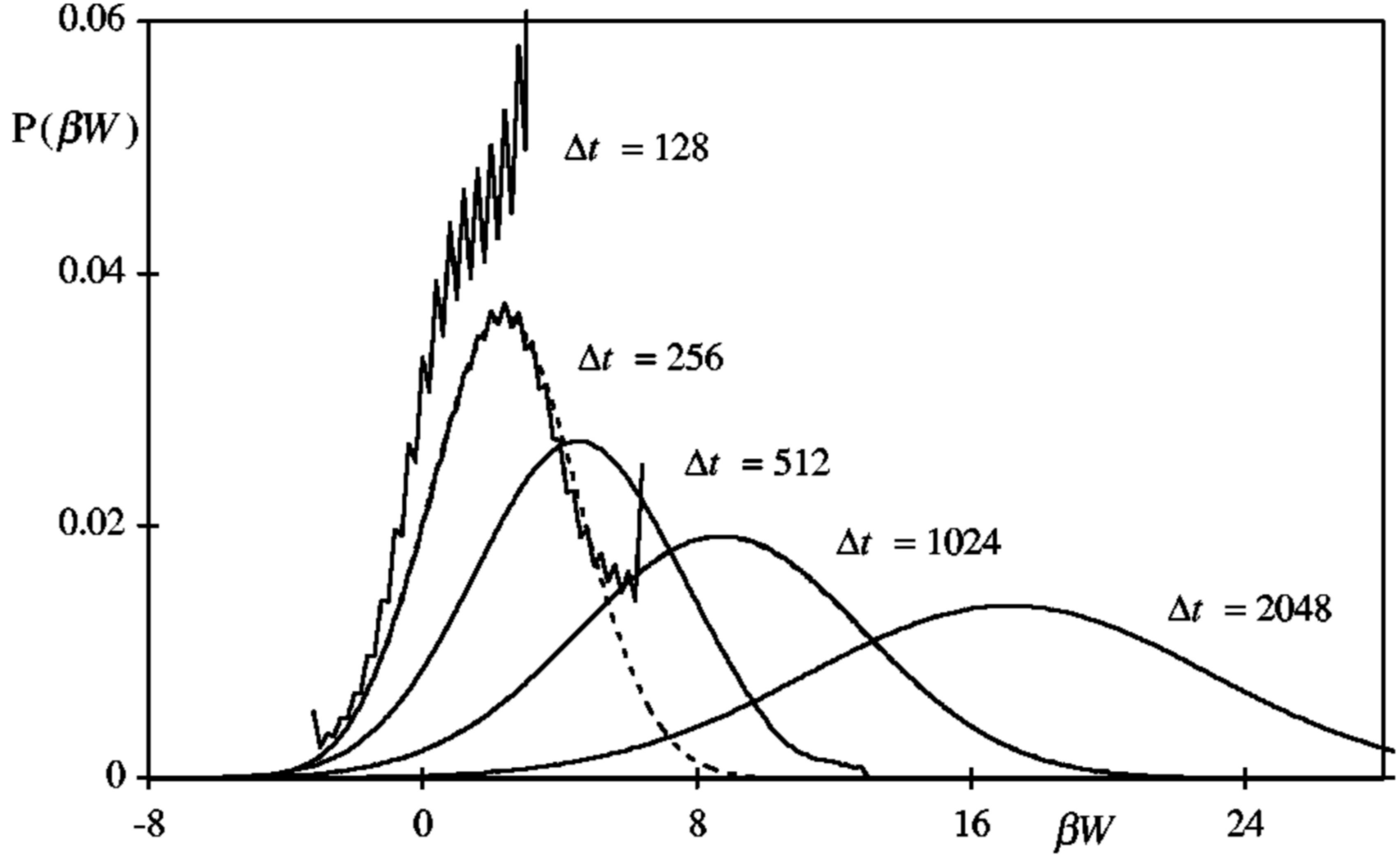
- Trade-offs: work, speed, size of the information-bearing degrees of freedom, fidelity, and storage robustness

Thermodynamic Computing

- Theory
- Design
- **Diagnosis**
- Experiment

Fluctuation Theorems

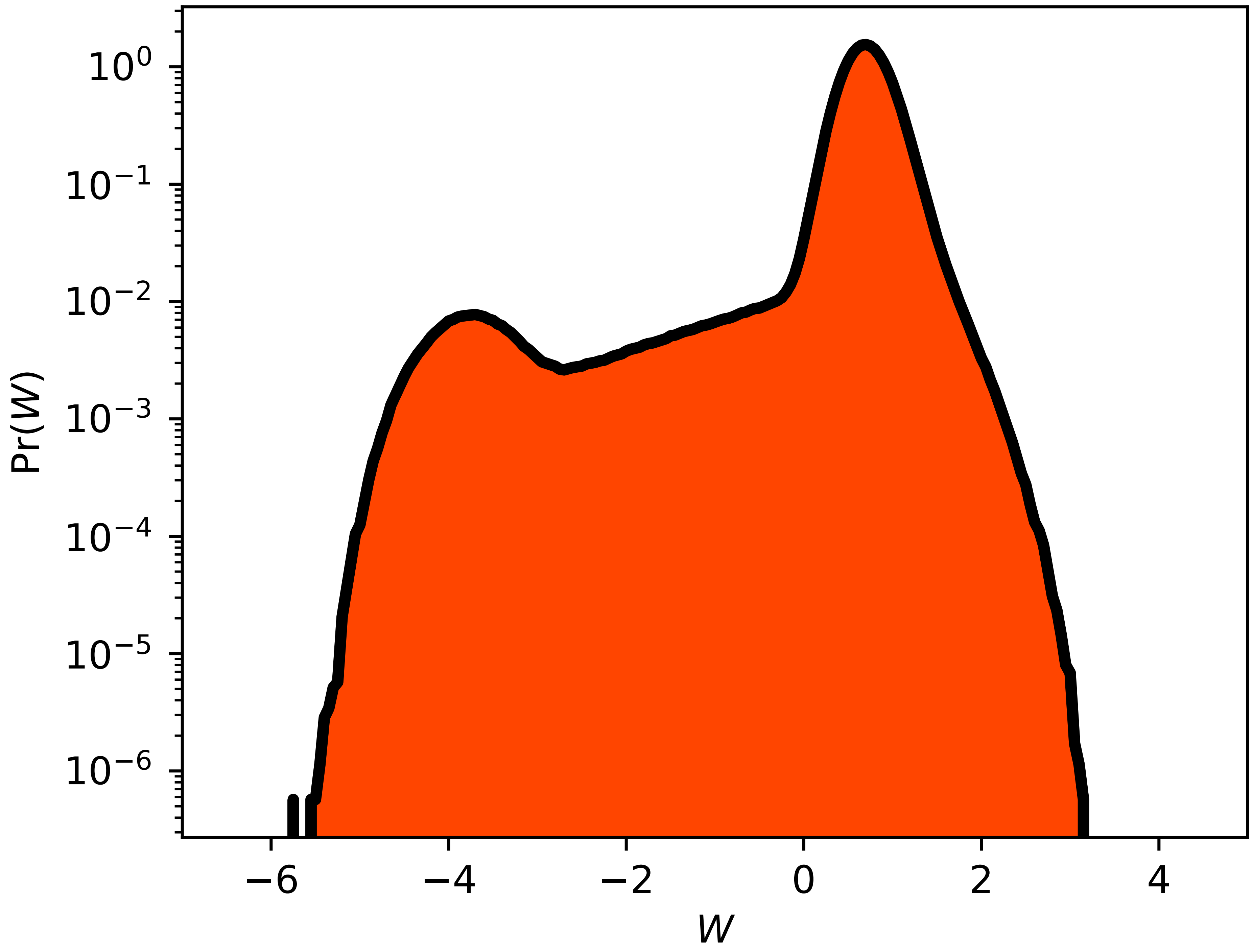
$$\frac{\mathrm{Pr}_{\mathrm{F}}(+\beta W)}{\mathrm{Pr}_{\mathrm{R}}(-\beta W)} = e^{\Delta F} e^{+\beta W}$$



Gavin E. Crooks, *Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences*, **Physical Review E** 60:3 (1999) 2721-2726.

Fluctuation Theorems

- Work distribution during erasure: Complex!



