DEMON DYNAMICS:

DETERMINISTIC CHAOS,
THE SZILARD MAP, & THE
INTELLIGENCE OF THERMODYNAMIC SYSTEMS

http://csc.ucdavis.edu/~cmg/

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1951

Abstract

We introduce a deterministic chaotic system—the Szilard Map—that encapsulates the measurement, control, and erasure protocol by which Maxwellian Demons extract work from a heat reservoir. Implementing the Demon's control function in a dynamical embodiment, our construction symmetrizes Demon and thermodynamic system, allowing one to explore their functionality and recover the fundamental trade-off between the thermodynamic costs of dissipation due to measurement and due to erasure. The map's degree of chaos—captured by the Kolmogorov-Sinai entropy—is the rate of energy extraction from the heat bath. Moreover, an engine's statistical complexity quantifies the minimum necessary system memory for it to function. In this way, dynamical instability in the control protocol plays an essential and constructive role in intelligent thermodynamic systems.

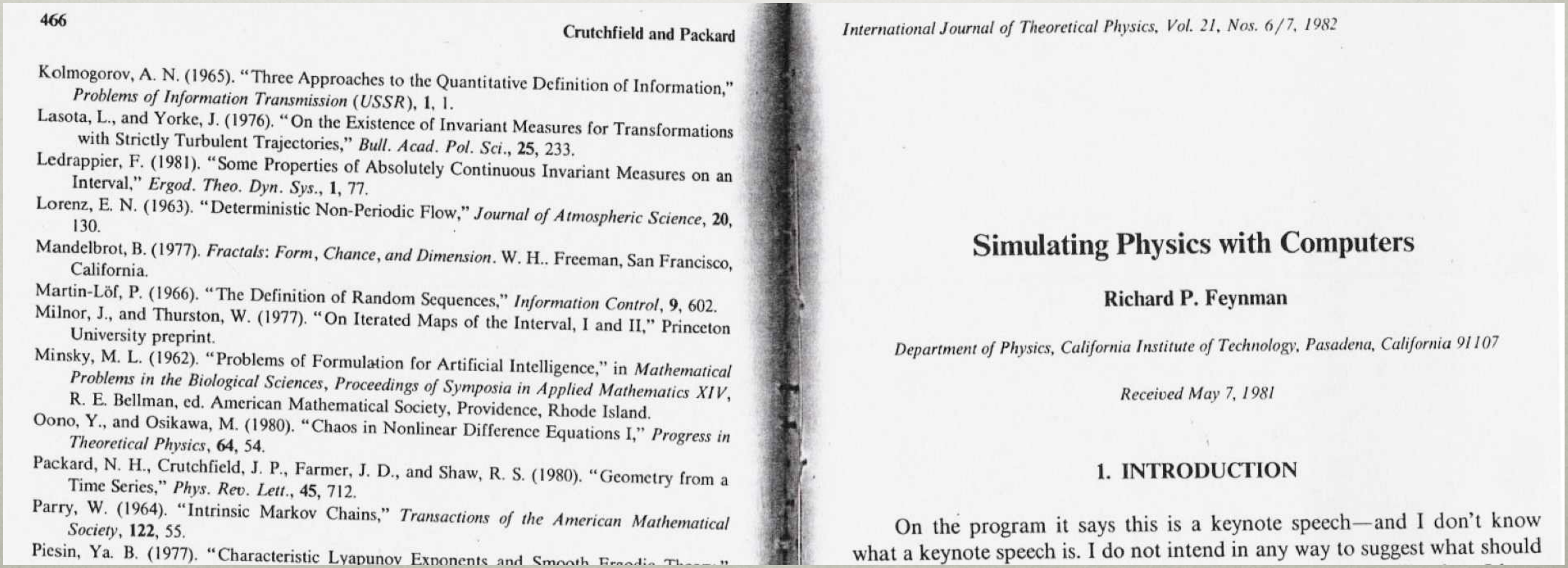
Joint work with Alexander B. Boyd.

HISTORY OF COMPUTING SUBSTRATES

- Analog: Mechanical gears
- Analog: Electron tube circuits
- Digital Gates: Electron tubes, semiconductors
- Digital Memory: Mercury delay lines, storage scopes, ...
- …
- Molecular Computing (1970s)
- “Physics & Computation” (MIT Endicott House 1981)
  - Quantum Computing: Feynman
  - Erasure cost: Bennett & Landauer
  - It from Bit: Wheeler
- Josephson Junction Computers (IBM 1980s)
- “Nanotech” (Drexler/Merkle XEROX PARC 1990s)
- Design goal: Useful computing

HISTORY OF INTRINSIC COMPUTING

- Nature already computes
- Information: $H(\Pr(X))$ (Shannon 1940s)
- In chaotic dynamics: h_μ (Kolmogorov 1950s)
- “Physics & Computation” (MIT Endicott House 1981)
  - “Intrinsic computing” there too!



466 **Crutchfield and Packard**

Kolmogorov, A. N. (1965). “Three Approaches to the Quantitative Definition of Information,” *Problems of Information Transmission (USSR)*, **1**, 1.
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Ledrappier, F. (1981). “Some Properties of Absolutely Continuous Invariant Measures on an Interval,” *Ergod. Theo. Dyn. Sys.*, **1**, 77.
Lorenz, E. N. (1963). “Deterministic Non-Periodic Flow,” *Journal of Atmospheric Science*, **20**, 130.
Mandelbrot, B. (1977). *Fractals: Form, Chance, and Dimension*. W. H.. Freeman, San Francisco, California.
Martin-Löf, P. (1966). “The Definition of Random Sequences,” *Information Control*, **9**, 602.
Milnor, J., and Thurston, W. (1977). “On Iterated Maps of the Interval, I and II,” Princeton University preprint.
Minsky, M. L. (1962). “Problems of Formulation for Artificial Intelligence,” in *Mathematical Problems in the Biological Sciences, Proceedings of Symposia in Applied Mathematics XIV*, R. E. Bellman, ed. American Mathematical Society, Providence, Rhode Island.
Oono, Y., and Osikawa, M. (1980). “Chaos in Nonlinear Difference Equations I,” *Progress in Theoretical Physics*, **64**, 54.
Packard, N. H., Crutchfield, J. P., Farmer, J. D., and Shaw, R. S. (1980). “Geometry from a Time Series,” *Phys. Rev. Lett.*, **45**, 712.
Parry, W. (1964). “Intrinsic Markov Chains,” *Transactions of the American Mathematical Society*, **122**, 55.
Piesin, Ya. B. (1977). “Characteristic Lyapunov Exponents and Smooth

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

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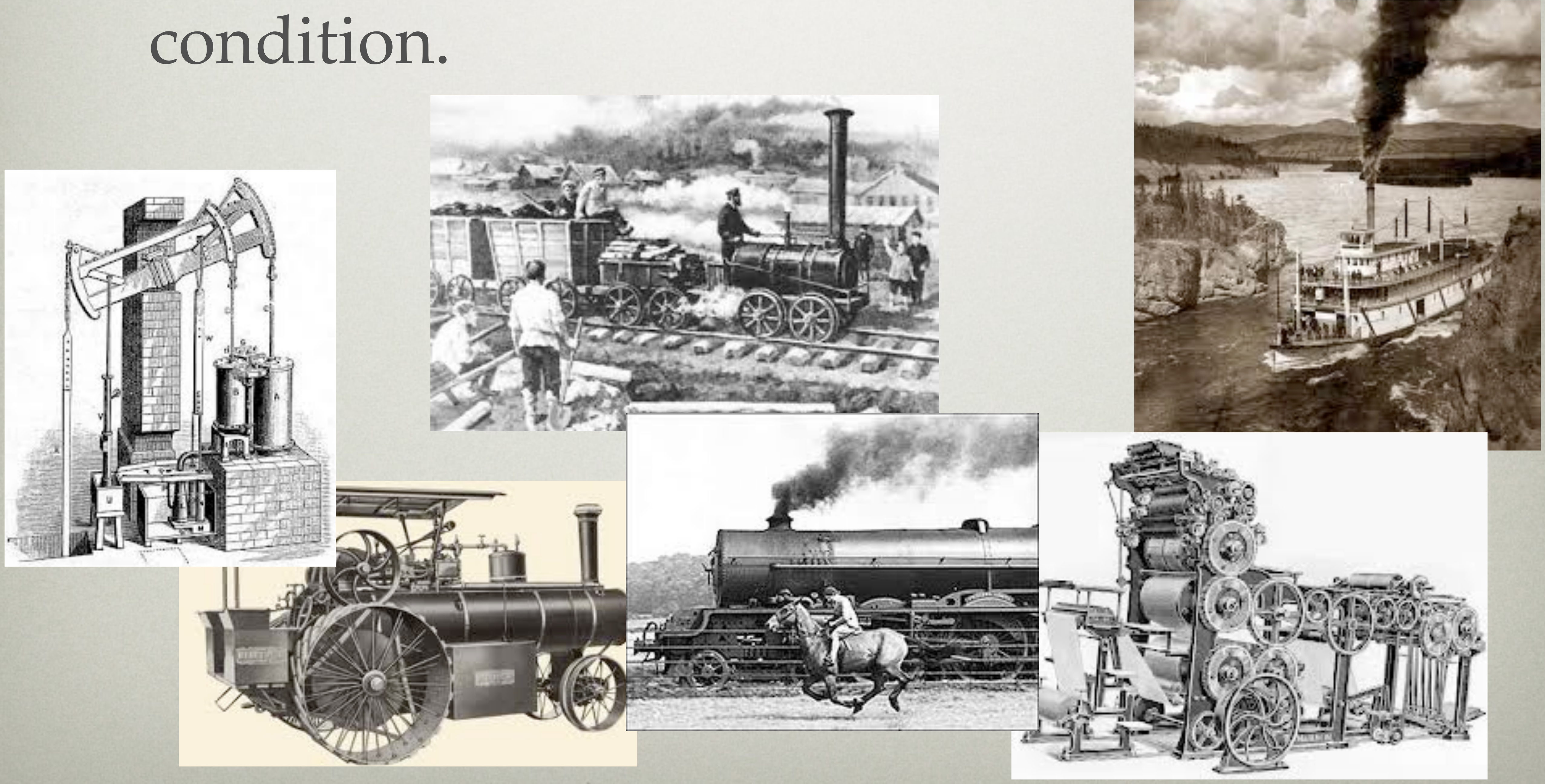
Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don’t know what a keynote speech is. I do not intend in any way to suggest what should

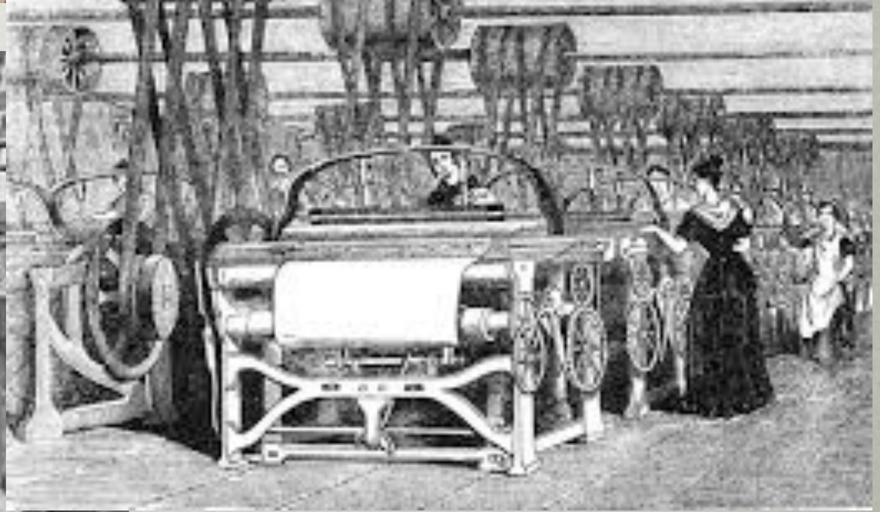
INDUSTRIAL REVOLUTION
1750s-1900

- Humans commandeer energy on vast scales.
- Unprecedented improvement in human condition.



INDUSTRIAL REVOLUTION 1750s-1900

- Urbanization, disease, ...
- Class inequality & oppression



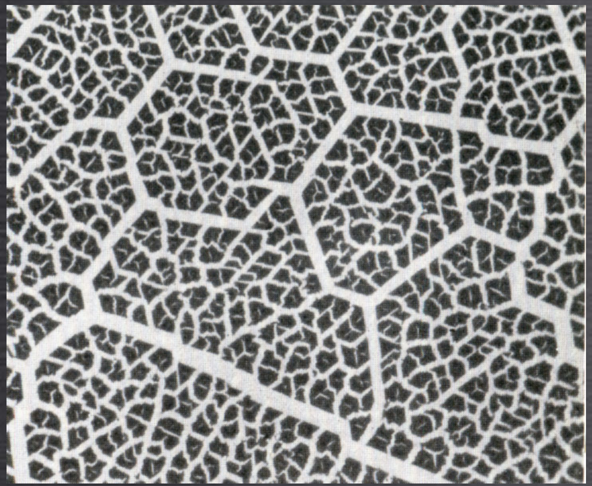
Agenda

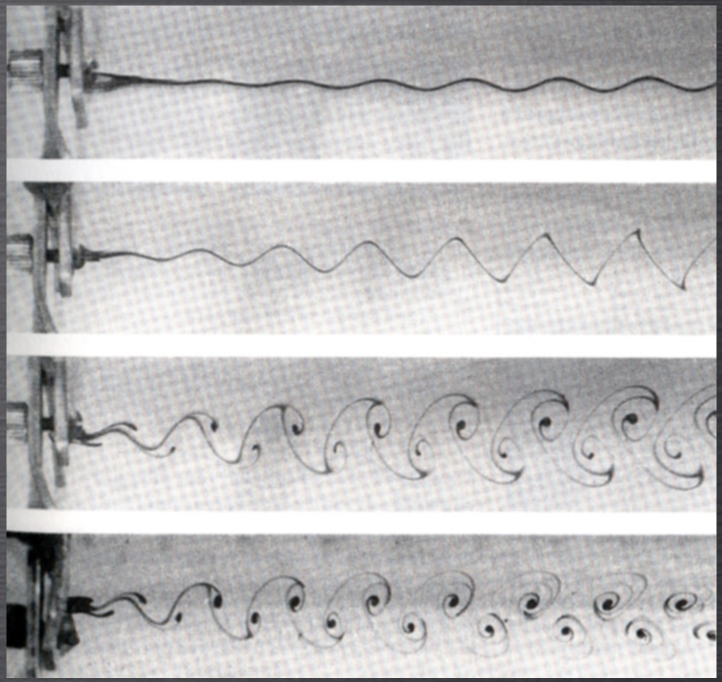
- intrinsic computation
- Maxwell's Demon
- Szilard's Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?

