Demon Dynamics:
Deterministic Chaos, the Szilard Map, & the Intelligence of Thermodynamic Systems

http://csc.ucdavis.edu/~cmg/

Jim Crutchfield
Complexity Sciences Center
Physics Department
University of California at Davis

Information Engines
Telluride Science Research Center
Telluride, Colorado
23 June 2016
Abstract

We introduce a deterministic chaotic system—the Szilard Map—that encapsulates the measurement, control, and erasure protocol by which Maxwellian Demons extract work from a heat reservoir. Implementing the Demon's control function in a dynamical embodiment, our construction symmetrizes Demon and thermodynamic system, allowing one to explore their functionality and recover the fundamental trade-off between the thermodynamic costs of dissipation due to measurement and due to erasure. The map's degree of chaos—captured by the Kolmogorov-Sinai entropy—is the rate of energy extraction from the heat bath. Moreover, an engine's statistical complexity quantifies the minimum necessary system memory for it to function. In this way, dynamical instability in the control protocol plays an essential and constructive role in intelligent thermodynamic systems.

Joint work with Alexander B. Boyd.
History of Computing Substrates

- Analog: Mechanical gears
- Analog: Electron tube circuits
- Digital Gates: Electron tubes, semiconductors
- Digital Memory: Mercury delay lines, storage scopes, ...
- ...
- Molecular Computing (1970s)
- “Physics & Computation” (MIT Endicott House 1981)
  - Quantum Computing: Feynman
  - Erasure cost: Bennett & Landauer
  - It from Bit: Wheeler
- Josephson Junction Computers (IBM 1980s)
- “Nanotech” (Drexler/Merkle XEROX PARC 1990s)
- **Design goal**: Useful computing
History Of Intrinsic Computing

- Nature already computes
- Information: $H(\Pr(X))$ (Shannon 1940s)
- In chaotic dynamics: $h_\mu$ (Kolmogorov 1950s)
- “Physics & Computation” (MIT Endicott House 1981)
- “Intrinsic computing” there too!
Industrial Revolution
1750s-1900

- Humans commandeer energy on vast scales.
- Unprecedented improvement in human condition.
Industial Revolution
1750s-1900

- Urbanization, disease, ...
- Class inequality & oppression
Agenda

- Intrinsic computation
- Maxwell’s Demon
- Szilard’s Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?
Agenda Structure!

- Intrinsic computation
- Maxwell’s Demon
- Szilard’s Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?
In what ways does nature organize?
(Phenomenology)

How does it organize?
(Mechanism)

Are these levels real or merely convenient?
(Objectivity)

Why does nature organize?
(Optimization versus chance versus ....)

Physics Today, March 2006
Foundations: Computational Mechanics

Causal Equivalence:
\[ \overrightarrow{x} \sim \overrightarrow{x}' \iff \Pr(\overrightarrow{X} | \overrightarrow{x}) = \Pr(\overrightarrow{X} | \overrightarrow{x}') \]

\( \varepsilon \)-Machine: Unique, minimal, & optimal predictor

Stored versus Generated Information

\[ C_\mu = -\sum_{\sigma \in S} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \text{versus} \quad h_\mu = -\sum_{\sigma \in S} \Pr(\sigma) \sum_{\sigma' \in S} \Pr(\sigma' | \sigma) \log_2 \Pr(\sigma' | \sigma) \]


Foundations: Computational Mechanics

\( \varepsilon \)-Machine: Unique, minimal, & optimal predictor

Causal Equivalence:
\[ \overleftarrow{x} \sim \overleftarrow{x}' \iff \Pr(\overleftarrow{X}|\overleftarrow{x}) = \Pr(\overleftarrow{X}|\overleftarrow{x}') \]

\( C_\mu = - \sum_{\sigma \in S} \Pr(\sigma) \log_2 \Pr(\sigma) \) \text{ versus } \( h_\mu = - \sum_{\sigma \in S} \Pr(\sigma) \sum_{\sigma' \in S} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma) \)

Structure versus Randomness

Randomness $(T, S, H, K, ...)$

Structural Complexity $C$
**Foundations: Computational Mechanics**

**ε-Machine: Unique, minimal, & optimal predictor**

Causal Equivalence:
\[
\overset{x}{\sim} \overset{x'}{\sim} \iff \Pr(\overset{x}{\downarrow}|\overset{x}{\uparrow}) = \Pr(\overset{x'}{\downarrow}|\overset{x'}{\uparrow})
\]

Stored versus Generated Information

\[
C_\mu = -\sum_{\sigma \in S} \Pr(\sigma) \log_2 \Pr(\sigma) \quad \text{VERSUS} \quad h_\mu = -\sum_{\sigma \in S} \Pr(\sigma) \sum_{\sigma' \in S} \Pr(\sigma'|\sigma) \log_2 \Pr(\sigma'|\sigma)
\]

**Intrinsic Computation:**

1. How much historical information does a process store?
2. In what architecture is it stored?
3. How is it used to produce future behavior?