Trajectory-Class Fluctuation Theorems

- Work distributions as a diagnostic

- Track microscopic information processing with only mesoscopic thermodynamics

- TCFTs for trajectory class C:

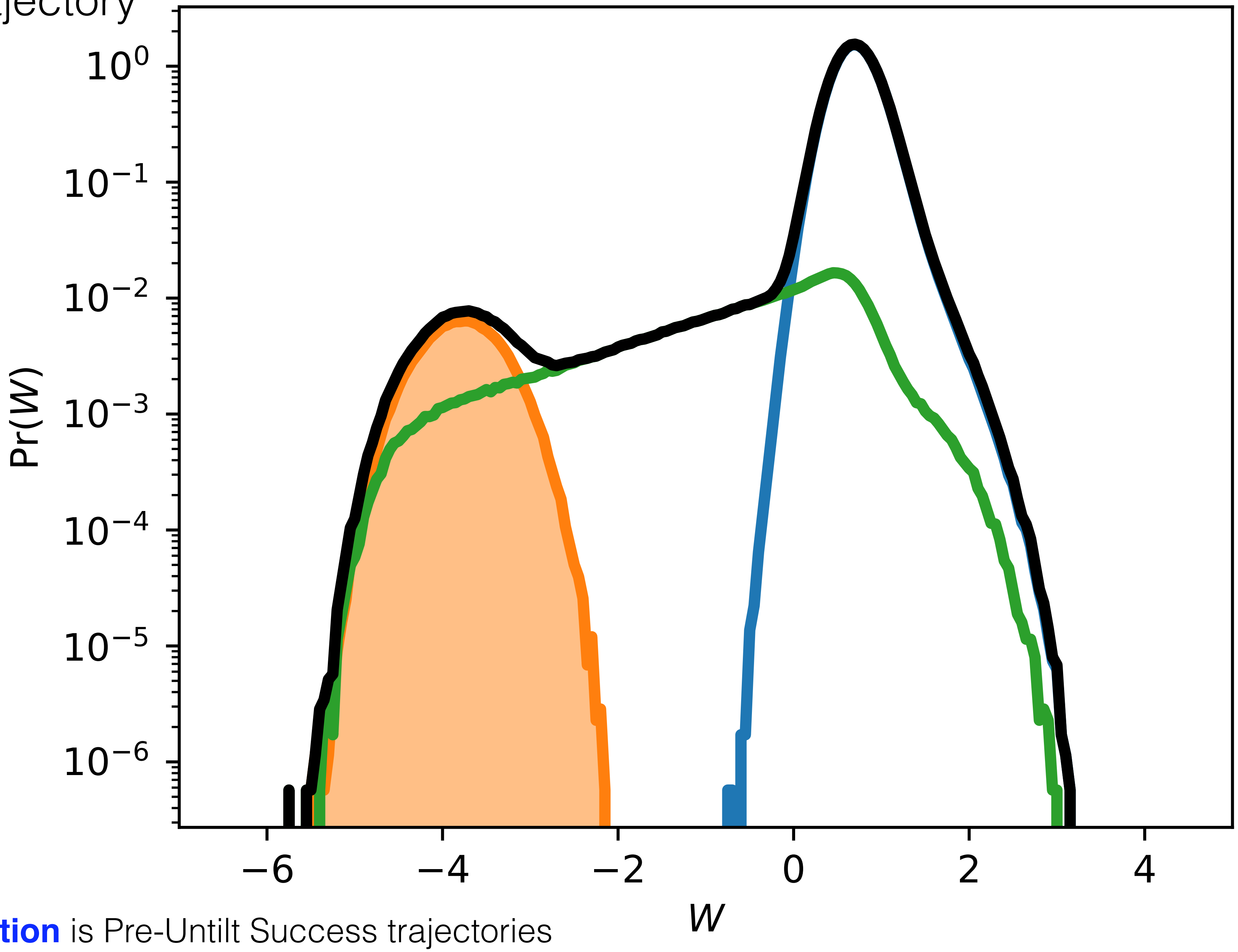
$$\frac{\Pr_R(C^R)}{\Pr(C)} = \langle e^{-\beta W} \rangle_C$$

- TCFTs interpolate btw integral and detailed FTs.

G. Wimsatt, O-P. Saira, A. B. Boyd, M. H. Matheny, S. Han, M. L. Roukes, and J. P. Crutchfield,
Harnessing Fluctuations in Thermodynamic Computing via Time-Reversal Symmetries, arxiv.org:1906.11973.

Trajectory-Class Fluctuation Theorems

Untilting trajectory partition



Blue distribution is Pre-Untilt Success trajectories

Orange is that of Pre-Untilt Fail trajectories

Green remaining Untilt Active trajectories.

Black curve is their sum, reconstructs the total work distribution.

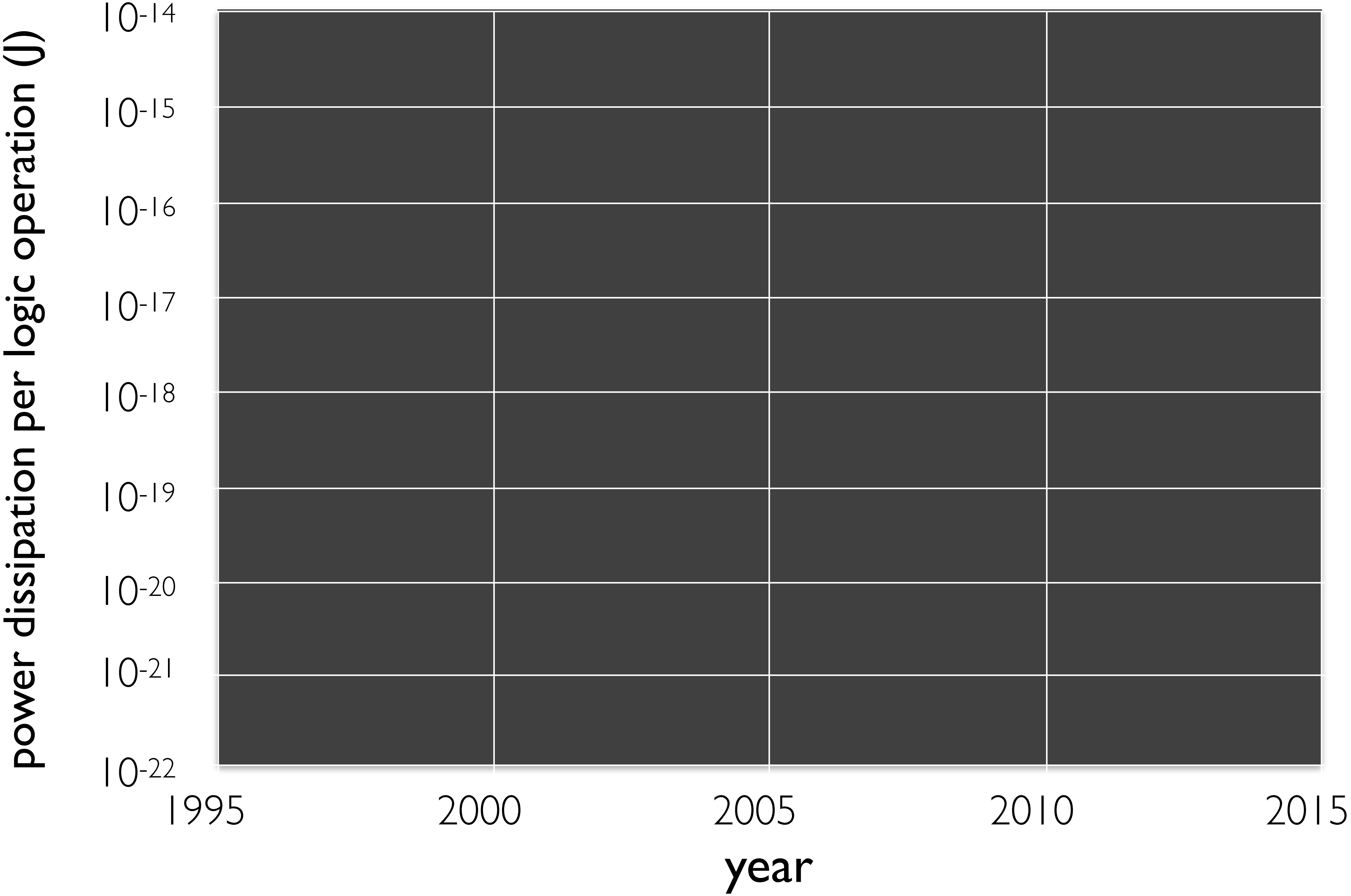
Thermodynamic Computing

- Theory
- Design
- Diagnosis
- **Experiment** (Sorry, not today: Roukes@Caltech)