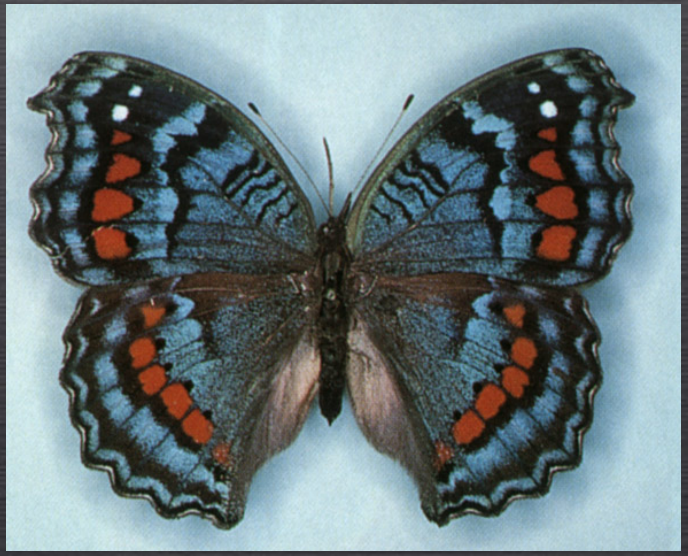
AGENDA STRUCTURE!

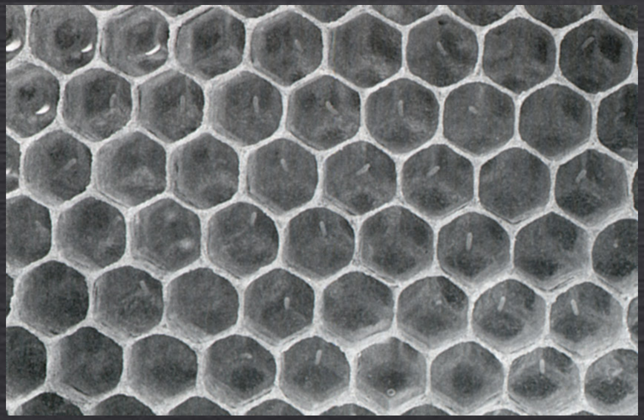
- INTRINSIC COMPUTATION
- MAXWELL'S DEMON
- SZILARD'S ENGINE
- THERMODYNAMICAL SYSTEMS
- APPLICATIONS: RATCHETS, ADAPTATION
- INTELLIGENCE?

STRUCTURE!

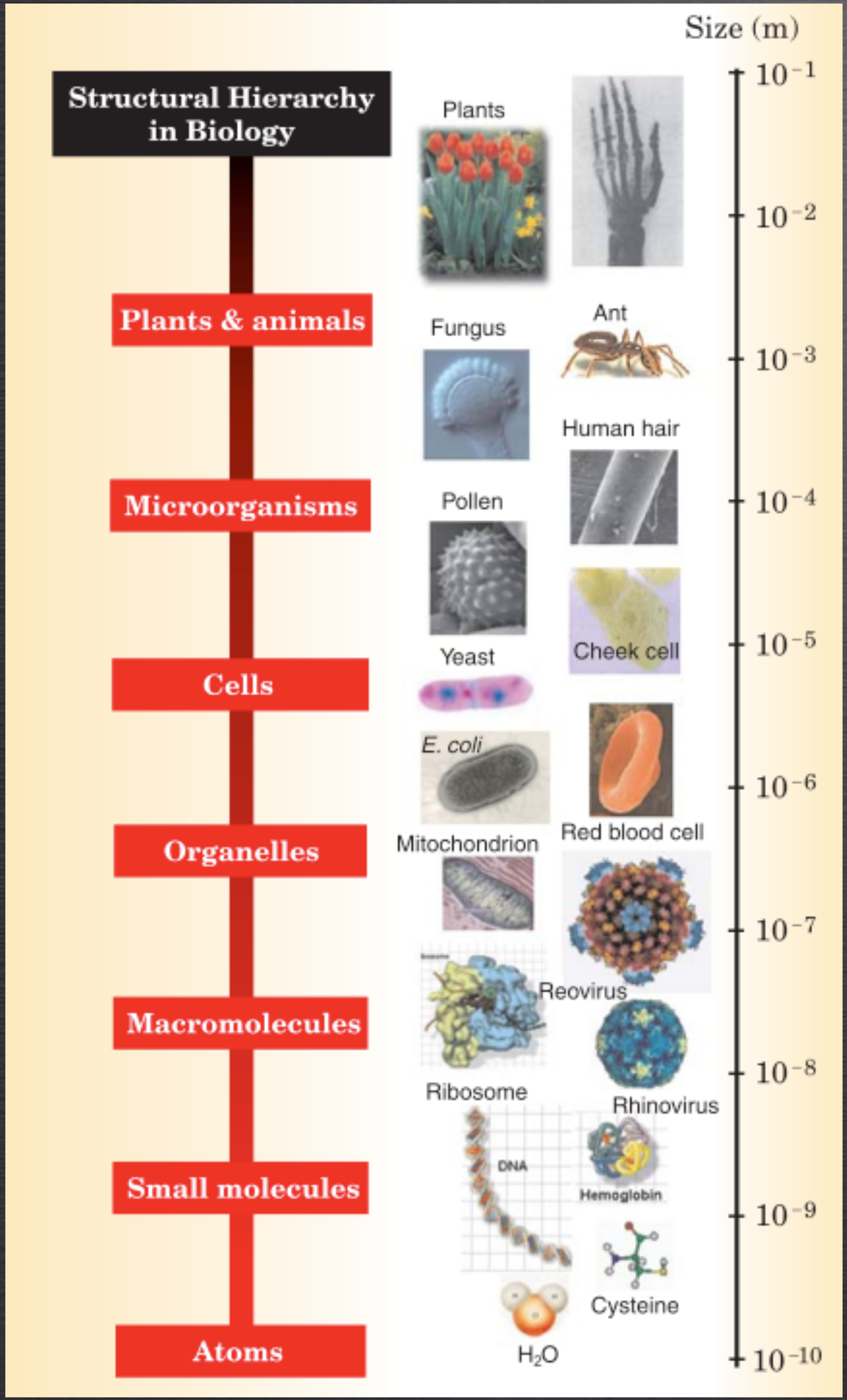








BIOLOGICAL STRUCTURE



In what ways does nature organize?
(Phenomenology)

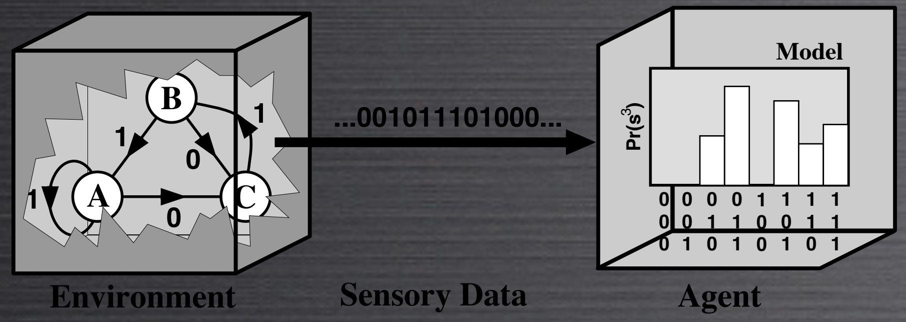
How does it organize?
(Mechanism)

Are these levels real or merely convenient?
(Objectivity)

Why does nature organize?
(Optimization versus chance versus ....)

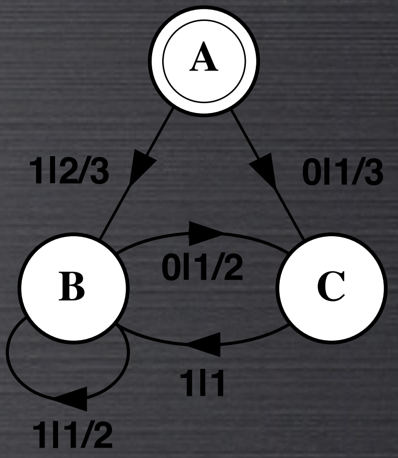
Physics Today, March 2006

FOUNDATIONS: COMPUTATIONAL MECHANICS



CAUSAL EQUIVALENCE:

$$\overleftarrow{x} \sim \overleftarrow{x}' \;\Leftrightarrow\; \Pr(\overrightarrow{X}|\overleftarrow{x}) = \Pr(\overrightarrow{X}|\overleftarrow{x}')$$



- ε-MACHINE: UNIQUE, MINIMAL, & OPTIMAL PREDICTOR

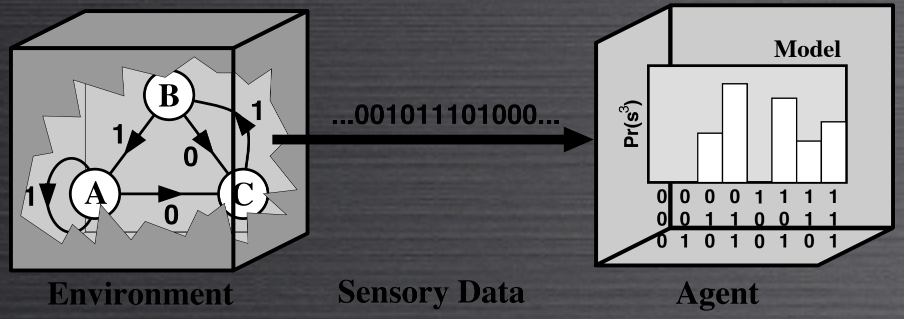
STORED VERSUS GENERATED INFORMATION

$$C_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \log_2 \Pr(\sigma)$$

VERSUS

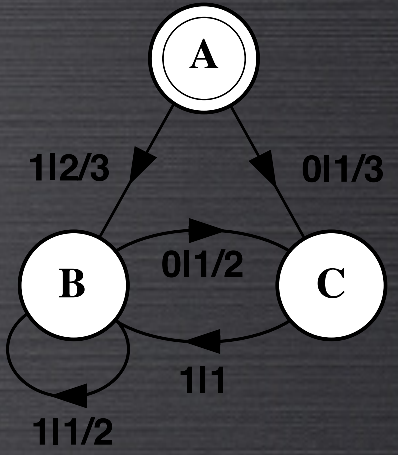
$$h_\mu = -\sum_{\sigma \in \boldsymbol{\mathcal{S}}} \Pr(\sigma) \sum_{\sigma' \in \boldsymbol{\mathcal{S}}} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma)$$

Foundations: Computational Mechanics



Causal Equivalence:

$$\overleftarrow{x} \sim \overleftarrow{x}' \;\Leftrightarrow\; \Pr(\overrightarrow{X}|\overleftarrow{x}) = \Pr(\overrightarrow{X}|\overleftarrow{x}')$$

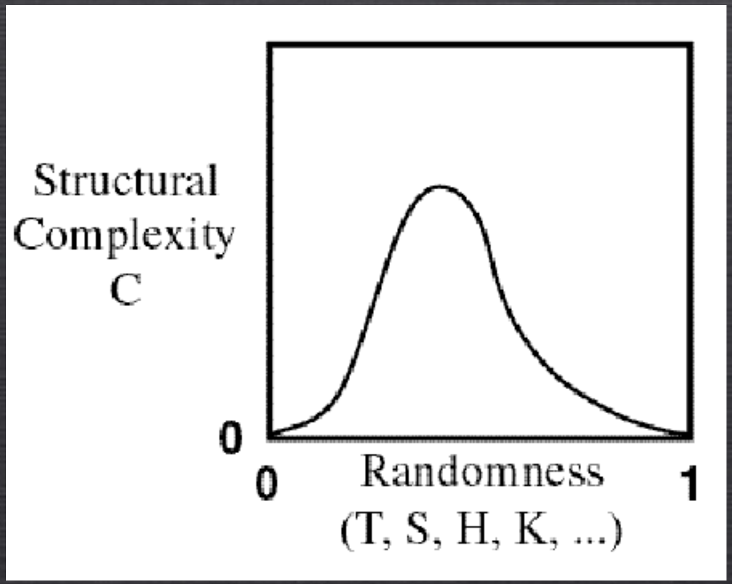


- ε-Machine: Unique, minimal, & optimal predictor

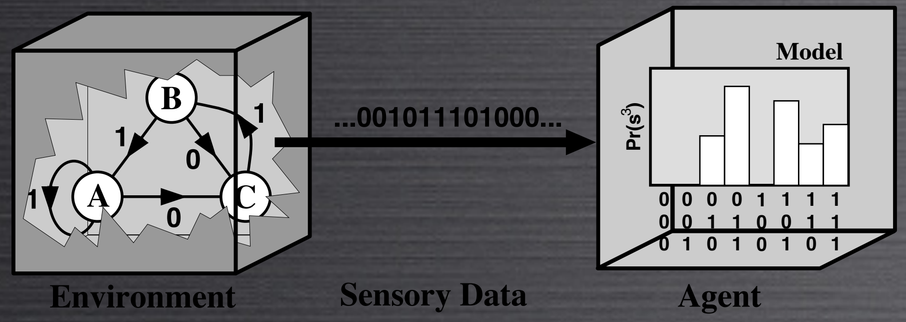
Stored versus Generated Information

$$C_\mu = -\sum_{\sigma \in \mathcal{S}} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \text{versus} \quad h_\mu = -\sum_{\sigma \in \mathcal{S}} \Pr(\sigma) \sum_{\sigma' \in \mathcal{S}} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma)$$