Problem Statement

= Ecological population dynamics of structurally complex adapting agents

+ Reproduction (evolutionary population dynamics)

AGENDA

- intrinsic computation
- Maxwell’s Demon
- Szilard’s Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?
... if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us. ... Now let us suppose that ... a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics. ...
Maxwell’s Demon

• Demon creates order out of chaos.
Maxwell’s Demon

- Demon creates order out of chaos.
Maxwell’s Demon

- Demon creates order out of chaos.
- Uses molecular information to convert heat to temperature difference & so to useful work.
Szilard’s Engine:

“ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM BY THE INTERVENTION OF INTELLIGENT BEINGS”,
Leo Szilard, Zeitschrift fur Physik 65 (1929) 840-866.

... we must conclude that the intervention which establishes the coupling between y and x, the measurement of x by y, must be accompanied by a production of entropy.
Szilard’s Single-Molecule Engine Cycle

1) Reset State

2) Insert Barrier

3) Demon Observes

4) Work Extraction

5) Remove Barrier

6) Reset

Single Particle
Left: A  Right: B

Demon

Measure

Control

Erase
... a simple inanimate device can achieve the same essential result as would be achieved by the intervention of intelligent beings. We have examined the ‘biological phenomena’ of a nonliving device and have seen that it generates exactly that quantity of entropy which is required by thermodynamics.”
... a Chaotic Dynamical System


Measure

Extract Energy from Heat Bath

Erase Demon Memory
... a Chaotic Dynamical System

The Szilard Map

Measure \( \mathcal{T}_M(x, y) = \begin{cases} (x, y) & x < \delta, y < \gamma \text{ or } x < \delta, y \geq \gamma, \\ (x, \gamma + y \frac{1-\gamma}{\gamma}) & x \geq \delta, y \leq \gamma, \\ (x, \gamma \frac{y-\gamma}{1-\gamma}) & x \geq \delta, y > \gamma. \end{cases} \)

Control \( \mathcal{T}_C(x, y) = \begin{cases} \left( \frac{x}{\delta}, y \right) & x < \delta, \\ \left( \frac{x-\delta}{1-\delta}, y \right) & x \geq \delta. \end{cases} \)

Erase \( \mathcal{T}_E^A(x, y) = \begin{cases} (x, y\delta) & y < \gamma, \\ (x, \delta \gamma + y \frac{1-\gamma}{1-\gamma} \gamma(1 - \delta)) & y \geq \gamma. \end{cases} \)

\( \mathcal{T}_{Szilard}(x, y) = \mathcal{T}_E^A(x, y) \circ \mathcal{T}_C(x, y) \circ \mathcal{T}_M(x, y) \)
Szilard Engine is a Chaotic Map

Measure Control Erase
Szilard Engine is a Chaotic Map

Measure Control Erase

System Under Study
The Szilard Map: Controller Symbolic Dynamics

$\epsilon$-Machine: Minimal Optimal Generator

$\Sigma_{in} = \{M, C, E\}$

Demon
Memory
Position

$\Sigma_{out} = \{A, B\} \times \{L, R\}$

Entropy Rate: $h_\mu = H(\delta)$ bits

Statistical Complexity: $C_\mu = H[Pr(\sigma)]$

$= \log_2 3 + H(\delta)$ bits
Agenda

- Intrinsic computation
- Maxwell’s Demon
- Szilard’s Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?
# The Szilard Engine: A Thermodynamical System

<table>
<thead>
<tr>
<th>Dynamics</th>
<th>Thermodynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_M(x, y)$ Measure</td>
<td>$\langle Q_{\text{measure}} \rangle = -k_B T (1 - \delta) \ln \left( \frac{(1 - \gamma)/\gamma}{1} \right)$</td>
</tr>
<tr>
<td>$T_C(x, y)$ Control</td>
<td>$\langle Q_{\text{control}} \rangle = -k_B T H(\delta) \ln 2$</td>
</tr>
<tr>
<td>$T_E^A(x, y)$ Erase</td>
<td>$\langle Q_{\text{erase}} \rangle = k_B T (1 - \delta) \ln \left( \frac{(1 - \gamma)/\gamma}{1} \right) + k_B T H(\delta) \ln 2$</td>
</tr>
</tbody>
</table>
Energy Dissipation: Erasure and Measurement!