Thermodynamic Computing

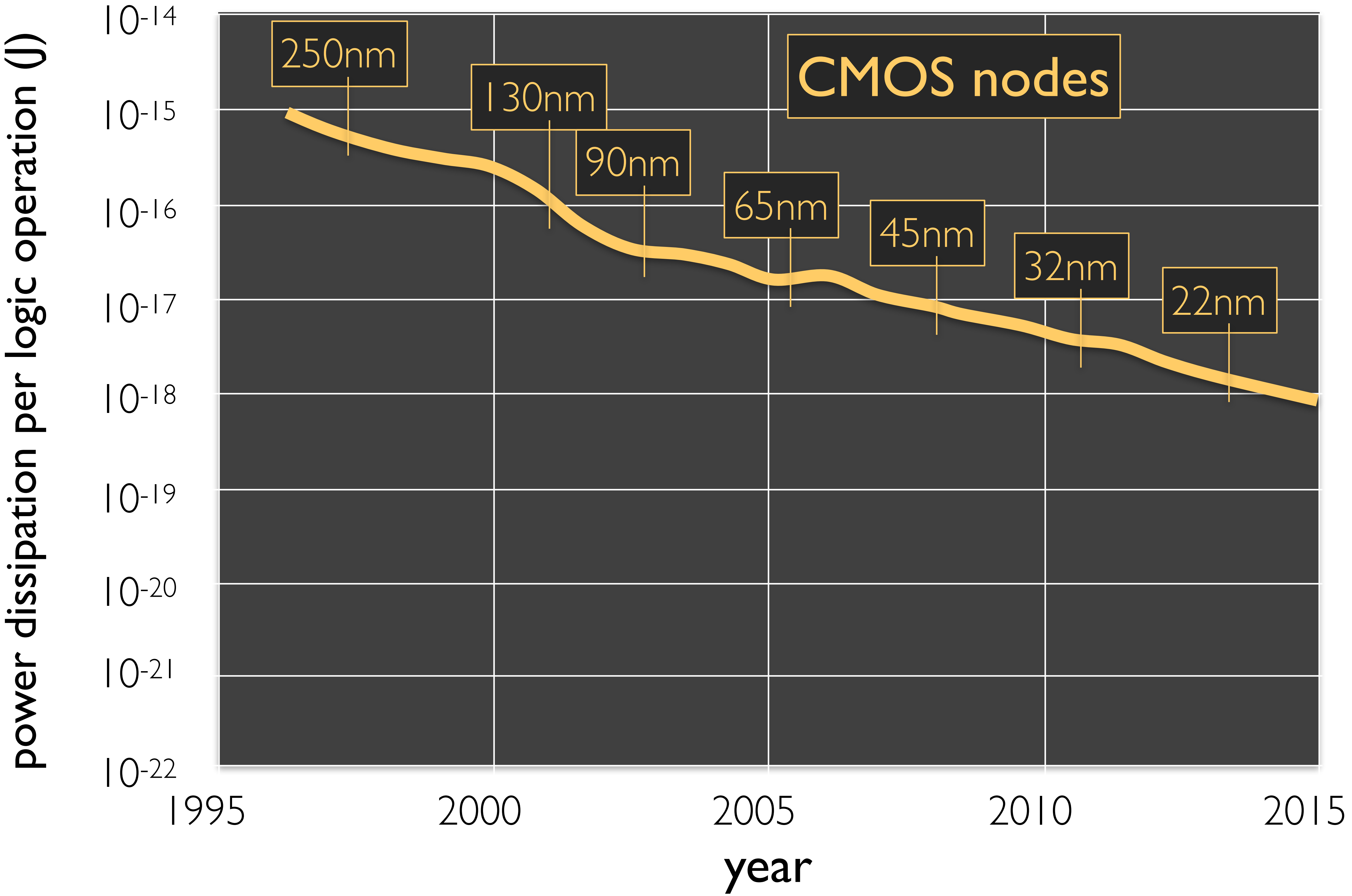
- Theory
- Design
- Diagnosis
- Experiment
- **Summary**

Nanoenergy Stack



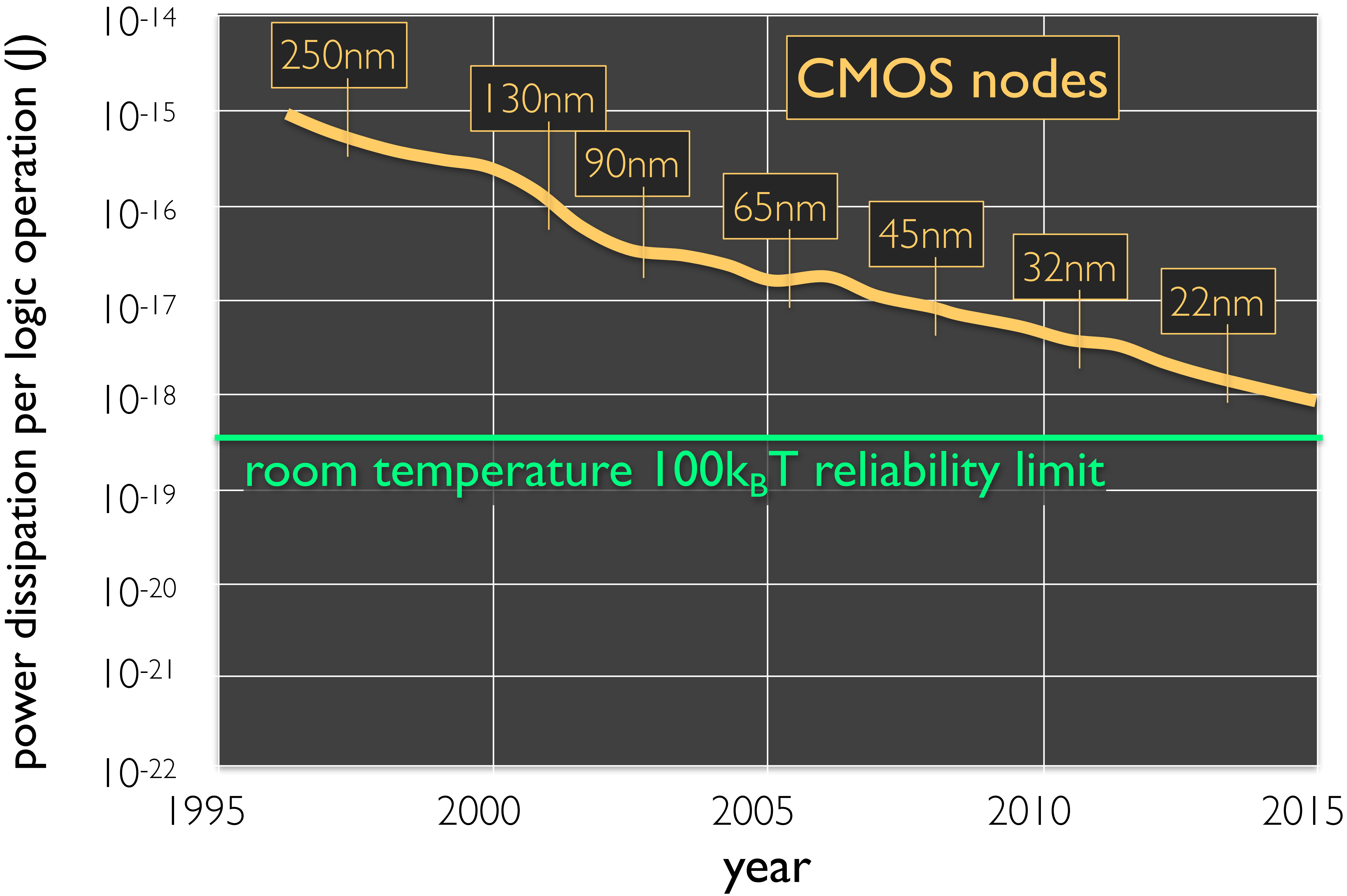
Michael Roukes, California Institute of Technology