Structure versus Randomness



Foundations: Computational Mechanics



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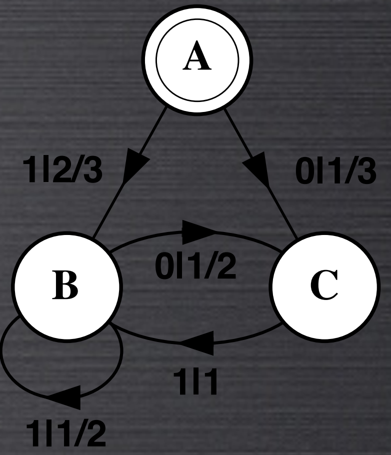

- ε-Machine: Unique, minimal, & optimal predictor

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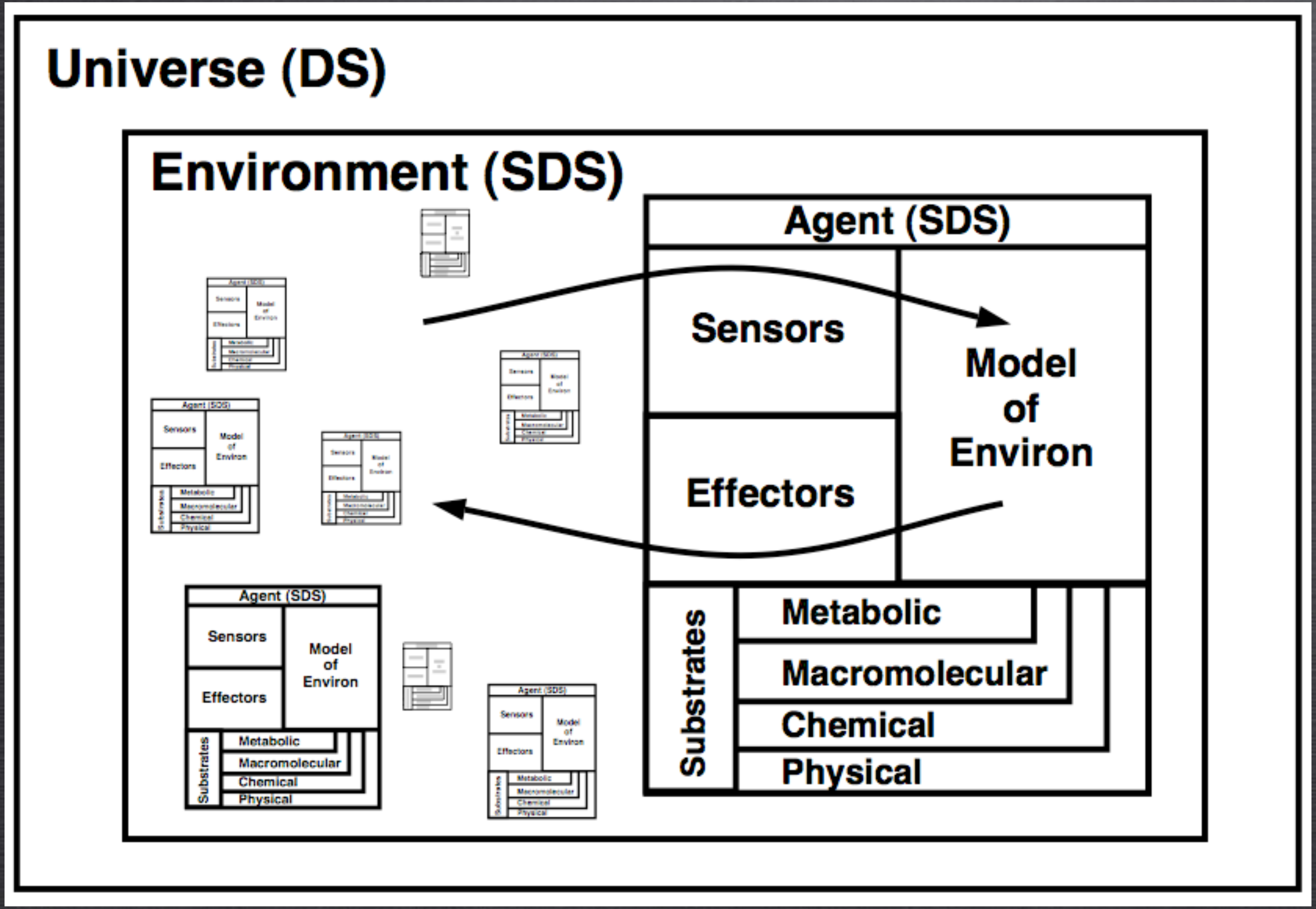
Intrinsic Computation:

1. How much historical information does a process store?
2. In what architecture is it stored?
3. How is it used to produce future behavior?

J.P. Crutchfield & K. Young, “Inferring Statistical Complexity”, Physical Review Letters 63 (1989) 105-108.

J.P. Crutchfield, “Between Order and Chaos”,
Nature Physics 8 (January 2012) 7-24.

PROBLEM STATEMENT



= Ecological population dynamics of structurally complex adapting agents

+ Reproduction (evolutionary population dynamics)

JP Crutchfield, "The Calculi of Emergence: Computation, Dynamics, and Induction", Physica D 75 (1994) 11-54.
In Proceedings of the Oji International Seminar:
Complex Systems—from Complex Dynamics to Artificial Reality 5 - 9 April 1993, Numazu, Japan.

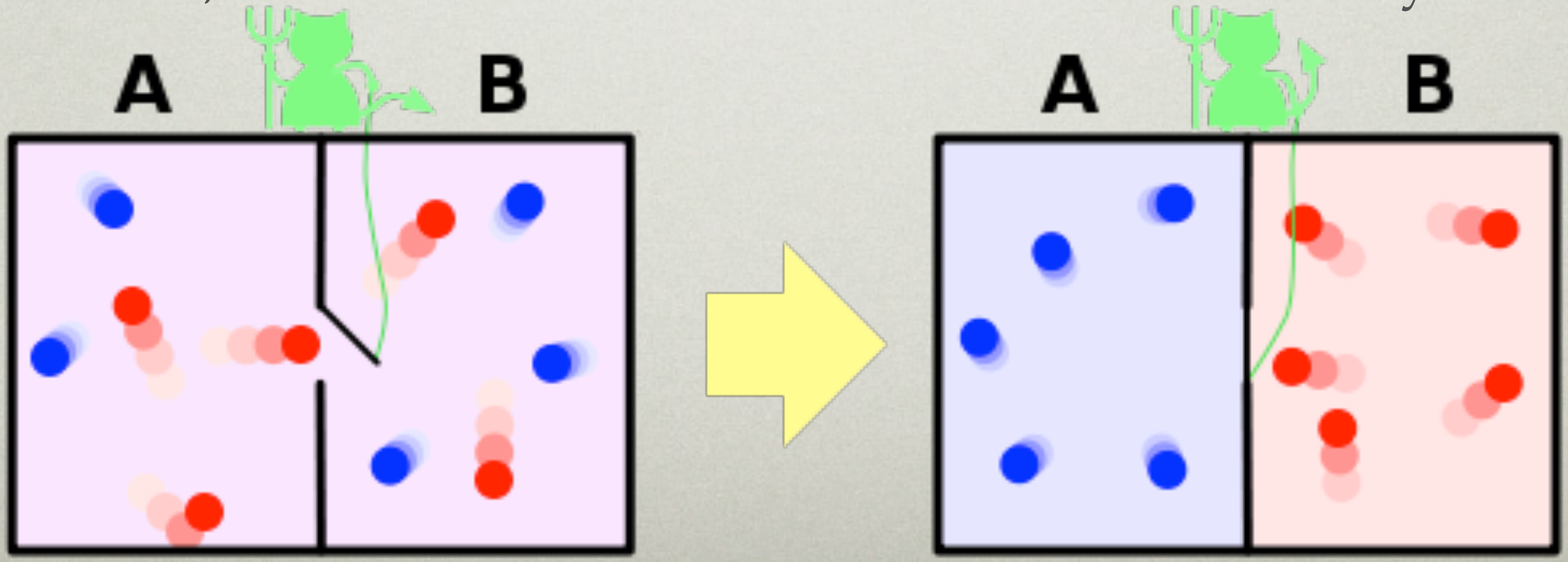
Agenda

- intrinsic computation
- Maxwell’s Demon
- Szilard’s Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?

MAXWELL'S DEMON

James Clerk &
Katherine Maxwell
(1865)

... if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us. ... Now let us suppose that ... a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics. ...



MAXWELL'S DEMON

- Demon creates order out of chaos.

MAXWELL'S DEMON

- Demon creates order out of chaos.

MAXWELL'S DEMON

- Demon creates order out of chaos.

- Uses molecular information to convert heat to temperature difference & so to useful work.

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SZILARD'S ENGINE:

"ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM BY THE INTERVENTION OF INTELLIGENT BEINGS",

LEO SZILARD, ZEITSCHRIFT FUR PHYSIK **65** (1929) 840-866.

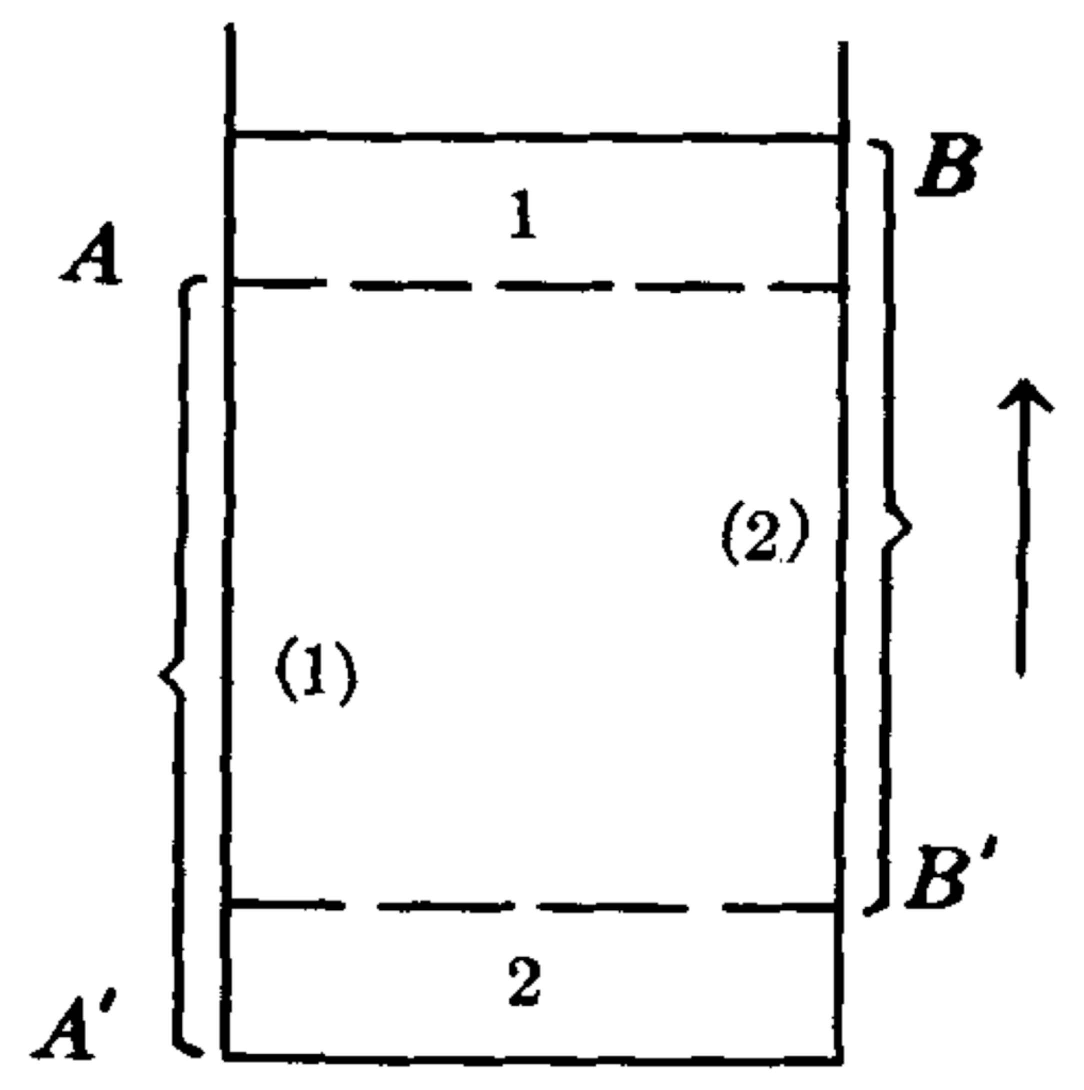
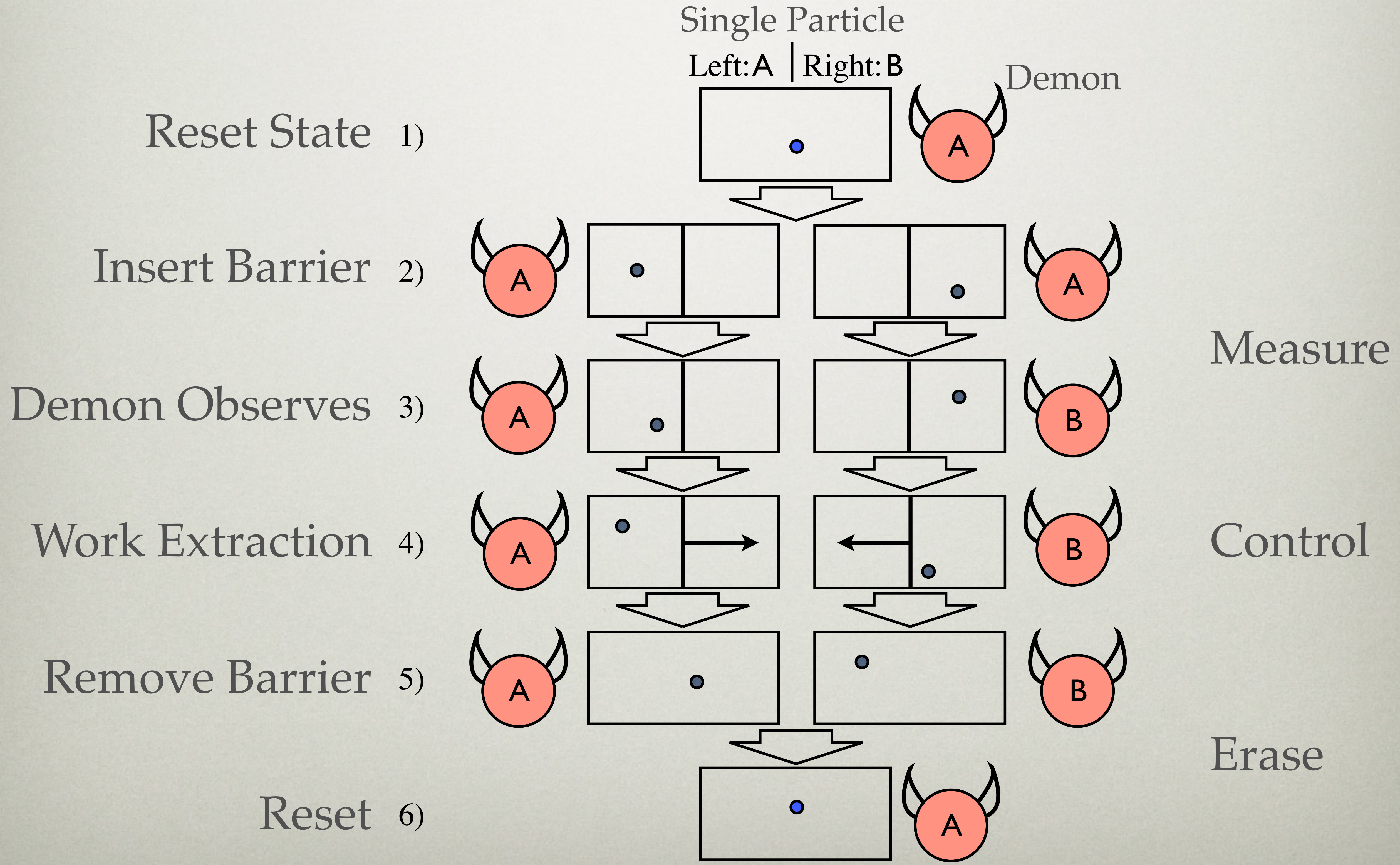