\[ Q_{\text{diss}} \ (kT) \]

\[ \delta = \frac{1}{2} \]

Erasure: \[ \text{red} \]
Measurement: \[ \text{blue} \]
Control: \[ \text{yellow} \]
Energy Dissipation: Erasure and Measurement!

\[ Q_{diss} \ (kT) \]

Landauer “Principle”

\[ \delta = \frac{1}{2} \]

Erasure:  
Measurement:  
Control:
Energy Dissipation: Erasure and Measurement!

\[
Q_{diss} \ (kT)
\]

\[
\delta = \frac{1}{2}
\]

Erasure: \( \text{Red} \)
Measurement: \( \text{Blue} \)
Control: \( \text{Yellow} \)

Anti-Landauer Landauer “Principle” “Principle”
An information engine is the dynamic over a joint state space of a thermodynamic system and a physically embodied controller. The causal states of the joint dynamics, formed from the predictive equivalence classes of system histories, capture its information processing and emergent organization. A necessary component of the engine’s effective “intelligence”, its memory, is given by its statistical complexity $C_\mu$. Its dissipation is given by the dynamical system negative LCEs, and the rate of energy extracted from the heat bath is governed by the Kolmogorov-Sinai entropy $h_\mu$. 
AGENDA

๏ INTRINSIC COMPUTATION
๏ MAXWELL’S DEMON
๏ SZILARD’S ENGINE
๏ THERMODYNAMICAL SYSTEMS
๏ APPLICATIONS: RATCHETS, ADAPTATION
๏ INTELLIGENCE?
Information Ratchets

Net Work Extraction!

Information Ratchets
Second Law for Intrinsic Computation

\[ \langle W \rangle \leq k_B T \ln 2 \left( h_\mu' - h_\mu \right) \]

Entropy Rates
\[ \Delta H(1) = H'(1) - H(1) \]
\[ \Delta h_\mu = h_\mu' - h_\mu \]
ENERGY HARVESTING AGENTS

(Paul Riechers and JPC, “Structural Thermodynamics of Adaptation”, In prep.)

Optimal agent

Stochastic environment

Suboptimal agent

Must sync to environment
ENERGY HARVESTING AGENTS

(Paul Riechers and JPC, “Structural Thermodynamics of Adaptation” (2016).)

The optimal agent can be derived from the driving MSP. The optimal behavior will involve deterministic behaviors given some measurement his-

Suboptimal agent

Optimal agent

Stochastic environment

Must sync to environment

Optimal synchronization for $\epsilon = 1/5$
ENERGY HARVESTING AGENTS

(Paul Riechers and JPC, “Structural Thermodynamics of Adaptation” (2016).)

The optimal agent can be derived from the driving MSP. The optimal behavior will involve deterministic behaviors given some measurement history.

Must sync to environment

Optimal agent

Suboptimal agent

Stochastic environment

Optimal synchronization for $\epsilon = 1/5$
Agenda

- intrinsic computation
- Maxwell’s Demon
- Szilard’s Engine
- Thermodynamical Systems
- Applications: Ratchets, Adaptation
- Intelligence?
Intelligence?

Maxwell's Demon:

An “intelligent” being?
Laplace’s Demon

The present state of the system of nature is evidently a consequence of what it was in the preceding moment, and if we conceive of an intelligence which at a given instant comprehends all the relations of the entities of this universe, it could state the respective positions, motions, and general affects of all these entities at any time in the past or future.

Pierre Simon de Laplace (1776)
Thanks!

http://informationengines.org/

- A. B. Boyd and J. P. Crutchfield,
  “Demon Dynamics: Deterministic Chaos, the Szilard Map, and the Intelligence of Thermodynamic Systems”,
  arxiv.org:1506.04327 [cond-mat.stat-mech]
- A. B. Boyd, D. Mandal, and J. P. Crutchfield,
  “Identifying Functional Thermodynamics in Autonomous Maxwellian Ratchets”,
  arxiv.org:1507.01537 [cond-mat.stat-mech]
- Paul Riechers and J. P. Crutchfield,