Nanoenergy Stack



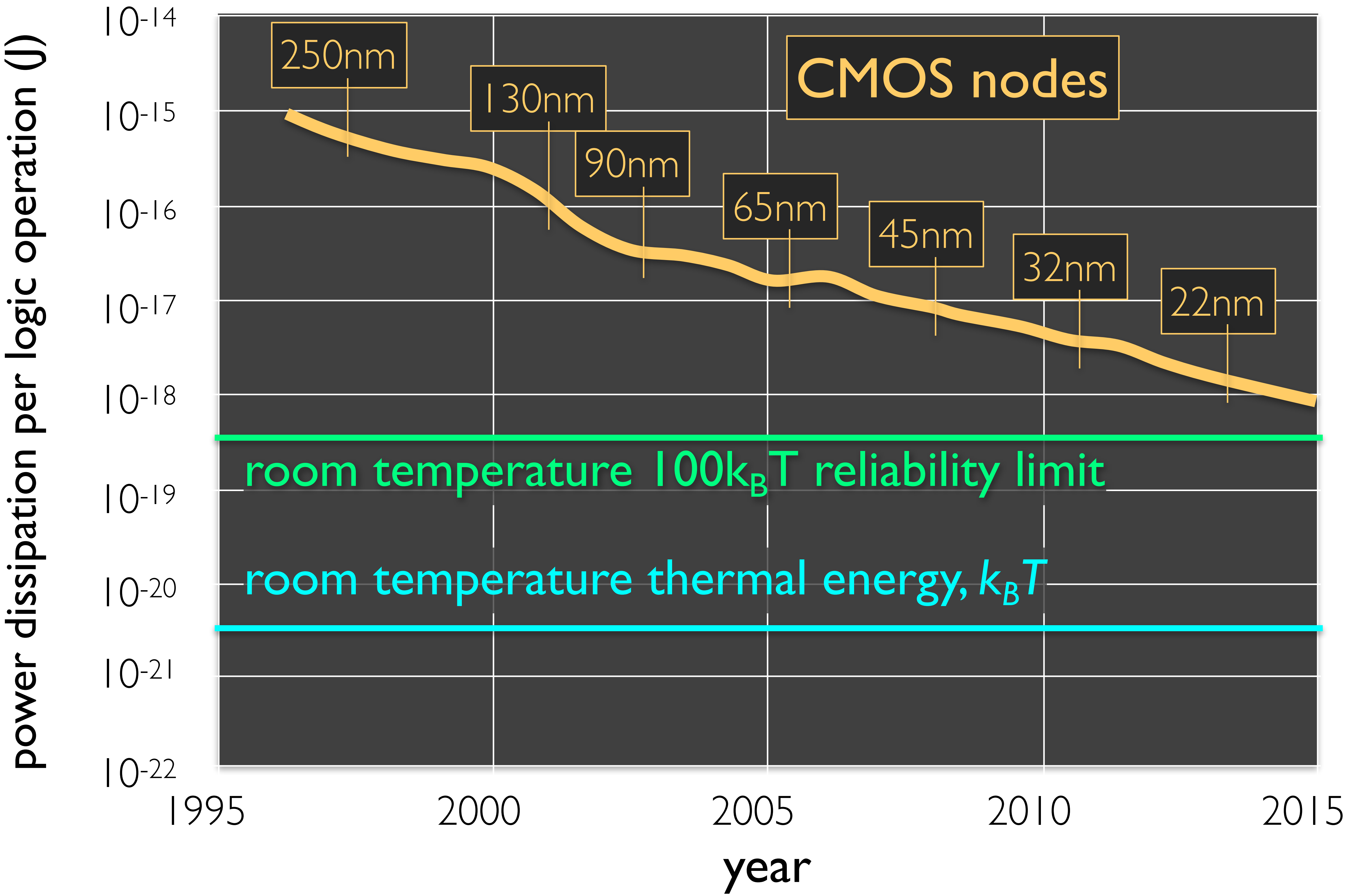
Michael Roukes, California Institute of Technology