FIG. 1

... we must conclude *that the intervention which establishes the coupling between y and x, the measurement of x by y, must be accompanied by a production of entropy.*

SZILARD'S SINGLE-MOLECULE ENGINE CYCLE



Reset State 1)

Insert Barrier 2)

Demon Observes 3)

Work Extraction 4)

Remove Barrier 5)

Reset 6)

Measure

Control

Erase

SZILARD'S ENGINE:

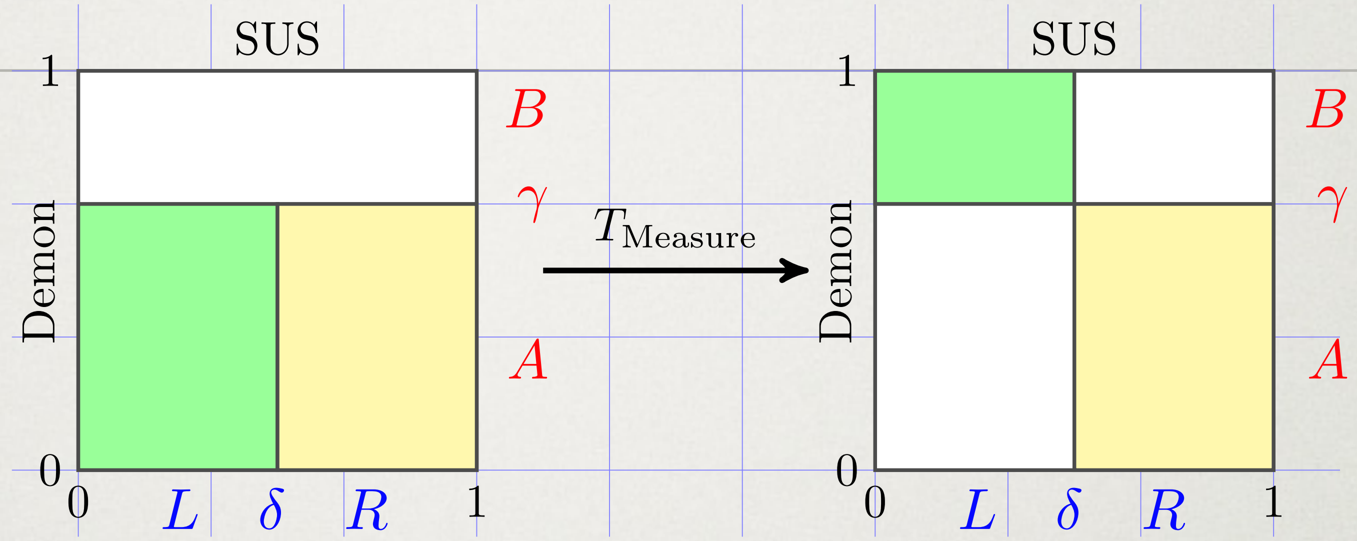
“ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM BY THE INTERVENTION OF INTELLIGENT BEINGS”,
LEO SZILARD, ZEITSCHRIFT FUR PHYSIK **65** (1929) 840-866.

“... a simple inanimate device can achieve the same essential result as would be achieved by the intervention of intelligent beings. We have examined the ‘biological phenomena’ of a nonliving device and have seen that it generates exactly that quantity of entropy which is required by thermodynamics.”

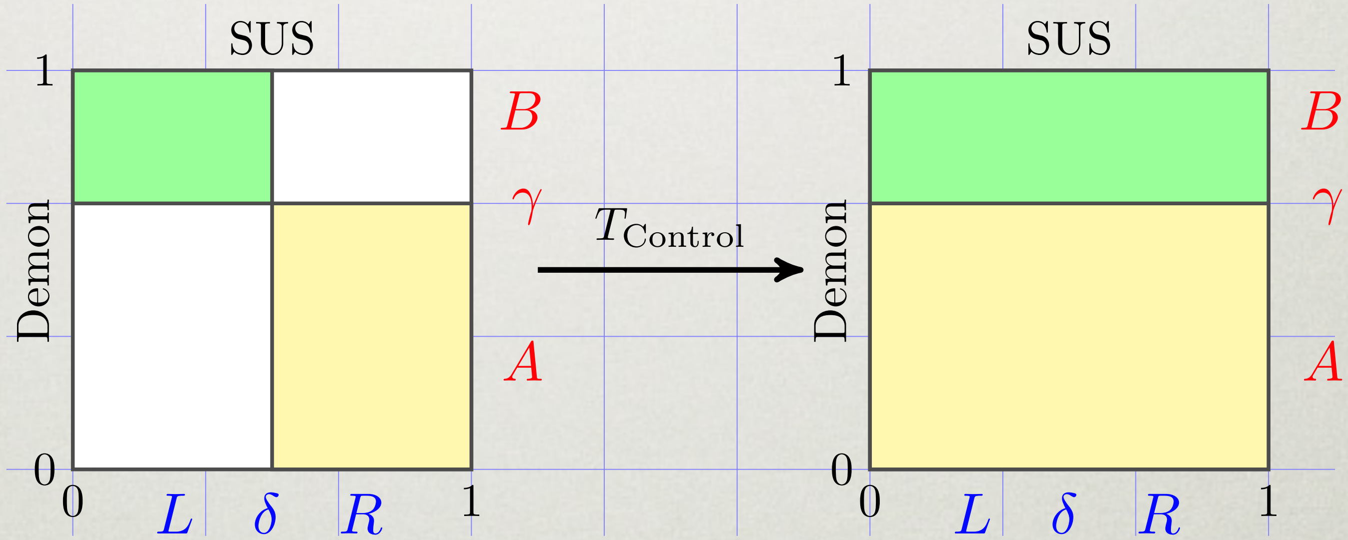
… A CHAOTIC DYNAMICAL SYSTEM

Alec Boyd and JPC, "Demon Dynamics: Deterministic Chaos, the Szilard Map, and the Intelligence of Thermodynamic Systems", Physical Review Letters **116** (2016) 190601. arxiv.org:1506.04327 [cond-mat.stat-mech].

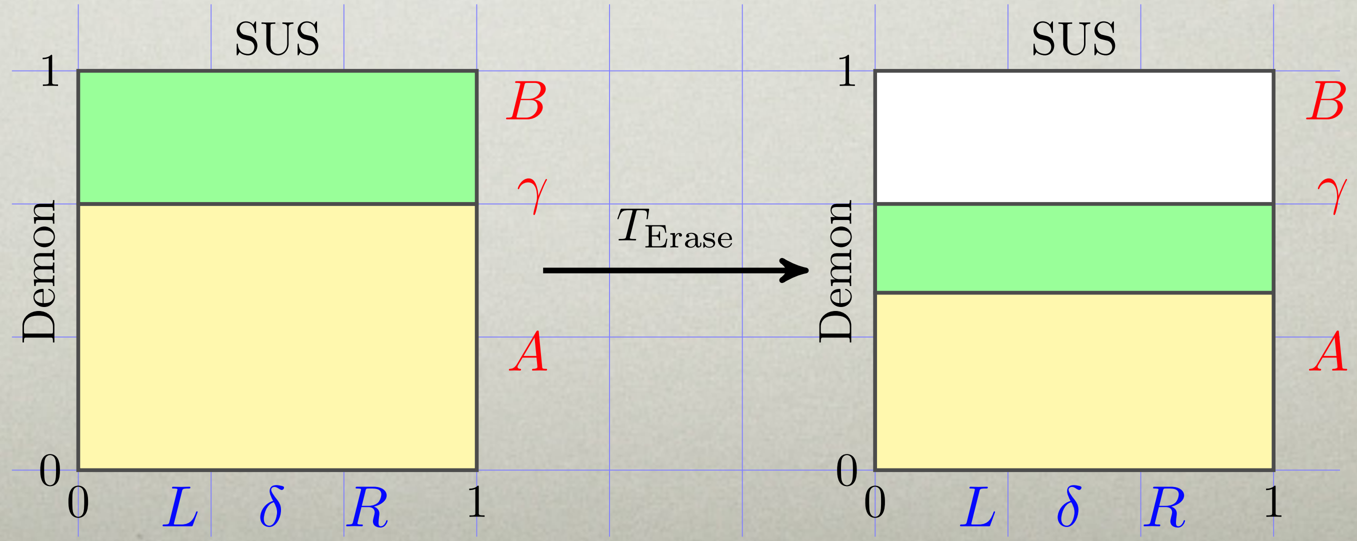
Measure



Extract Energy from Heat Bath



Erase Demon Memory



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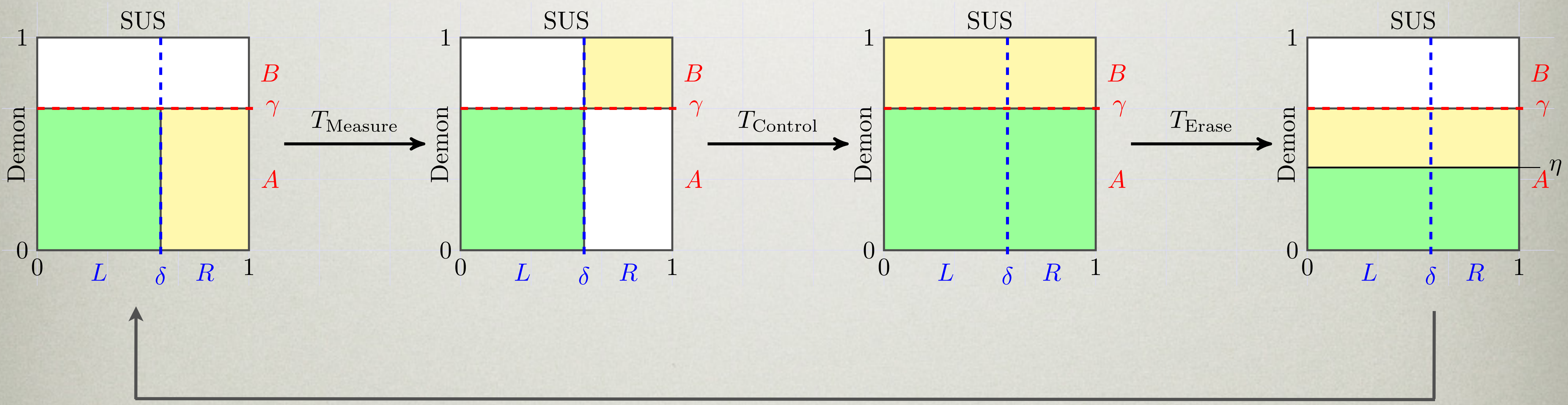


THE SZILARD MAP

Measure $$\mathcal{T}_{\mathrm{M}}(x,y) = \begin{cases} (x,y) & x<\delta, y<\gamma \text{ or } x<\delta, y\geq\gamma\ , \\ \left(x, \gamma + y\frac{1-\gamma}{\gamma}\right) & x\geq\delta, y\leq\gamma\ , \\ \left(x, \gamma\frac{y-\gamma}{1-\gamma}\right) & x\geq\delta, y>\gamma\ . \end{cases}$$

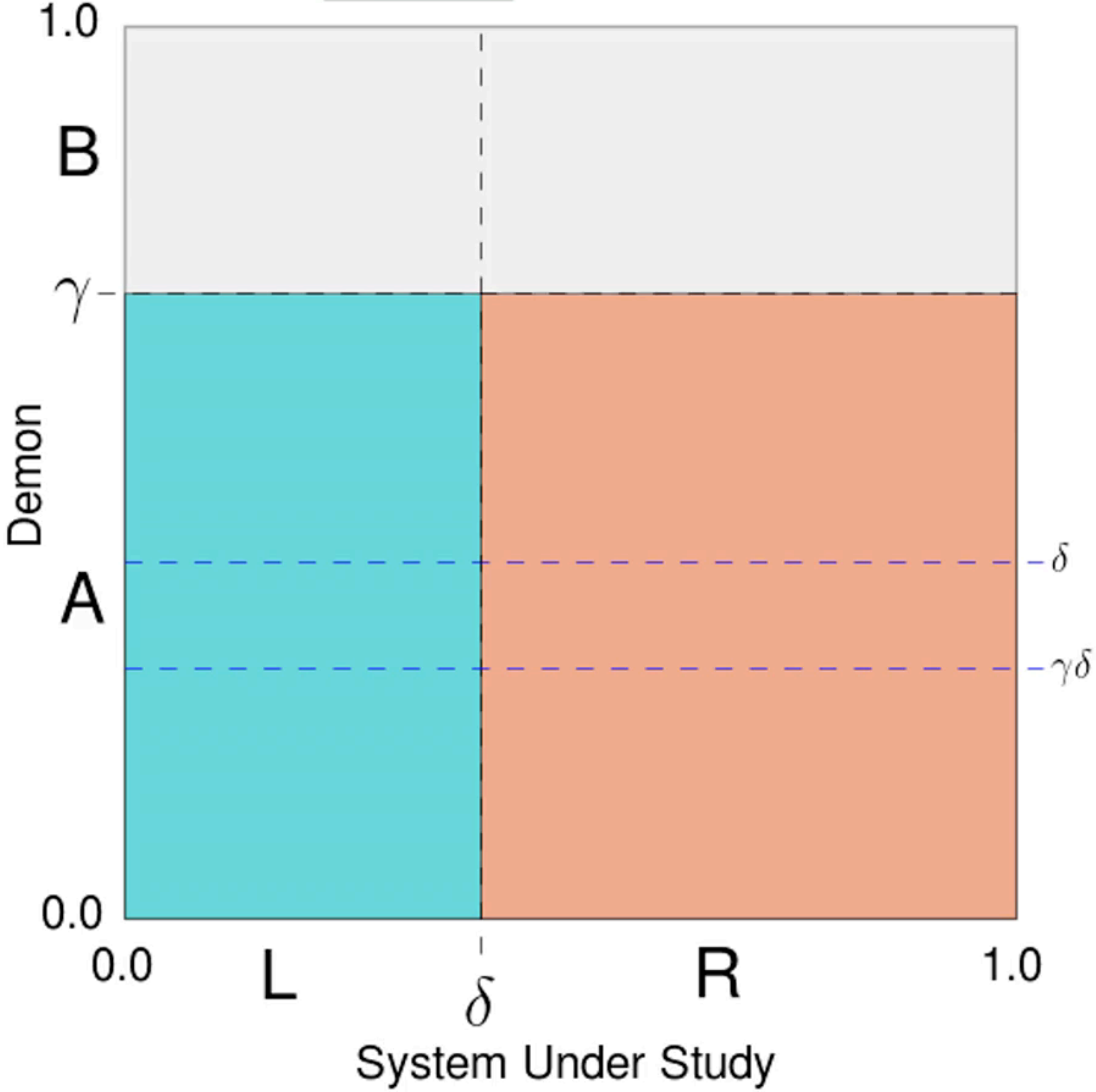
Control $$\mathcal{T}_{\mathrm{C}}(x,y) = \begin{cases} (\frac{x}{\delta}, y) & x<\delta\ , \\ \left(\frac{x-\delta}{1-\delta}, y\right) & x\geq\delta\ . \end{cases}$$

Erase $$\mathcal{T}_{\mathrm{E}}^{A}(x,y) = \begin{cases} (x, y\delta) & y<\gamma\ , \\ \left(x, \delta\gamma + \frac{y-\gamma}{1-\gamma}\gamma(1-\delta)\right) & y\geq\gamma\ . \end{cases}$$

$$\mathcal{T}_{\mathrm{Szilard}}(x,y) = \mathcal{T}_{\mathrm{E}}^{A}(x,y) \circ \mathcal{T}_{\mathrm{C}}(x,y) \circ \mathcal{T}_{\mathrm{M}}(x,y)$$

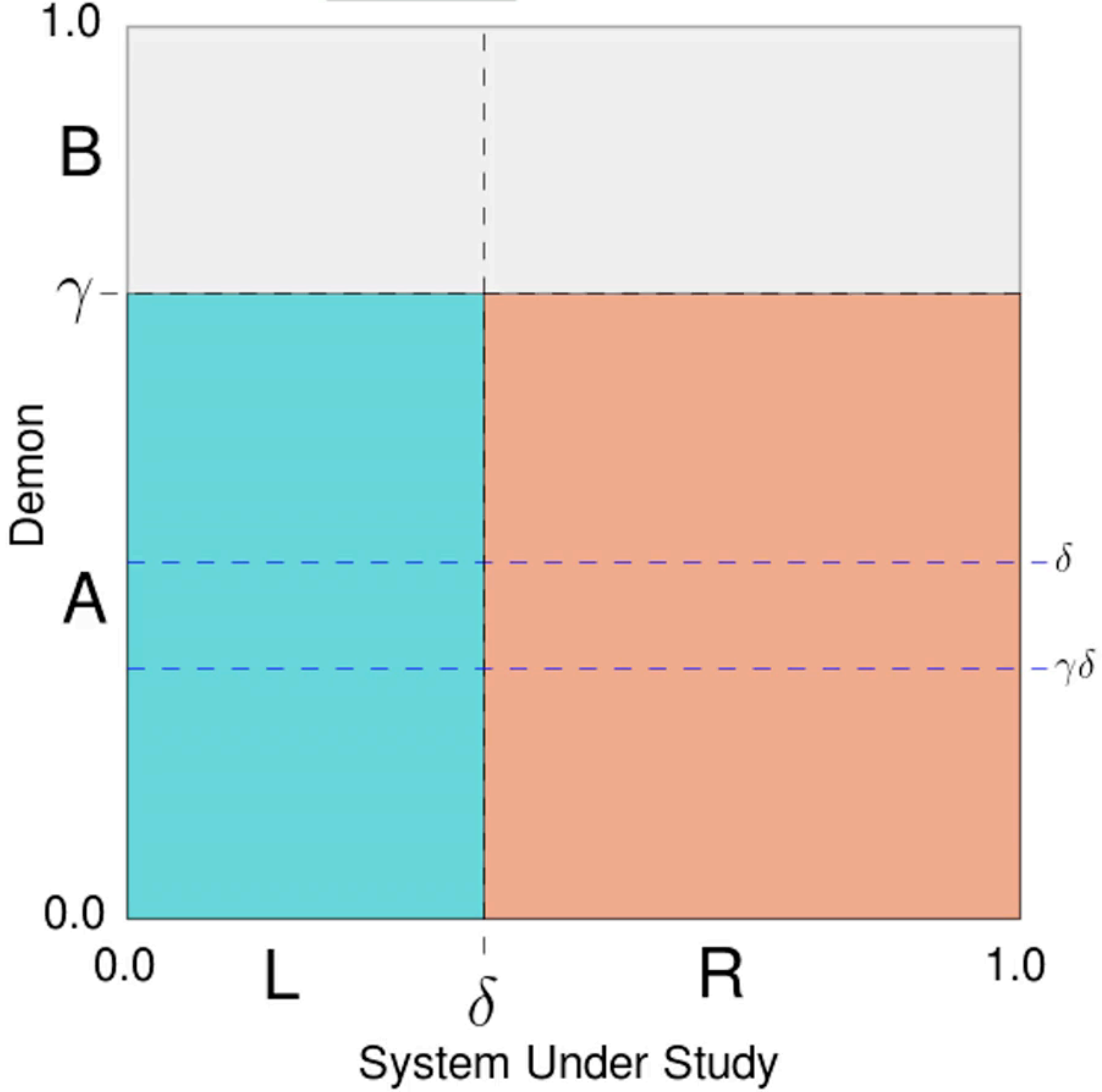
Szilard Engine is a Chaotic Map

Measure Control Erase



Szilard Engine is a Chaotic Map

Measure Control Erase



THE SZILARD MAP:

CONTROLLER SYMBOLIC DYNAMICS

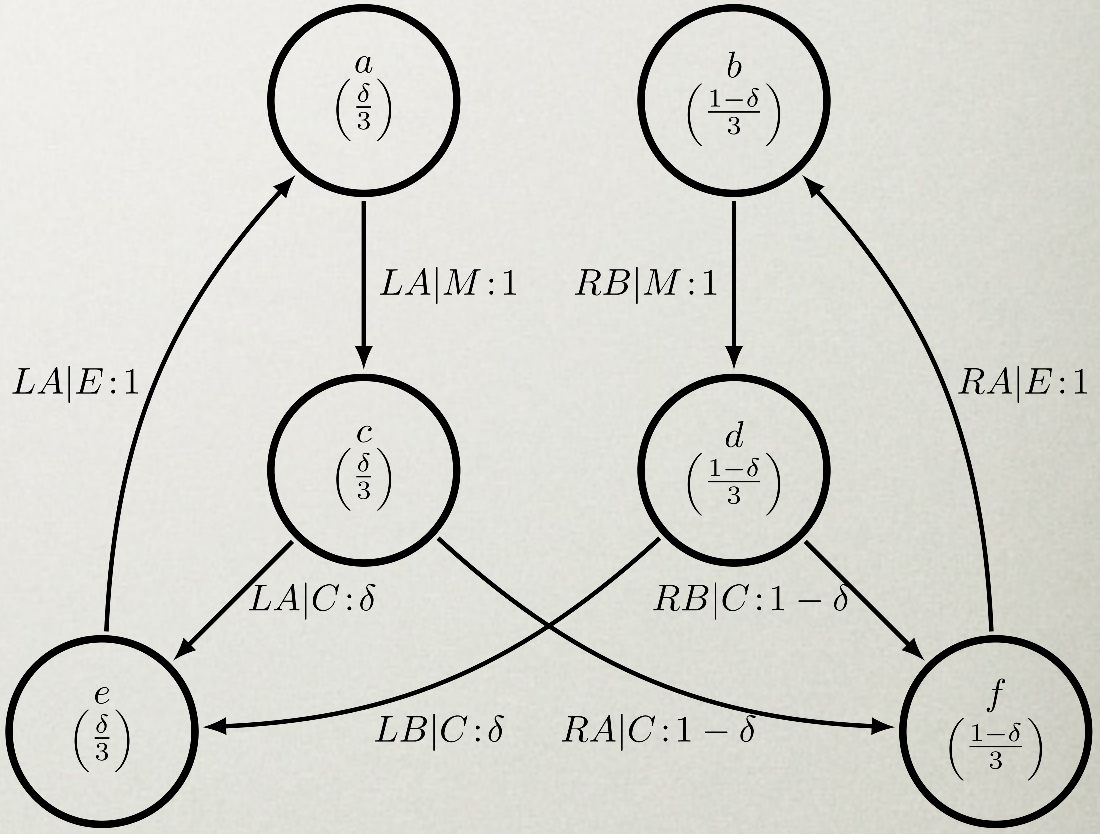
ϵ-Machine: Minimal Optimal Generator

$$\Sigma_{\text{in}} = \{M, C, E\}$$

Demon Memory

Molecule Position

$$\Sigma_{\text{out}} = \{A, B\} \times \{L, R\}$$



Entropy Rate: $h_\mu = \mathrm{H}(\delta)$ bits

Statistical Complexity: $C_\mu = \mathrm{H}[\Pr(\sigma)]$

$$= \log_2 3 + \mathrm{H}(\delta) \text{ bits}$$

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THE SZILARD ENGINE: A THERMODYNAMICAL SYSTEM

Dynamics

Thermodynamics

$\mathcal{T}_{\mathrm{M}}(x, y)$

Measure

$$\langle Q_{\mathrm{measure}} \rangle = \\ - k_B T (1 - \delta) \ln \left((1 - \gamma)/\gamma \right)$$

$\mathcal{T}_{\mathrm{C}}(x, y)$

Control

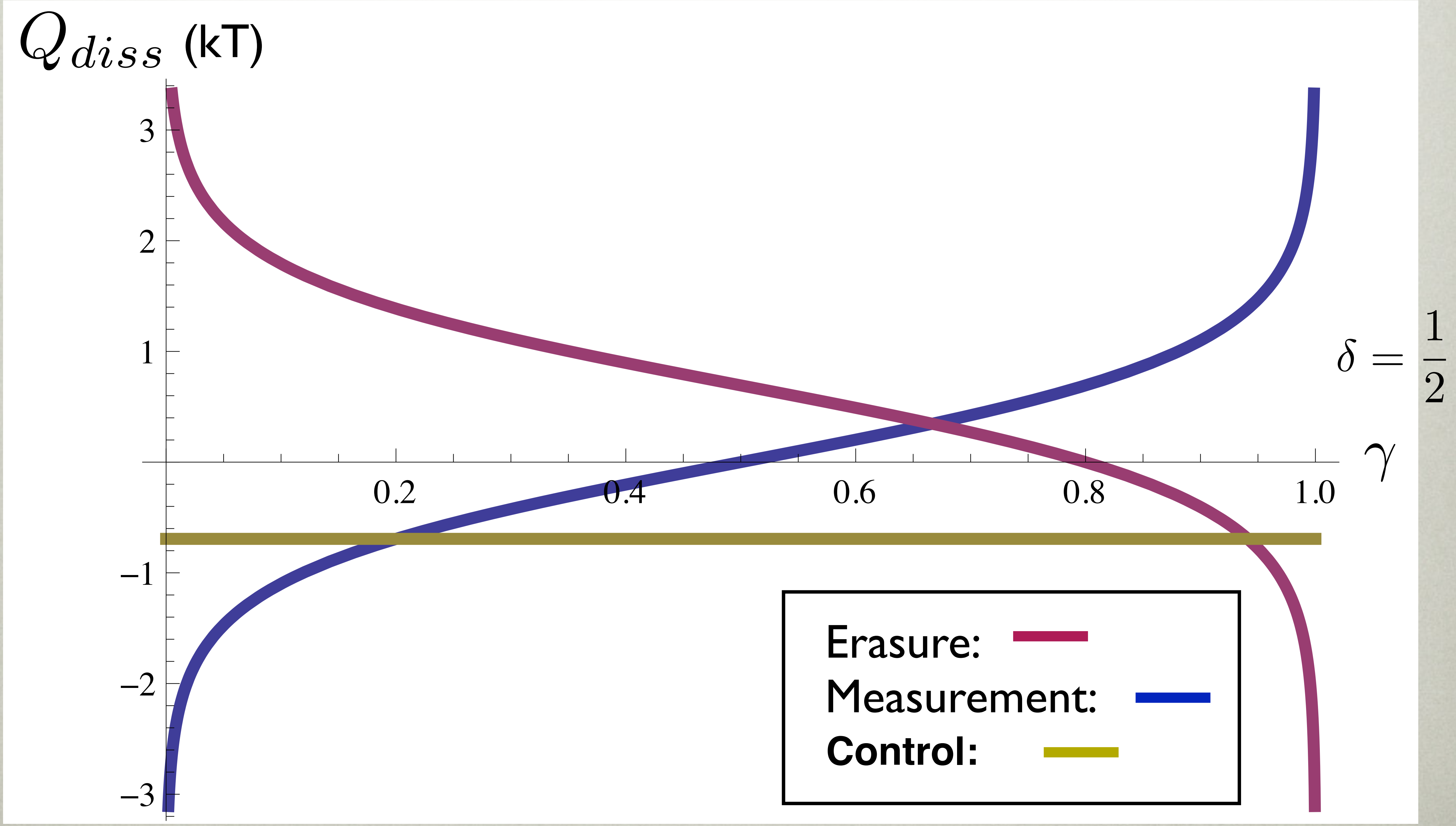
$$\langle Q_{\mathrm{control}} \rangle = -k_B T \, \mathrm{H}(\delta) \ln 2$$