Nanoenergy Stack



Michael Roukes, California Institute of Technology

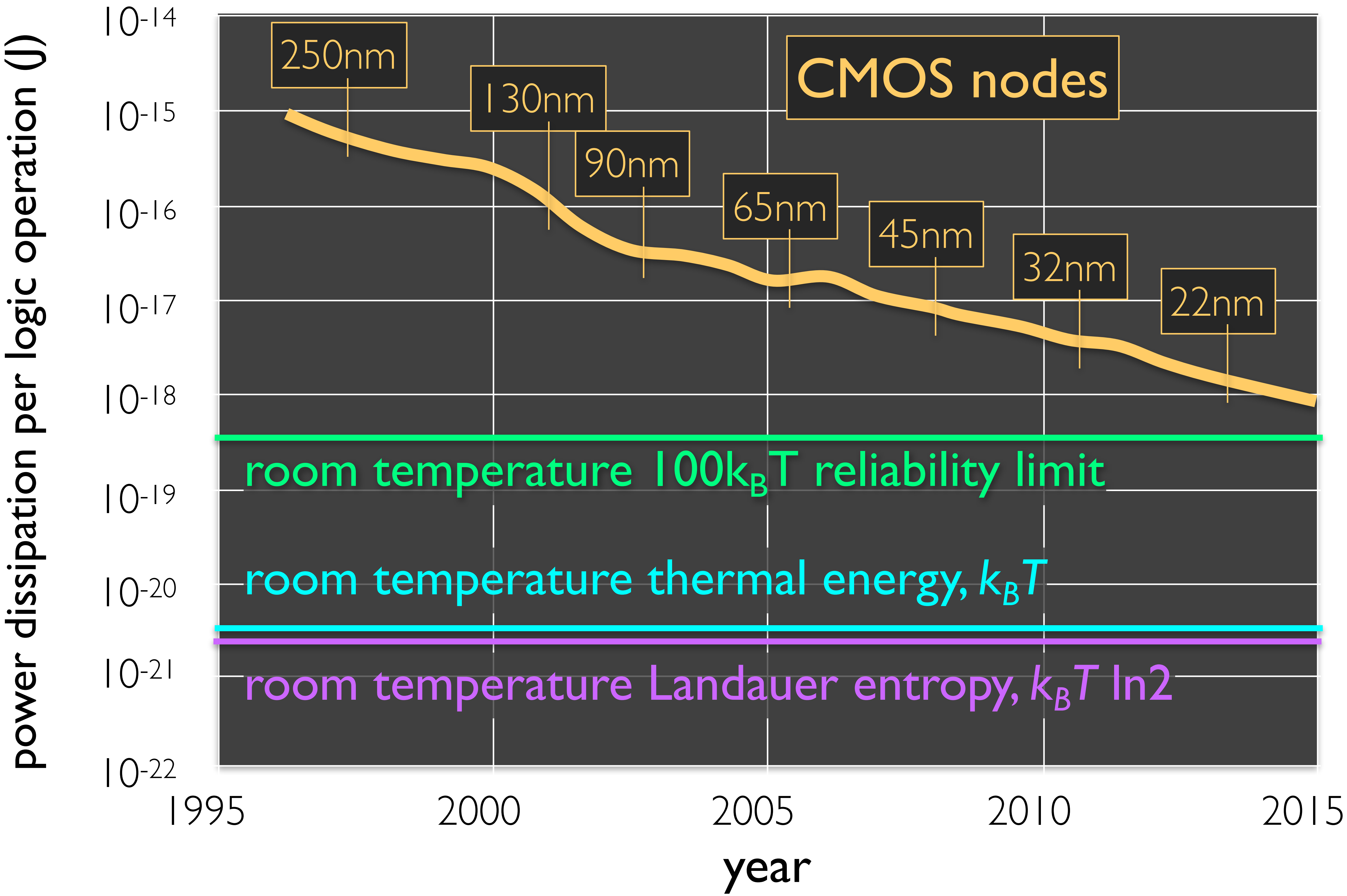
Nanoenergy Stack



Michael Roukes, California Institute of Technology

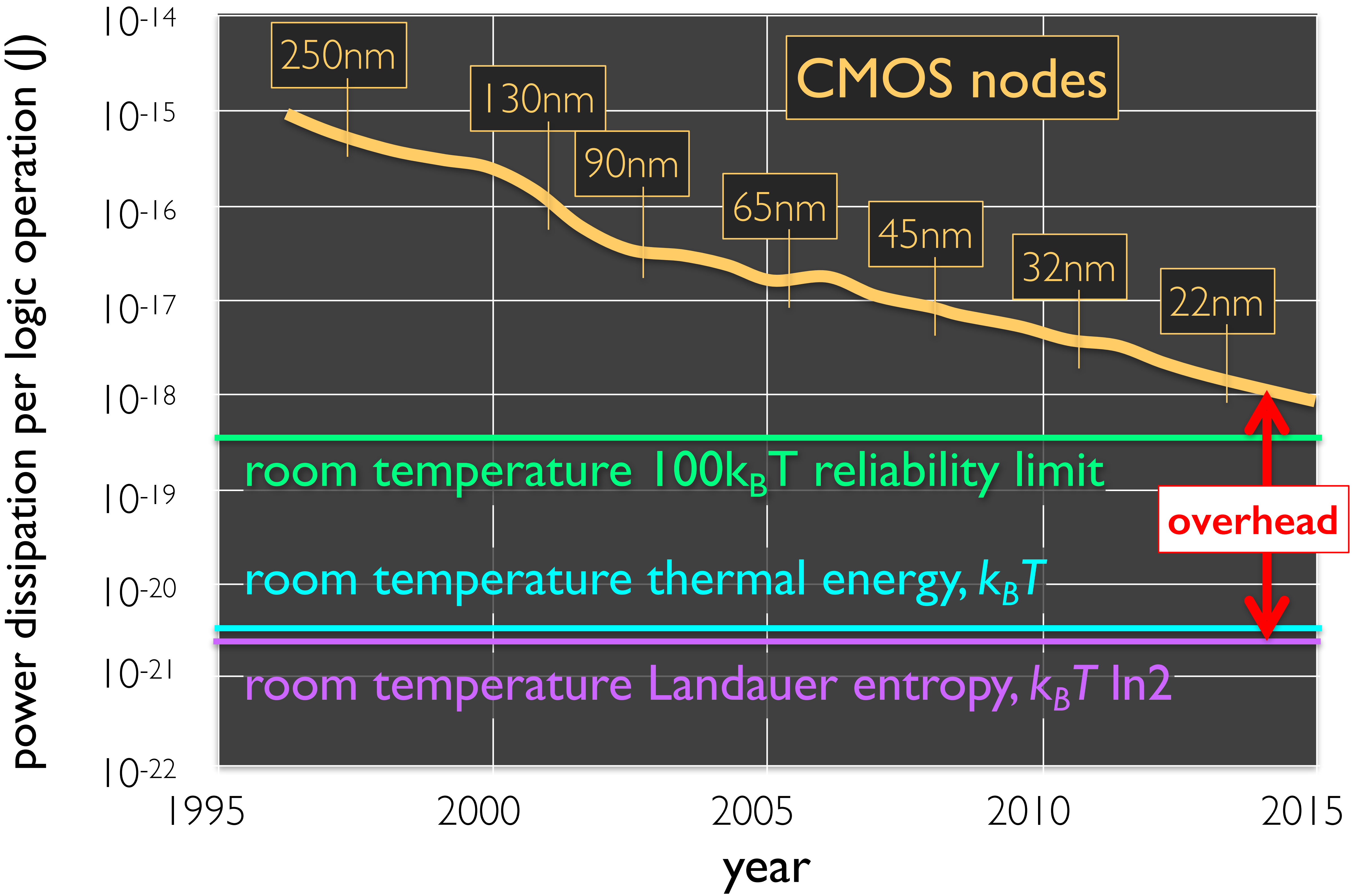
© Caltech 2015

Nanoenergy Stack



Michael Roukes, California Institute of Technology

Nanoenergy Stack

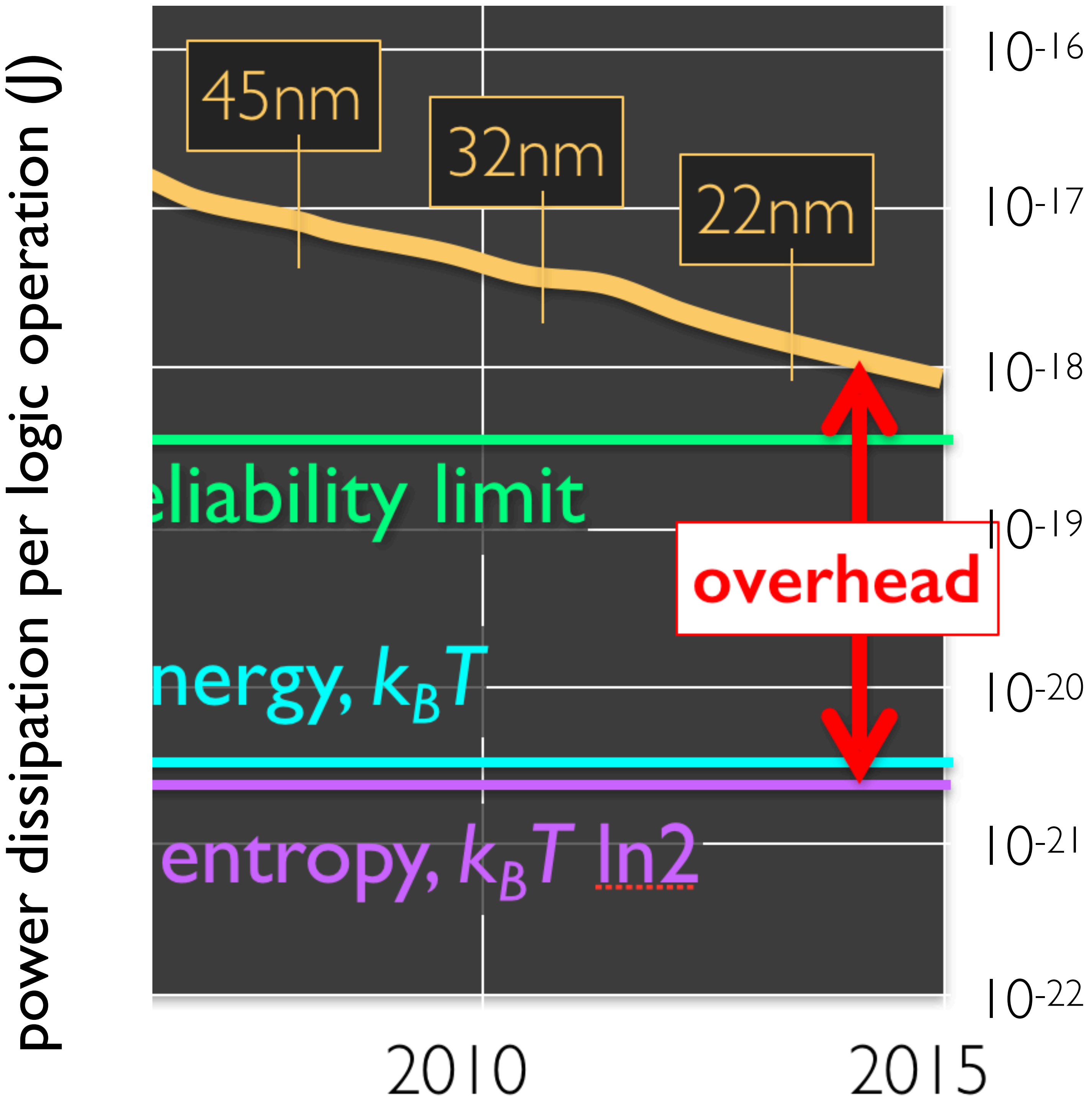


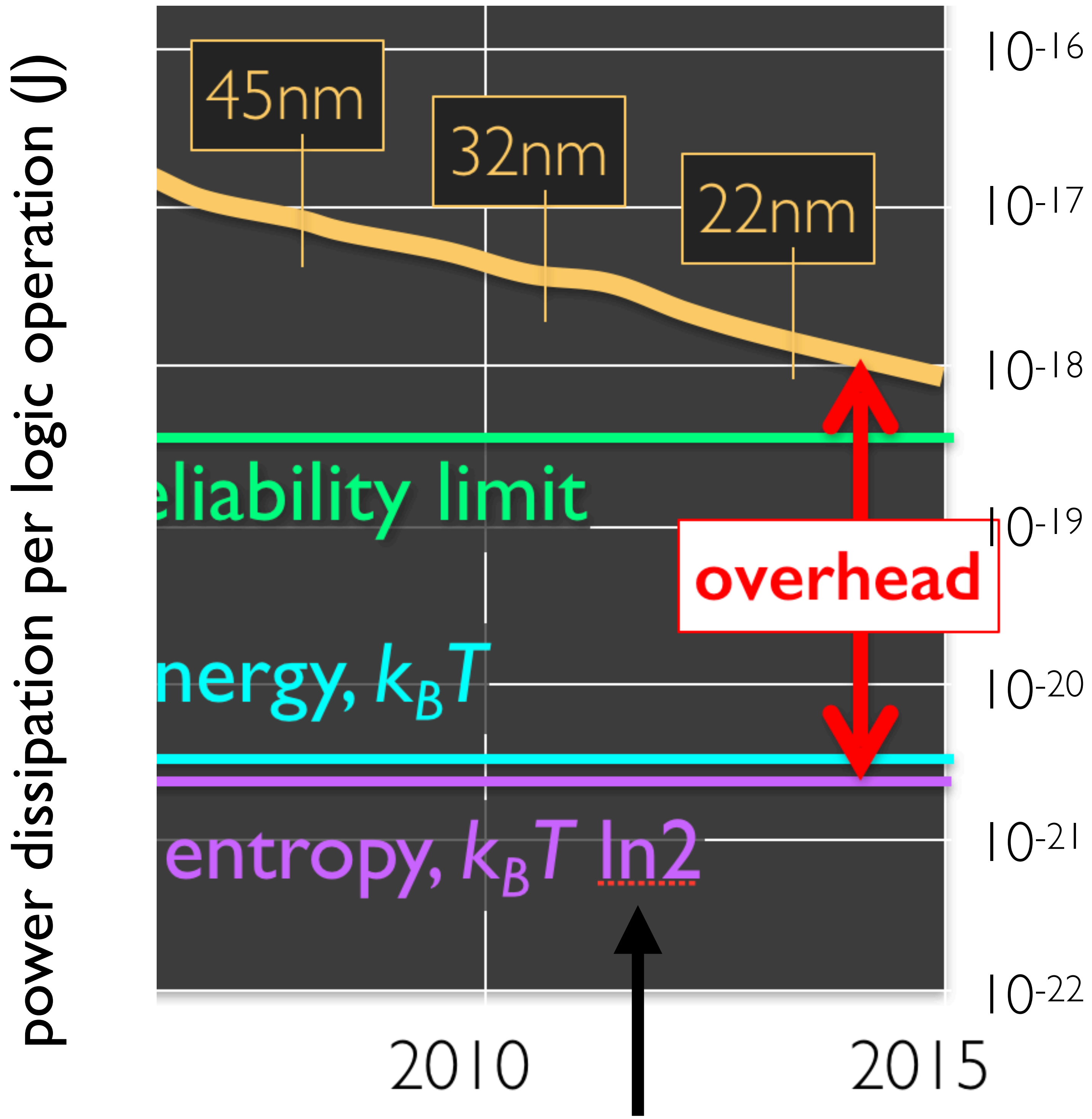
Michael Roukes, California Institute of Technology

Principles of Thermodynamic Computing

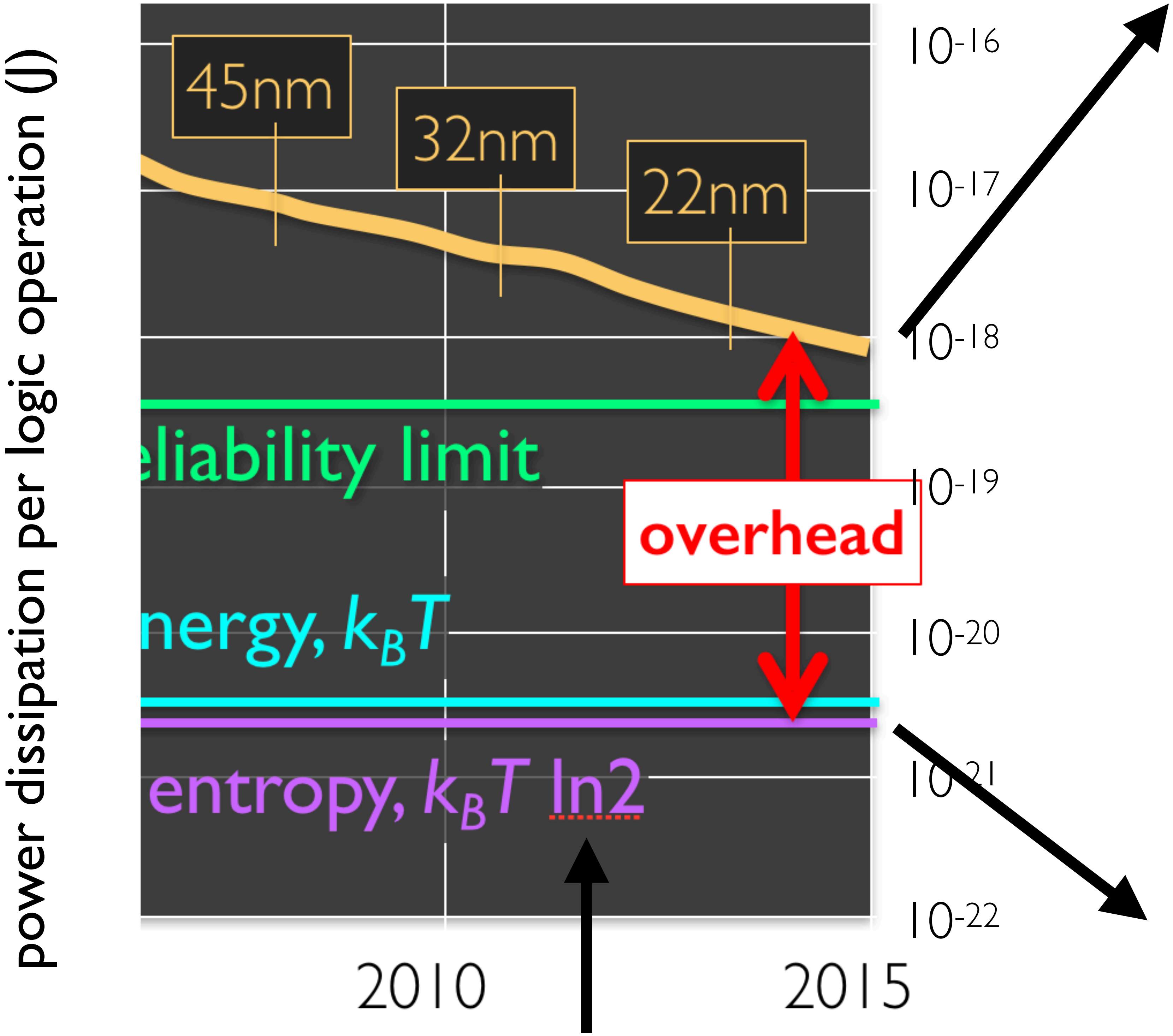
Principle	Meaning
Information Destruction $\langle W \rangle \geq k_B T \ln(2)$	Logically irreversible operations dissipate energy (Landauer, 1961)
Reciprocity $\langle W_{min}^{t-sym} \rangle = k_B T \langle \Psi \rangle - \langle W \rangle$	Logically nonreciprocal operations dissipate energy
Information Creation $\dot{Q} \geq k_B T \ln(2)\left(h_\mu - h'_\mu / \hat{L}\right)$	Creating information dissipates heat (Aghamohammdi & Crutchfield, 2017)