$\mathcal{T}_{\mathrm{E}}^{A}(x, y)$

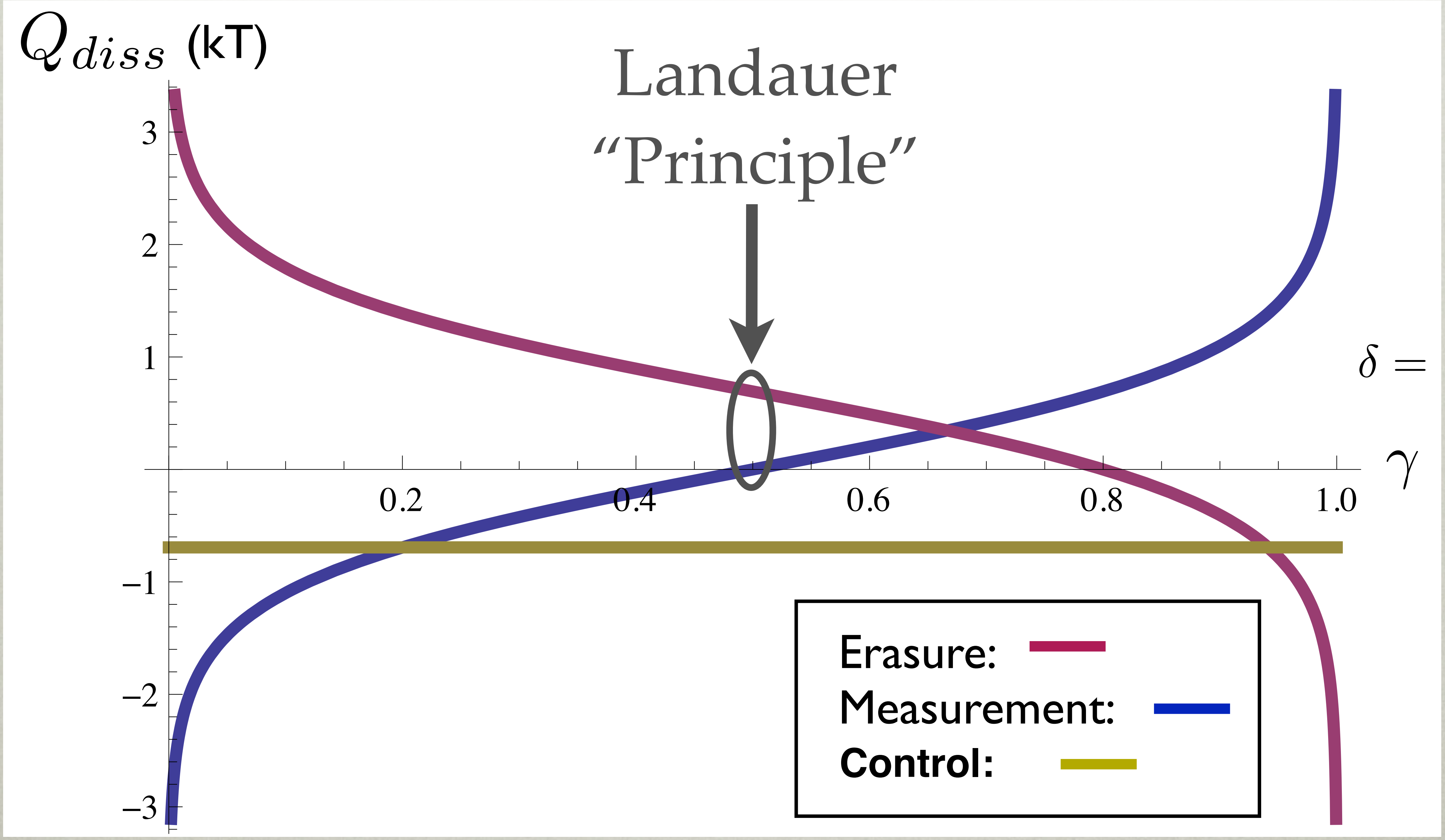
Erase

$$\langle Q_{\mathrm{erase}} \rangle = \\ k_B T (1 - \delta) \ln \left((1 - \gamma)/\gamma \right) \\ + k_B T \, \mathrm{H}(\delta) \ln 2$$

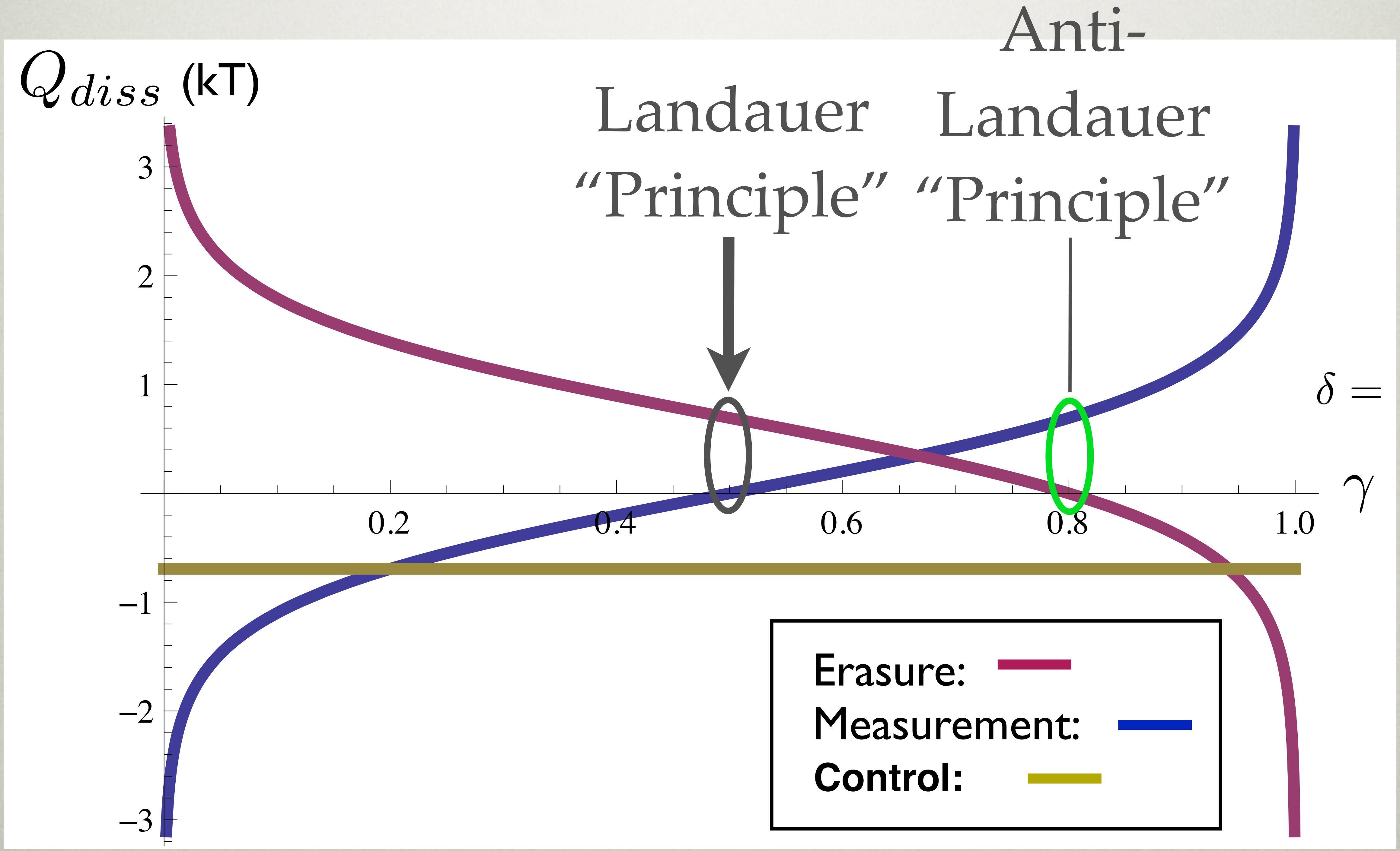
ENERGY DISSIPATION: ERASURE **AND** MEASUREMENT!



ENERGY DISSIPATION: ERASURE AND MEASUREMENT!



ENERGY DISSIPATION: ERASURE **AND** MEASUREMENT!



INFORMATION ENGINES: AN ANALYTICAL STRATEGY

(i) An information engine is the dynamic over a joint state space of a thermodynamic system and a physically embodied controller.

(ii) The causal states of the joint dynamics, formed from the predictive equivalence classes of system histories, capture its information processing and emergent organization.

(iii) A necessary component of the engine's effective "intelligence", its memory, is given by its statistical complexity C_μ.

(iv) Its dissipation is given by the dynamical system negative LCEs, and

(v) the rate of energy extracted from the heat bath is governed by the Kolmogorov-Sinai entropy h_μ.

Agenda

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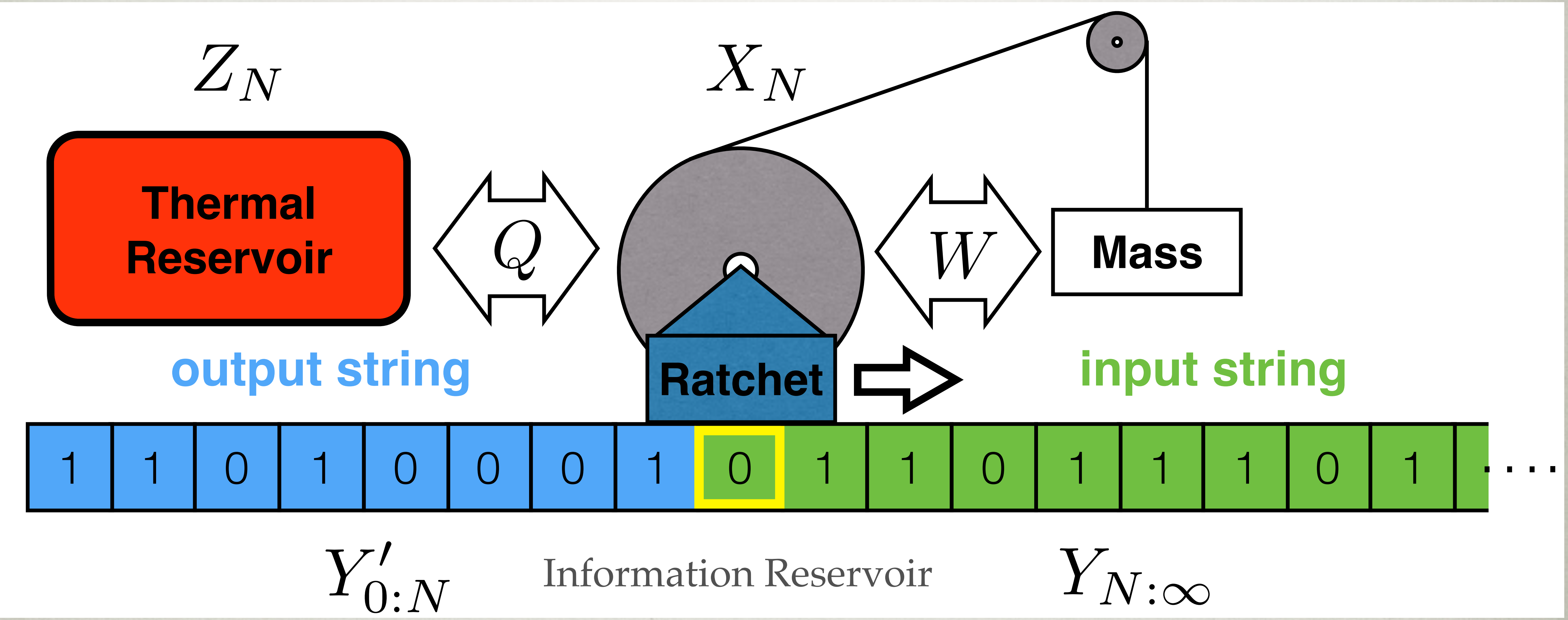
INFORMATION RATCHETS

Net Work Extraction!



Output $h_\mu{}'$

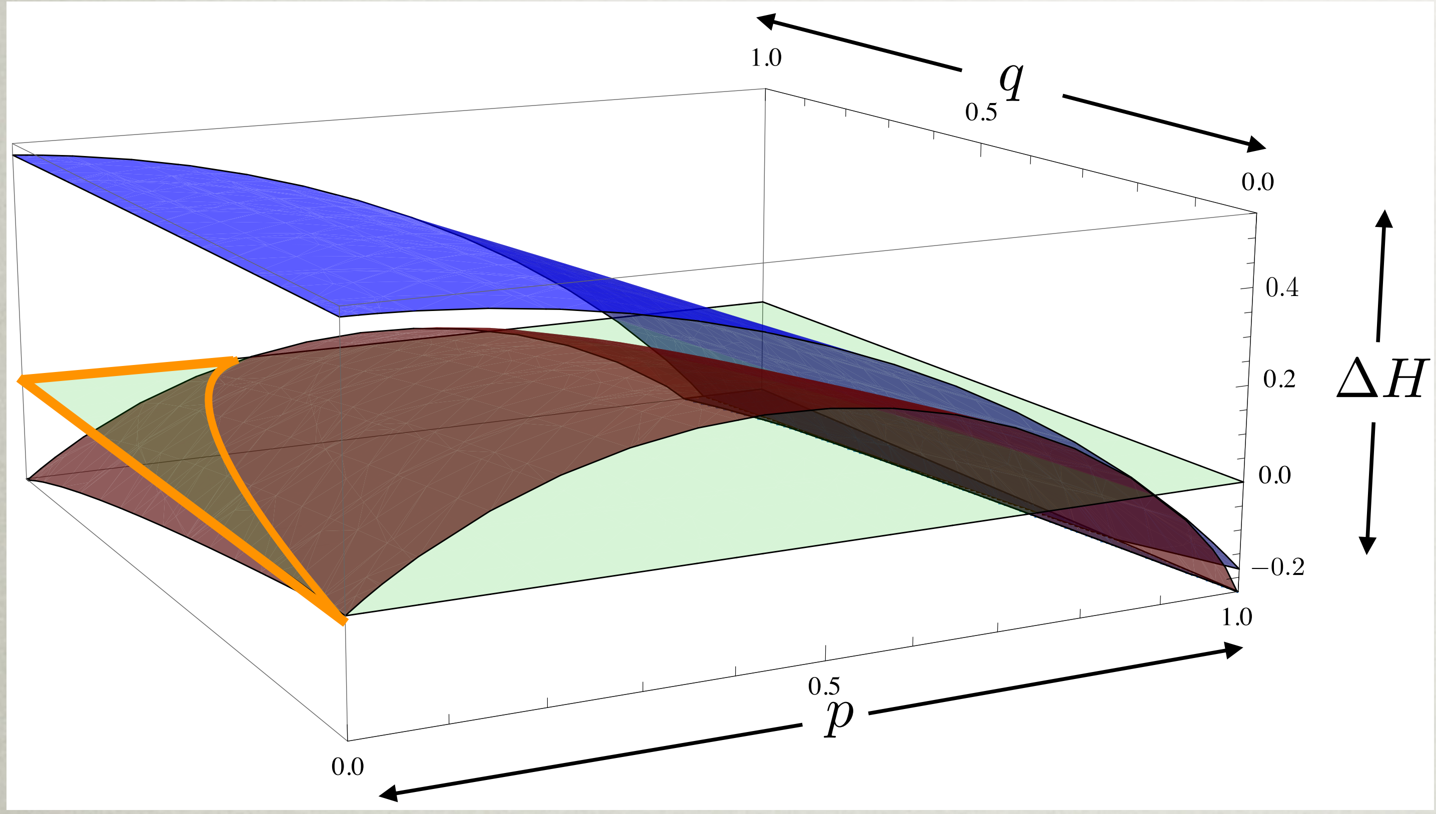
h_μ Input

A. Boyd, D. Mandal, and JPC, "Identifying Functional Thermodynamics in Autonomous Maxwellian Ratchets ". New Journal of Physics **18** (2016) 023149. arxiv.org:1507.01537 [cond-mat.stat-mech]

Information Ratchets

Second Law for Intrinsic Computation

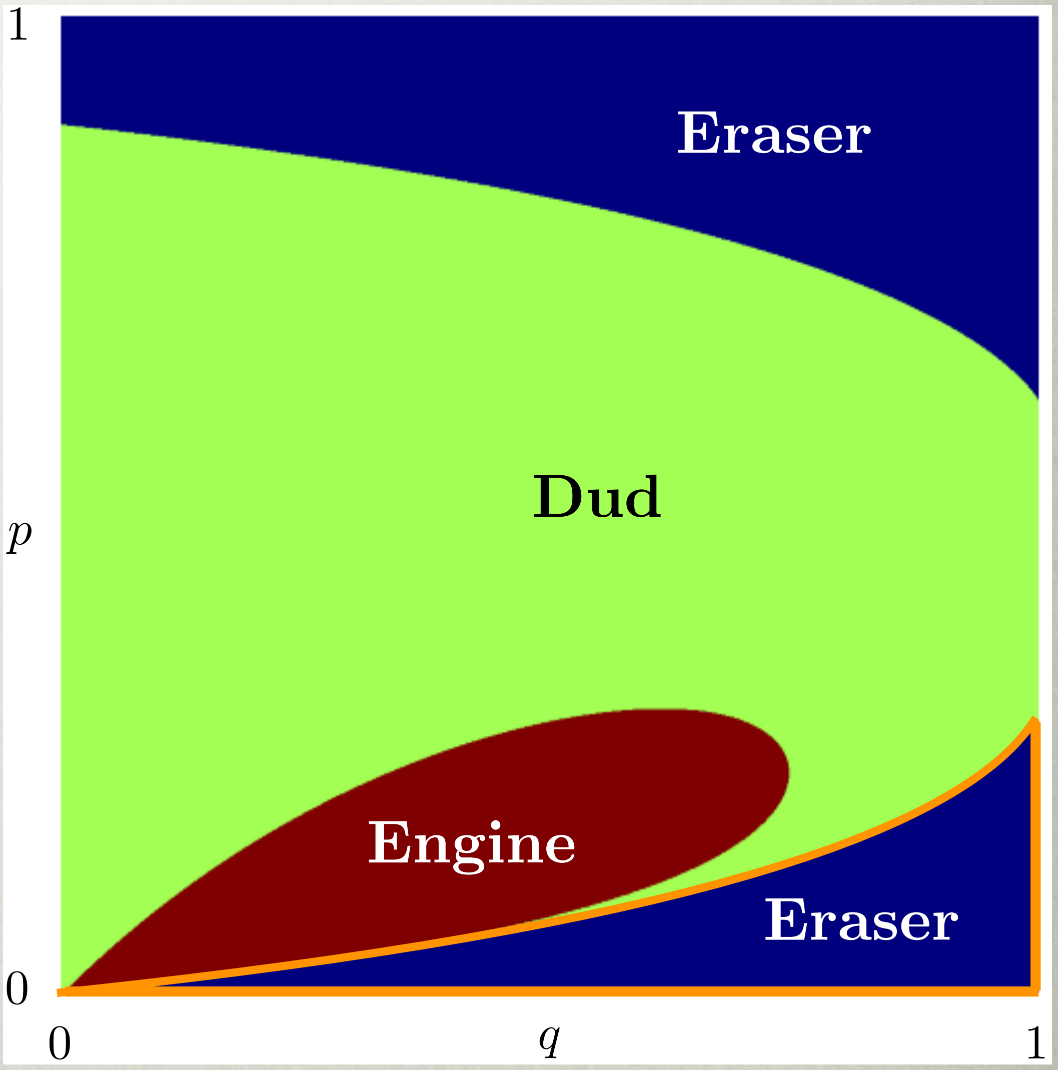
$$\langle W \rangle \ \leq \ k_{\mathrm{B}} T \ln 2 \ \left({h_\mu}' - h_\mu \right)$$



Entropy Rates

$\Delta \, \mathrm{H}(1) = \mathrm{H}'(1) - \mathrm{H}(1)$

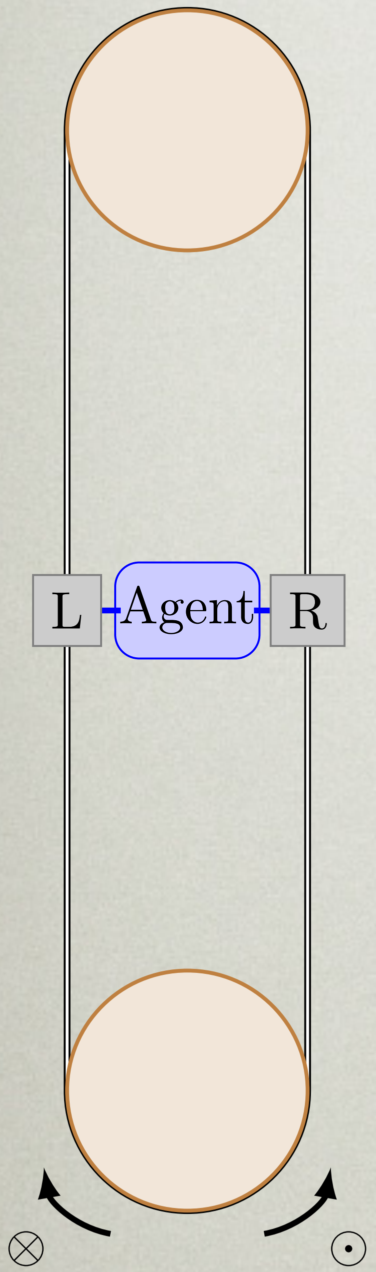
$\Delta h_\mu = {h_\mu}' - h_\mu$



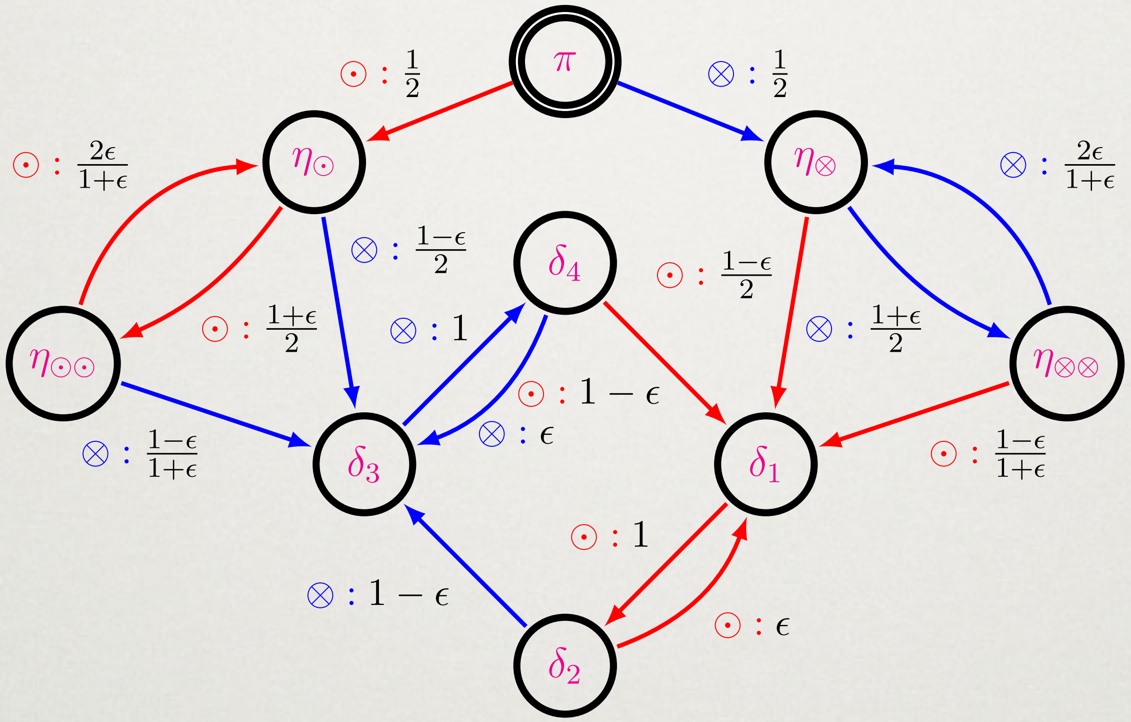
Thermodynamic Functions

ENERGY HARVESTING AGENTS

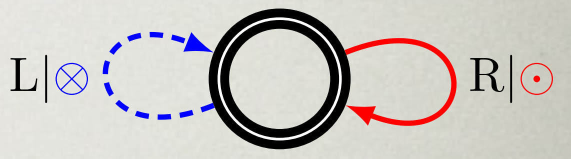
(Paul Riechers and JPC, “Structural Thermodynamics of Adaptation”, In prep.