Information Process Second Law $\langle W \rangle \geq k_B T \left(h'_\mu - h_\mu\right)$	Work to drive (or energy dissipated) during computation (Boyd, Mandal, & Crutchfield, 2016a)
Requisite Complexity $\langle W \rangle \leq k_B T \ln(2) \min\{\Delta H_1, \Delta h_\mu\}$	Advantage maximized when controller matches environment (Boyd, Mandal, & Crutchfield, 2016b)
Synchronization & Error Correction $\langle Q^{tran} \rangle_{min} \geq k_B T \ln(2)\, \mathrm{I}\left[X_0 : \overleftarrow{Y}'\right] - \boldsymbol{E}'$	Work to correct errors or synchronize to environment (Boyd, Mandal, & Crutchfield, 2017)
Modularity $\langle \Sigma_{t \to t+\tau}^{mod} \rangle_{min} = k_B T \ln(2)\, \Delta I_{t \to t+\tau}$	Controller modularity is thermodynamically expensive (Boyd, Mandal, & Crutchfield, 2018)
Information Dynamics $\lambda > 0$	Maxwellian demons are chaotic dynamical systems (Boyd & Crutchfield, 2016)
Steady-State Transitions $\Pr(W_{ex}, \Psi) / \Pr(-W_{ex}, -\Psi) = e^{\Psi} e^{\beta W_{ex}}$	Work to drive transitions between information storage states (Riechers & Crutchfield, 2017)
Functional Fluctuations $Iu{=}\beta{-}1{-}1h\mu P\beta{-}\beta{-}1\log2\lambda\; I(u) =$ $(\beta^{-1} - 1) h_\mu(P_\beta) - \beta^{-1} \log_2(\hat{\lambda}_\beta)$	Engine functionality fluctuates in small systems, short times (Crutchfield & Aghamohammdi, 2016)
Control tradeoffs $\dot{\Sigma} = f(1/\tau, L^2)$	Counterdiabatic control dissipation design (Campbell & De, 2017) (Boyd, Patra, Jarzynski, & Crutchfield, 2018)
Reliability $\Sigma^{\cdot} = f(-\ln(\epsilon))$	Dissipation costs of high-reliability information processing
Trajectory-class fluctuations $\langle e^{-W/k_B T} \rangle_C = R(C^{\mathrm{R}}) / P(C)\; e^{-\Delta F / k_B T}$	Success and failure have thermodynamic signatures (Wimsatt, et al., 2019)

Table 2: Nonequilbrium thermodynamics of information processing in classical physical systems. For notation, refer to the cited works.


power dissipation per logic operation (J)
45nm
32nm
22nm
eliability limit
overhead
nergy, $k_B T$
entropy, $k_B T$ ln2
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
2010
2015

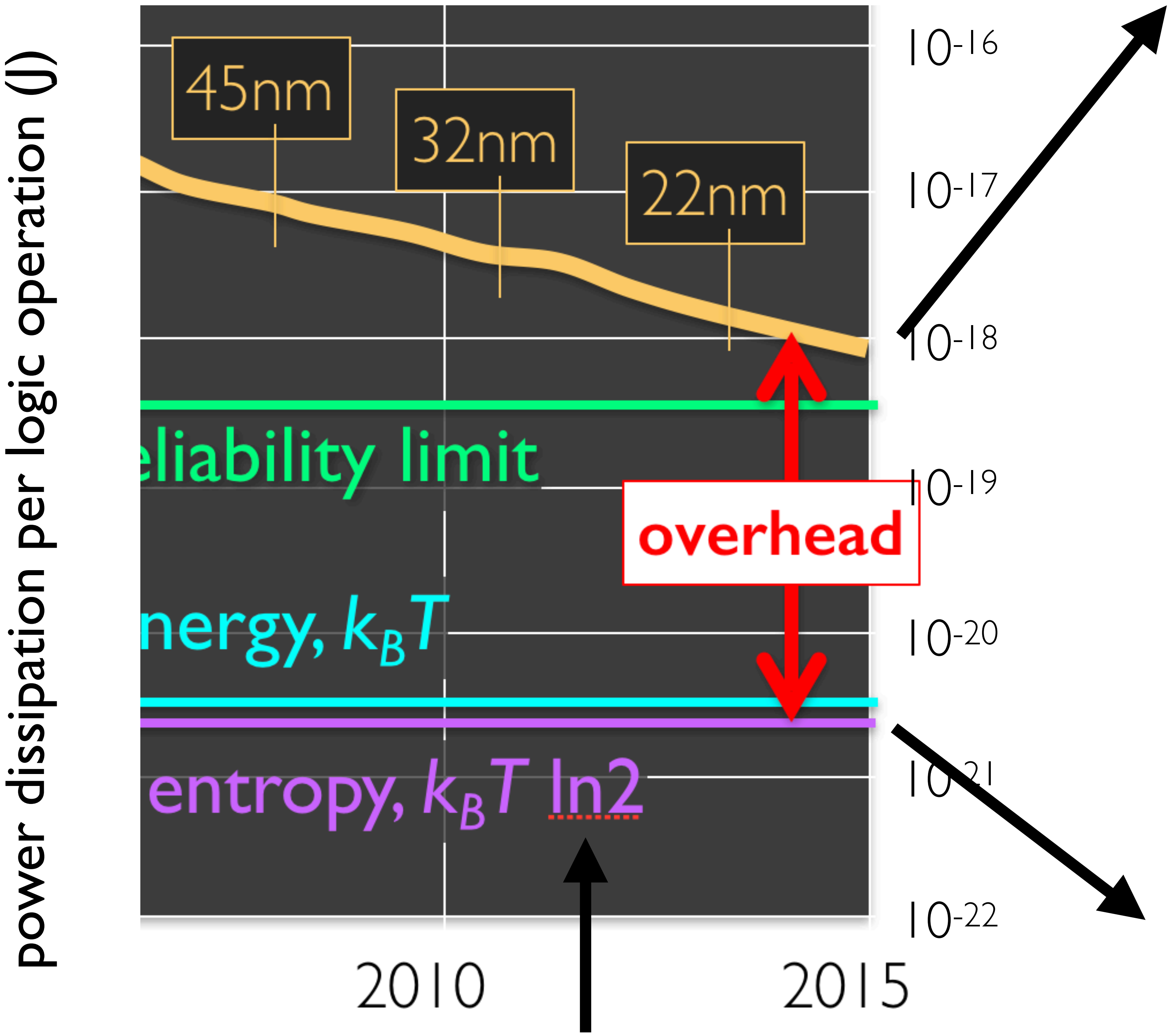


Logical
irreversibility



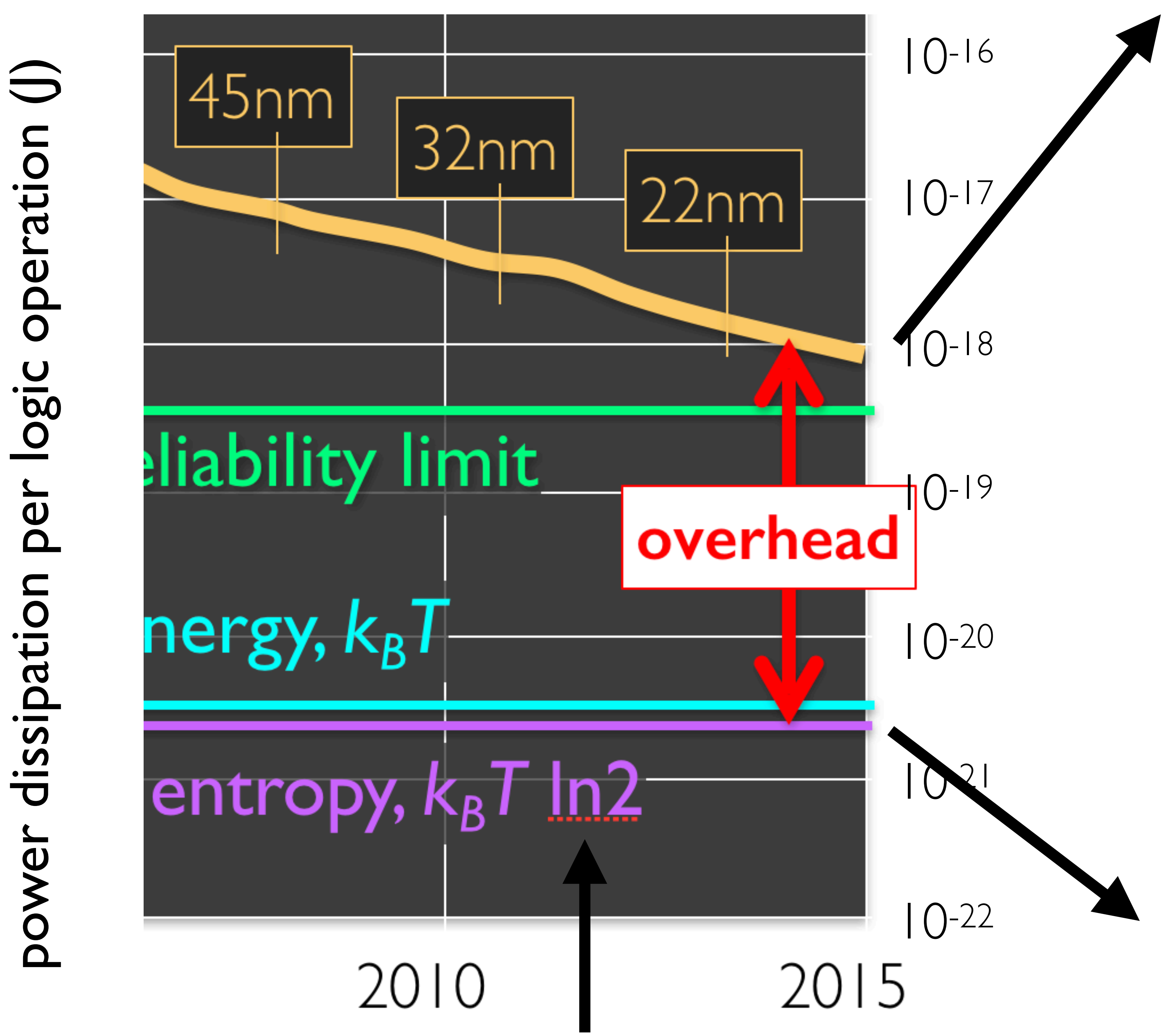
Logical
irreversibility

Landauer's Stack



Logical
irreversibility

Landauer’s Stack



Logical
irreversibility

Nonreciprocity

IPSL: $\langle W \rangle \leq k_{\mathrm{B}} T \ln 2 \ (h_\mu{}' - h_\mu)$

Rate: $1/\tau$

Size: L^2

Reliability: $-\ln \epsilon$

Storage stability

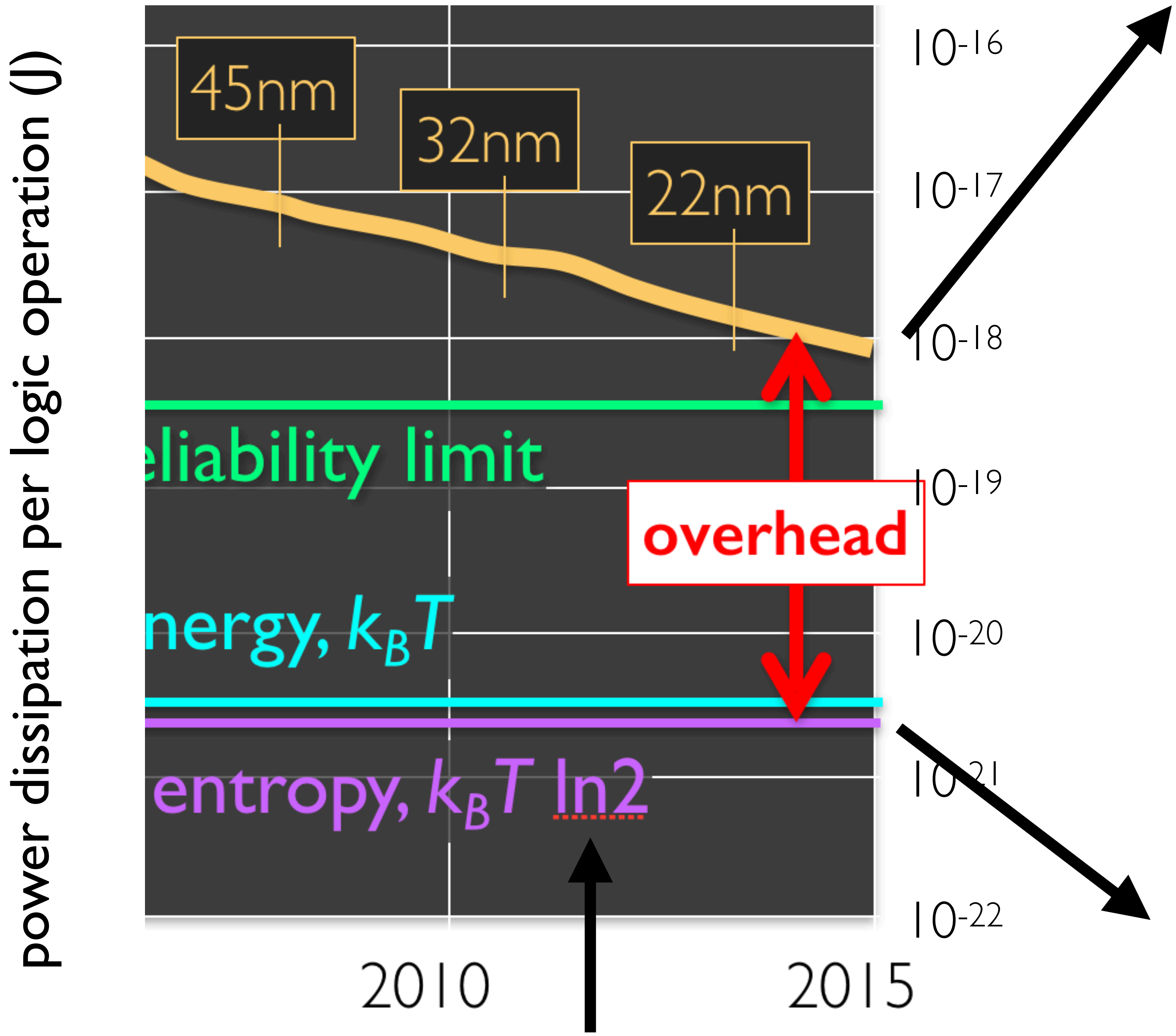
Circuit modularity

NESS transitions

Trajectory-class FTs

Quantum Coherence

Landauer's Stack



Logical irreversibility

10^{-24}

Nonreciprocity

IPSL: $\langle W \rangle \leq k_{\mathrm{B}} T \ln 2 \; ({h_\mu}' - h_\mu)$

Rate: $1/\tau$

Size: L^2

Reliability: $-\ln \epsilon$

Storage stability

Circuit modularity

NESS transitions

Trajectory-class FTs

Quantum Coherence

Experiment thermal energy resolution

Landauer’s Stack

- Fundamental costs of information processing?
  - Account for all currently-identified thermodynamic costs:

    Landauer irreversibility, nonreciprocity, IPSL, requisite complexity, randomness creation, prediction, functional fluctuations, sensing, synchronization, error correction, modularity, high reliability, …
  - Predict total stack cost—energetics of extant computing
- If not, how far are we from doing so?
- What might we be missing?

Thermodynamic Computing Futures

Thermodynamic Computing Roadmap To Date

- **Design**
  - Comparative optimality: Many ways (now) to design control protocols (geometric control, counterdiabatic, …)
  - Trade-offs: Speed v. energy v. accuracy v. …

- **Diagnosis**
  - Mesoscopics (e.g., work distributions) to diagnose success & failure

- **Experiment**
  - Flux qubits: an attractive platform
  - Thermodynamics of bit reset via a continuously-monitored flux cbit
  - Test current predictions
  - Work distributions display universal features of functional microscopic trajectories

Thermodynamic Computing Challenges

- Beyond overdamped: Fully underdamped dynamics
- Beyond detailed balance
- Beyond linear response
- Beyond Markovian: Hidden Markov dynamics, long-range correlations
- Beyond memoryless: Memoryful information processing, transducers
- Multivariate information theory: Beyond single bit, two-point mutual information, transfer entropy, …
- Information-bearing degrees of freedom?
- Strong system+environment & system+system coupling
- Nonlinear dynamics of information processing: Beyond NESS
- New analytics: Nonnormal, nondiagonalizable, …
- Information-engine circuits and lattices

Thanks!