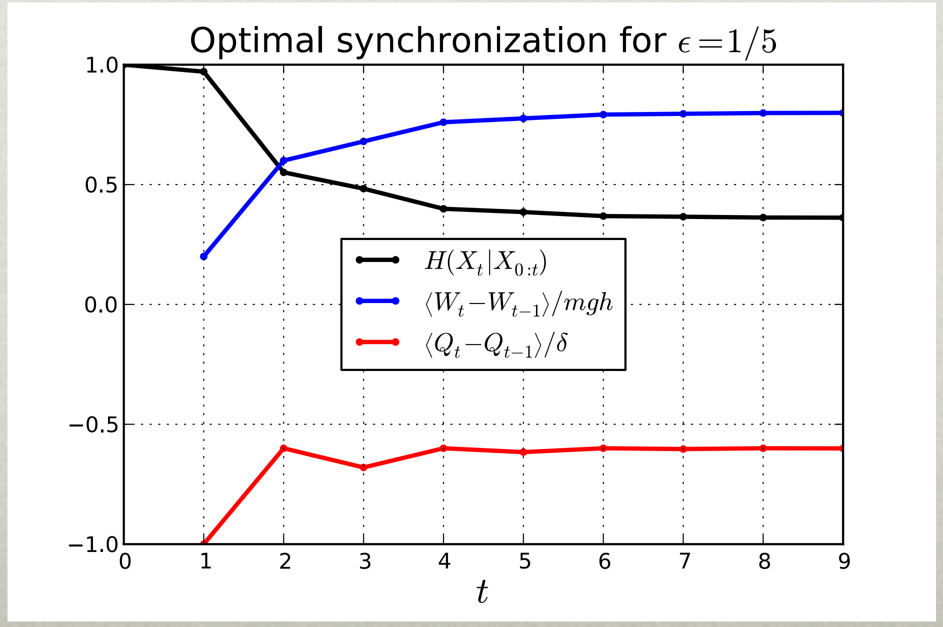
Stochastic environment



Optimal agent



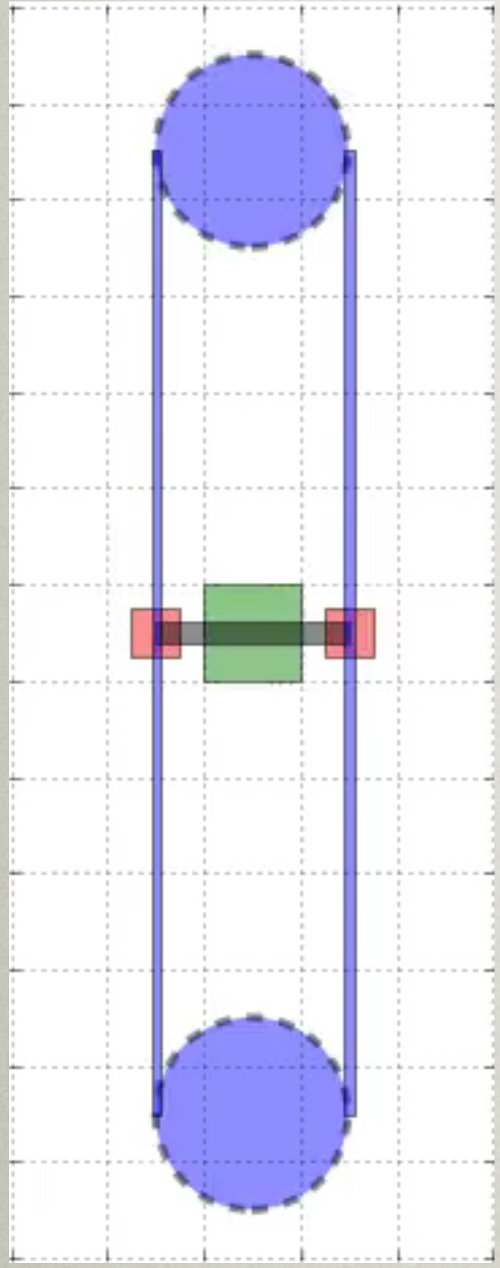
Suboptimal agent



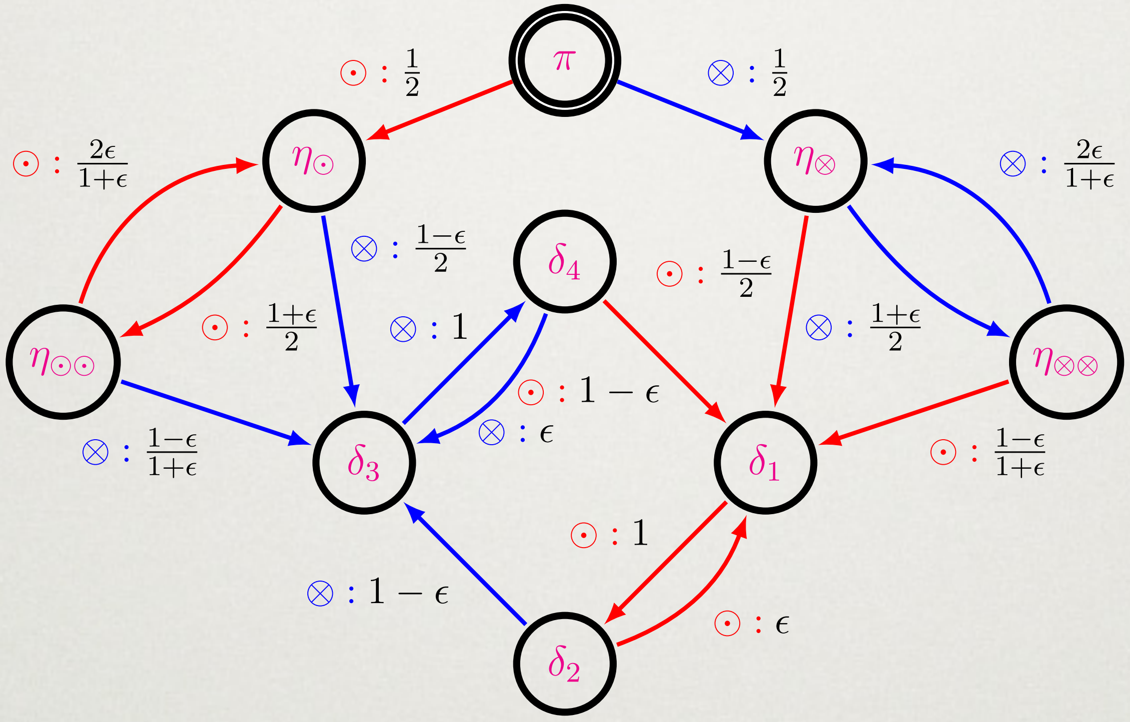
Must sync to environment

ENERGY HARVESTING AGENTS

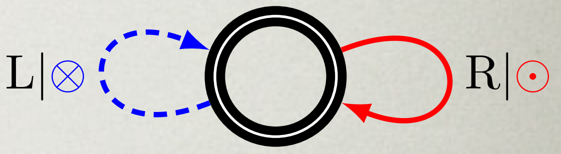
(Paul Riechers and JPC, “Structural Thermodynamics of Adaptation” (2016).)



Stochastic environment



Optimal agent



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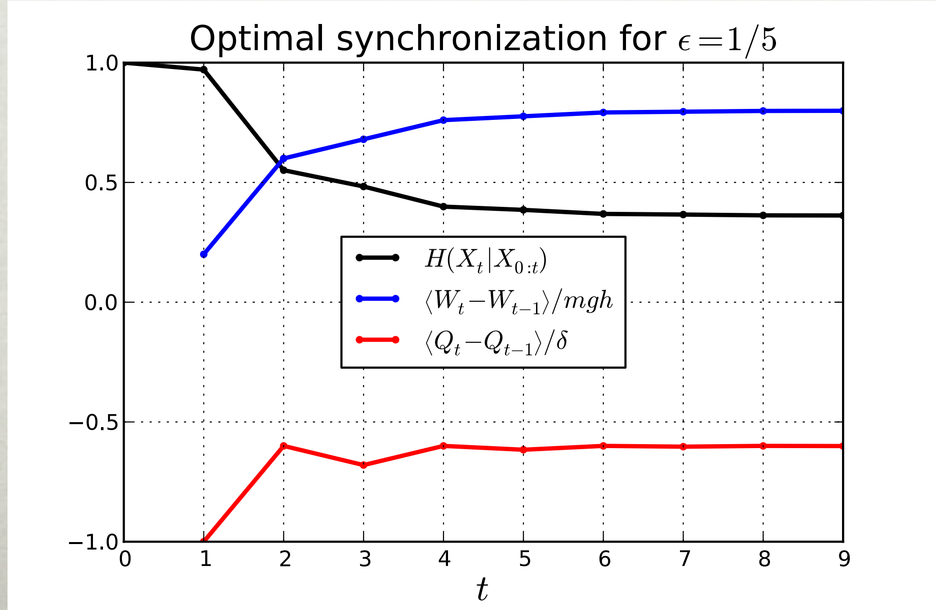

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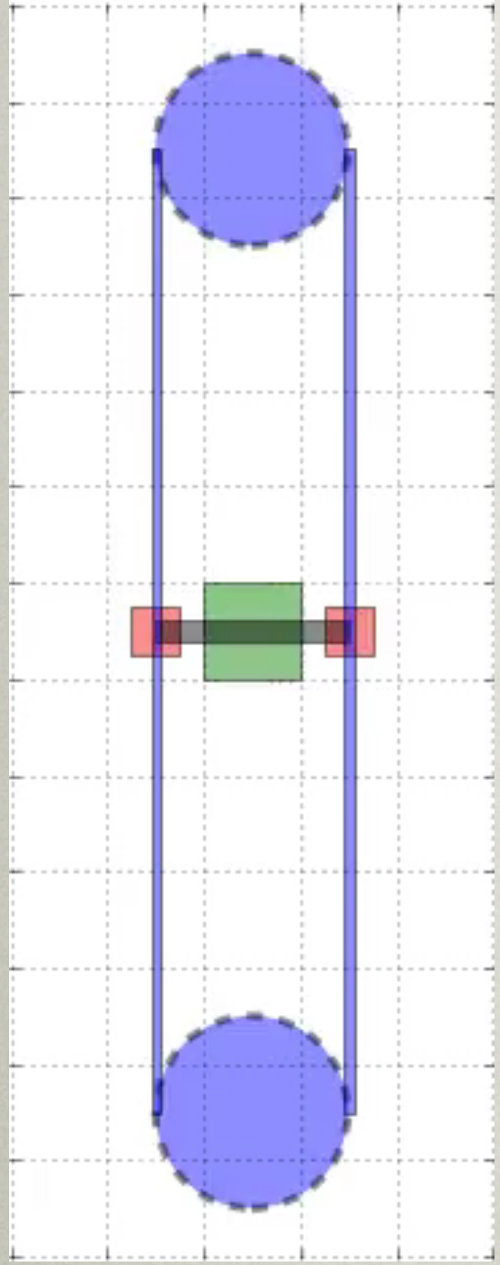
ENERGY HARVESTING AGENTS

(Paul Riechers and JPC, “Structural Thermodynamics of Adaptation” (2016).)



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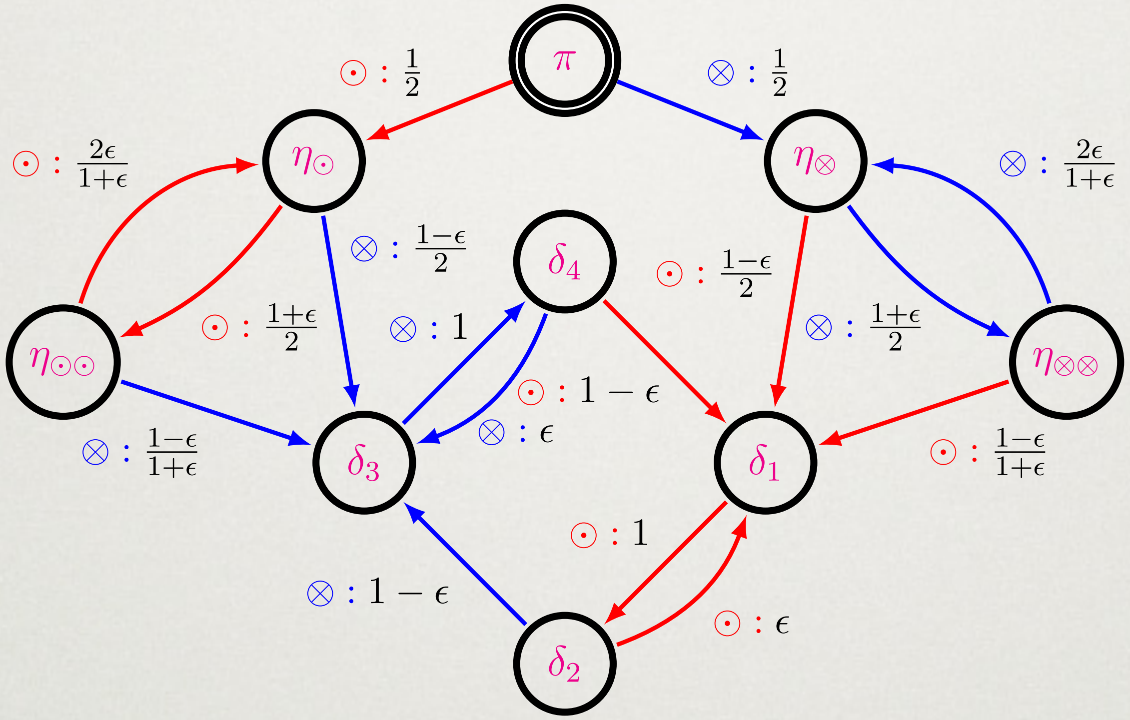

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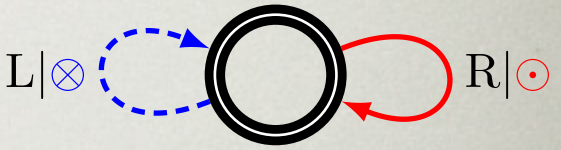

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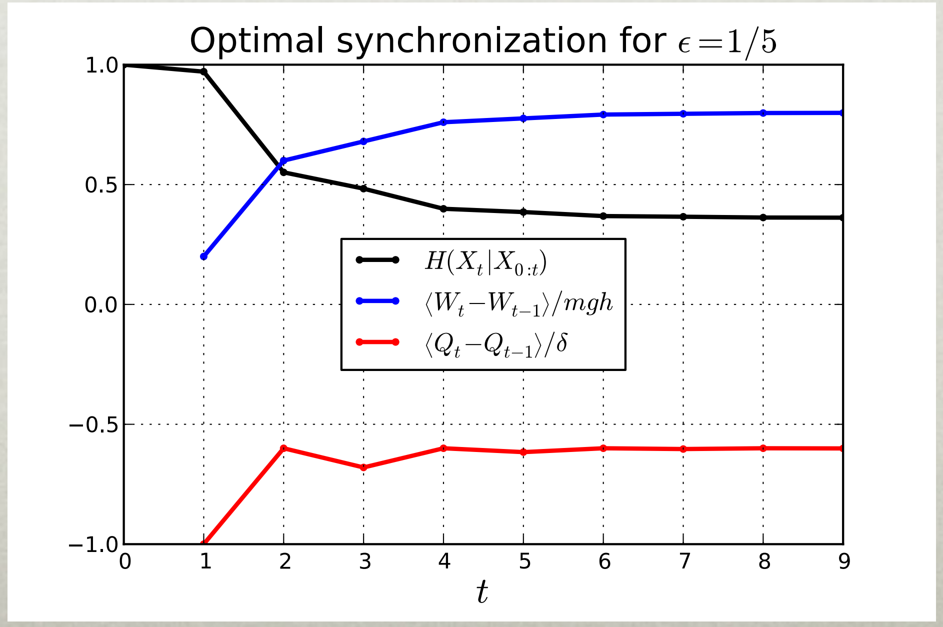

Must sync to environment

AGENDA

- INTRINSIC COMPUTATION
- MAXWELL'S DEMON
- SZILARD'S ENGINE
- THERMODYNAMICAL SYSTEMS
- APPLICATIONS: RATCHETS, ADAPTATION
- INTELLIGENCE?

INTELLIGENCE?

Maxwells' Demon:

An "intelligent" being?

Laplace's Demon

The present state of the system of nature is evidently a consequence of what it was in the preceding moment, and if we conceive of an **intelligence** which at a given instant comprehends all the relations of the entities of this universe, it could state the respective positions, motions, and general affects of all these entities at any time in the past or future.

PIERRE SIMON DE LAPLACE (1776)

THANKS!

http://informationengines.org/

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  arxiv.org:1507.01537 [cond-mat.stat-mech]
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  "Structural Thermodynamics of Adaptation" (2016) in preparation.